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DEFENSE COMMUNICATIONS AGENCY

SPECTRUM UTILIZATION CONCEPTS FINAL REPORT

30 November 1988

CENTER FOR COMMAND AND CONTROL AND COMMUNICATIONS SYSTEMS

EXCELLENCE IN
C^3 SYSTEMS FOR
NATIONAL DEFENSE

C^4S

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Introduction

I. INTRODUCTION

The support provided to the DCA/MSO under this task was focused on preparation for the World Administrative Radio Conference held in September 1988. This conference represented the first opportunity in 9 years (since WARC-1979) to introduce changes to the International Radio Regulations. Of particular concern was the introduction of changes that could result in improvements that would facilitate the coordination and obtaining of recognition of the Defense Communication Satellite Systems in the geostationary orbit.

The objective of this task was to provide support for the development and assessment of DoD proposals, positions and technical analyses related to the WARC-ORB-88 Treaty Conference on Space Telecommunications.

Concepts addressed under this task included:

1) Interference analysis of geostationary satellites in inclined orbits

2) Bandwidth averaging

3) Comparison of interference from multiple satellites versus a single satellite

4) Multiband satellite networks.

Investigations concerning Inclined Orbit GSO Satellites are contained in Section II. The concept development work for the Bandwidth Averaging was carried out mainly in the context of ensuring that this concept received acceptance in the U.S. Advisory Committee during preparation for the Conference and the Joint Study Groups of the ITU/CCIR that provide the technical bases for the Conference. The results of this effort are in Section III.

Section IV concerns the work done in connection with developing a better statistical basis for the ratio of multiple to single entry interference into a satellite network.

Finally, Section V is a conceptual description of the problems encountered when a single satellite platform contains frequencies subject to more than one procedure.

The technical approach to these activities is described in Figure I-1, and the milestones for accomplishing it are shown in Figure I-2. This report also has several separate appendices associated with the work in Sections II and IV.

	Bandwidth Averaging	Inclined Orbit	Multiband Satellites	Multiple/Single Entry Interference
• Prepare Technical Analysis	X	X		X
• Submit Technical Material for U.S. Conference	X	X		
• Prepare Proposal	X	X	X	
• Conduct Domestic (U.S.) Coordination	X	U.S. Delegation	X	
• Conduct International Coordination	JIWP, CCIR	JIWP, CCIR	Draft Resolution	JIWP, CCIR

M88-4.011

Figure I-1 Spectrum Support Technical Approach

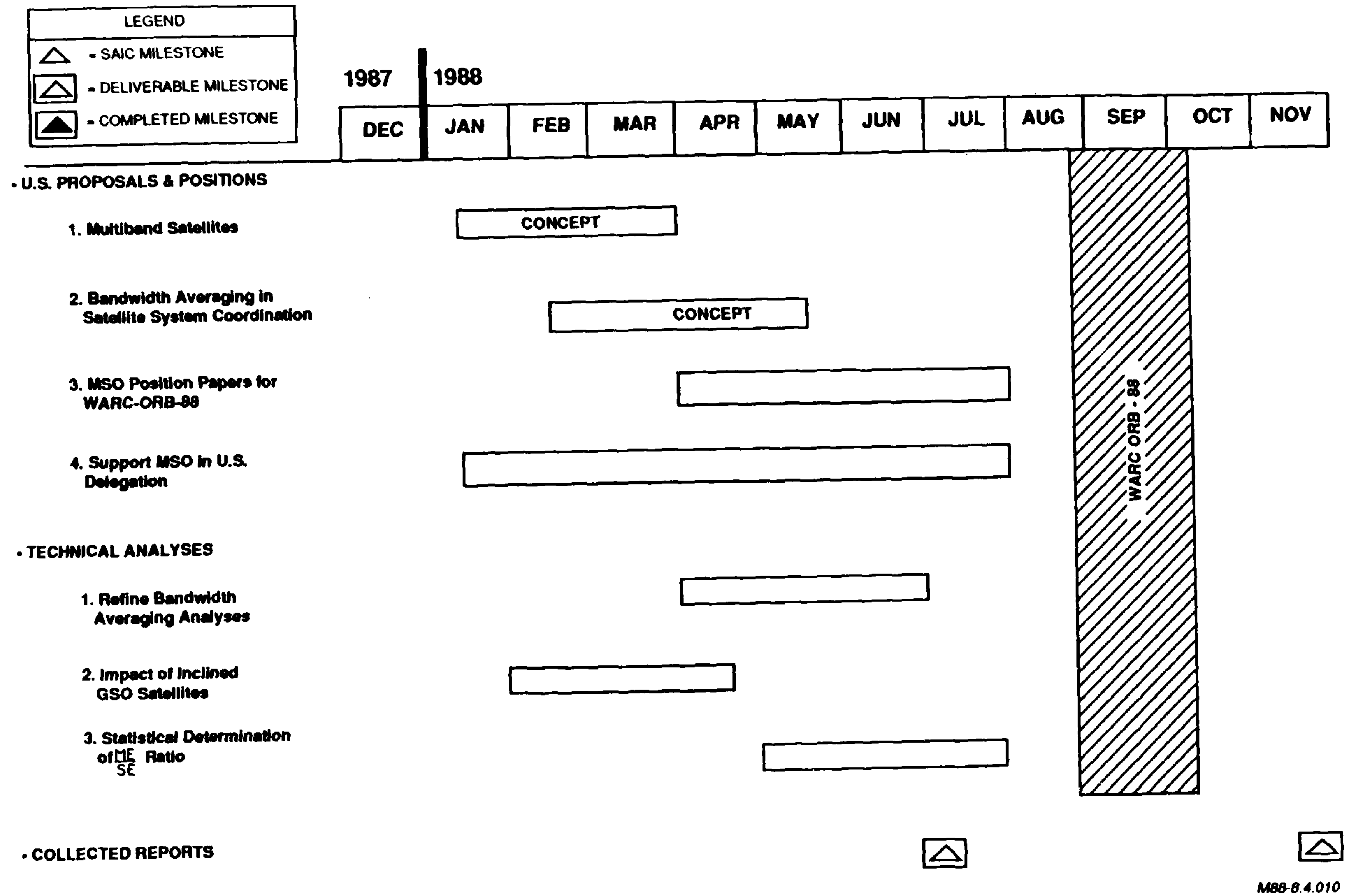

Figure I-2 MSO SETA TASK 88-4 (SPECTRUM SUPPORT)

Section II
Interference Analysis of Geosynchronous Satellites in Inclined Orbits

II. GEOSTATIONARY SATELLITES IN INCLINED ORBITS

A. Introduction

Section 3.13.11 of the CCIR Report to the Second Session of the World Administrative Radio Conference, ORB '88, discusses the effect of inclined orbit and geostationary satellites. It notes that "this is a matter of great potential importance, because geostationary satellites and inclined geosynchronous satellites share the same space/spectrum resource if the geosynchronous satellites are not made subject to RR2613".

It further observes that the useful life of current communication satellites can be extended significantly through judicious use of fuel, provided that the satellite increases its inclination. However, it points out that "the interference geometry of inclined geosynchronous satellites is considerably more complex than that of geostationary, and has not been studied in detail." It suggests that the areas of particular concern are: interference between satellite networks, coordination between earth stations and terrestrial stations, and sharing of constraints to limit interference between satellites and terrestrial stations.

This section addresses the angular separation between two geostationary satellites in inclined orbits which, in turn, affect the earth station antenna discrimination achievable between the two networks.

B. Background

The principal effect of the gravitational fields of the Sun and the Moon on a geostationary satellite is to change the angle of inclination of the orbital plane. For satellites in the equatorial plane, the initial rate of change of inclination is between 0.75 and 0.95° (currently about 0.86° per year) (Report 556-3). As inclination increases, the rate of change declines, until a maximum inclination of about 15° is reached after about 26 years. Thus, a natural limit exists, which would appear to be a reasonable upper limit for a geostationary satellite.

The inclination of an orbit is the angle determined by the plane containing the orbit and the plane of the Earth's equator. The orbital plane carries the satellite above and below the equatorial plane for non-zero inclination angles. At the same time, the Earth is rotating so that a geosynchronous satellite appears to be stable over a single point (the sub-satellite point) when the inclination angle is zero. With an inclination angle, the satellite traces a figure-eight pattern about the sub-satellite point.

Figure II-1 illustrates the figure-eight pattern for two adjacent satellites. The horizontal scale is longitude and the vertical scale is latitude. The center of the figure-eight pattern is the sub-satellite point. The equatorial plane separation angle is the longitudinal difference between the sub-satellite points. The nodal phase angle is the angular difference between the satellites along the figure-eight pattern. The nodal phase angle is also the angular difference in ascending nodes. Finally, the total separation angle is the total angle between the two satellites measured in a geocentric coordinate system.

C. Change in Separation Angle Due to Inclination Angle

The worst case change in total separation angle (in degrees) between two adjacent satellites is given approximately by:

$$\Delta \theta_g = \pi/360 \; I_1 \; I_2 \qquad (1)$$

where I_1 is the inclination angle of satellite 1 in degrees and I_2 is the inclination angle of satellite 2 in degrees. The worst case occurs for a nodal phase angle of 270°, as shown in Figure II-1. The approximations involve less than 1% error for inclination angles less than 15°.

The change in total separation angle (in degrees) for any nodal phase angle is given approximately by:

$$\Delta \theta = \pi/360 \; I_1 \; I_2 \; \mathrm{Sin} \; \delta_2 \qquad (2)$$

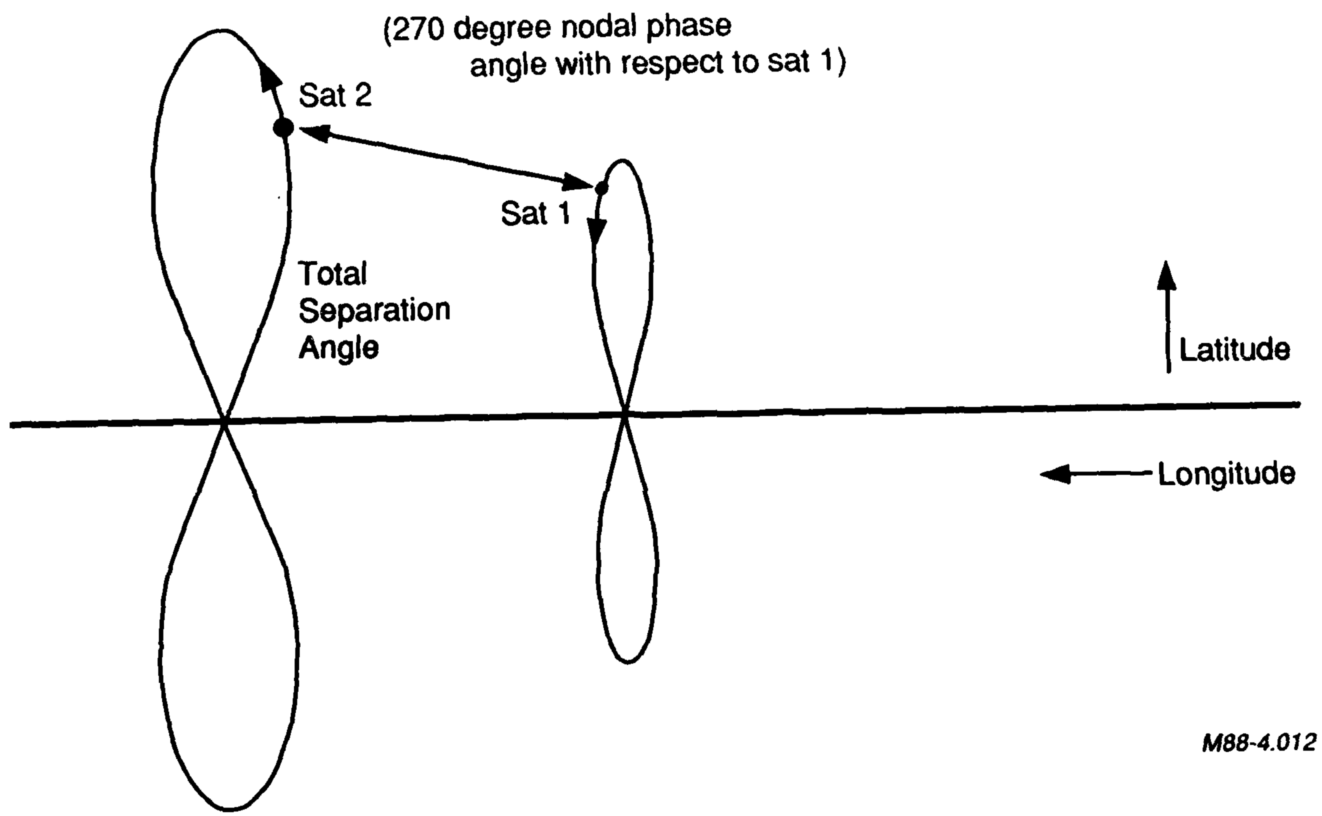

Figure II-1: Latitude and longitude motion of satellites due to inclination angles

where δ_2 is the nodal phase angle. Figures II-2(a) through II-2(c) present plots of the minimum total separation angle as a function of the nodal phase angle for satellite 2 inclination angles of 5, 10, and 15 degrees. Each plot shows the variation with the inclination angle of satellite 1 from 0 to 15 degrees. The bases for these figures may be found in Appendix A. The smallest total separation angle occurs at a nodal phase angle of 270^o, while a nodal phase angle of 90^o results in an increase in total separation angle. In general, when both satellites have some inclination, nodal phase angles between 0^o and 180^o result in an increase in separation angle for any inclination angles as compared to their nodal point separation, while nodal phase angles between 180 and 360 degrees result in a decrease in the separation angle for short periods of time varying from 0 at nodal phase angles of 180 and 360 degrees to a maximum at 270 degrees.

The percent of time that the separation angle can be less than the nodal point value is relatively small (about 8% maximum for a 2^o nodal point angle) for equal inclination angles. For non-equal inclination angles, the percent of time decreases.

In summary, use of inclined geostationary orbits might increase the interference potential, since the separation angle between the satellites may be less than the nodal point separation angle.

D. Effect on Determination of Need to Coordinate

Insofar as earth station discrimination is affected in the Appendix 29 calculations, the change in geocentric satellite separation angle can be neglected as long as $\Delta\theta_g$ is small compared to θ_g, the angular separation of the nodal points. For instance, if $\Delta\ \theta g/\theta g = 0.1$, the effect on earth station discrimination is about 1 dB on a 25 log θ envelope slope. With this assumption, the change in angular separation could be neglected when $I_1 I_2/\theta g < 10$. When this condition is not met, the equation (1) could be used to determine the minimum possible angular spacing (θg) to be used in the Appendix 29 calculations; i.e.;

$$\bar{\theta}g=\theta g-(\pi/360)I_1 I_2 \text{ (degrees)} \quad (3)$$

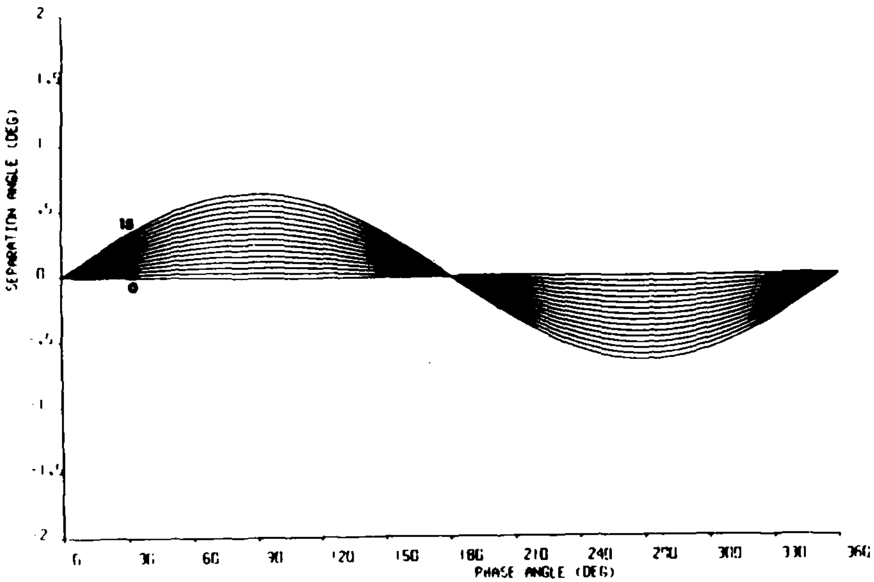

Figure II-2 (a) Minimum total separation angle as a function of nodal phase angle. Satellite 2 inclination angle is 5 degrees.

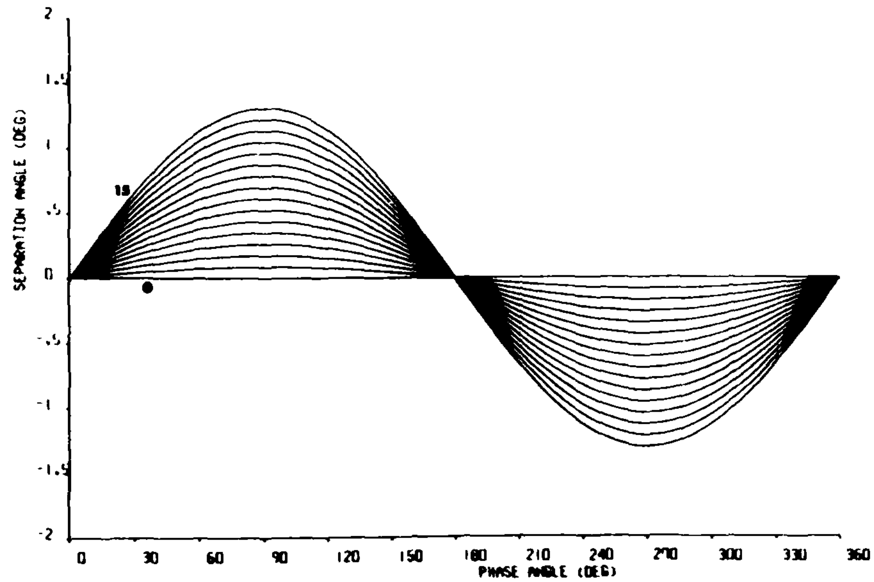

Figure II-2 (b) Minimum total separation angle as a function of nodal phase angle. Satellite 2 inclination angle is 10 degrees.

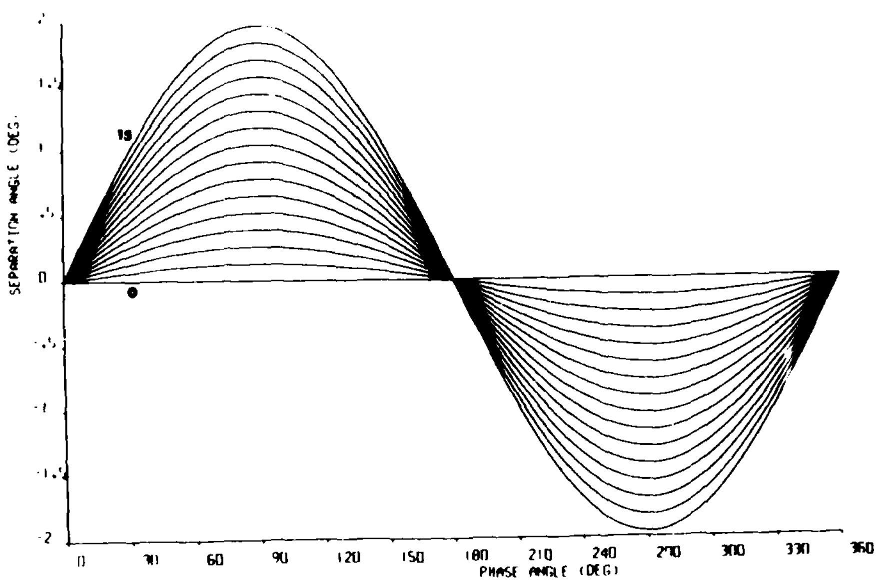

Figure II-2 (c) Minimum total separation angle as a function of nodal phase angle. Satellite 2 inclination angle is 15 degrees.

Thus, the effects of fairly large inclinations could be neglected for large satellite spacings (i.e., inclination angle effects on minimum satellite spacing would generally only need to be taken into account for close spacings).

The most likely case in current operation of satellites in the GSO is that a satellite with a large inclination angle will be adjacent to a satellite with a small inclination. For Equation (3), when one satellite has 0° inclination, the smallest separation is the nodal point separation. If one satellite has a 1° inclination angle and the other a 15° inclination, the nodal angle separation could be used in Appendix 29 calculations as long as this angle was greater than 1.5°. Thus, it would be expected that in most cases the nodal angle separation angle could be used in Appendix 29 calculations. Even if a correction is necessary for the satellite separation angle, Appendix 29 can still be used to determine the need to coordinate, and coordination can be achieved in accordance with Section II of Article 11.

Section III
Bandwidth Averaging

III. BANDWIDTH AVERAGING

From work done in the previous year, a concept had been developed which has the potential for improving the satellite network coordination process. This technique is known as bandwidth averaging. The material which had been previously developed was submitted by the U.S. to the Joint Meeting of CCIR Study Groups meeting in December of 1987 to provide the Technical Bases for the WARC-ORB-88.

This effort was successful in obtaining recognition for the possible use of this technique in both the part of the report dealing with multilateral coordination and in determining the need to coordinate in the nonplanned fixed satellite bands.

Excerpts from the report of this meeting are in Annex III-1.

Annex III-1
Bandwidth Averaging Technique in the CCIR Report to WARC-ORB-88

CCIR

INTERNATIONAL RADIO CONSULTATIVE COMMITTEE

CCIR REPORT TO THE SECOND SESSION OF THE WORLD ADMINISTRATIVE RADIO CONFERENCE ON THE USE OF THE GEOSTATIONARY-SATELLITE ORBIT AND THE PLANNING OF THE SPACE SERVICES UTILIZING IT (WARC-ORB(2))

PART I (Chapters 1 to 4)

INTERNATIONAL TELECOMMUNICATION UNION

3.4.1.4 Example Approach D: Power density-averaging bandwidth method

A. Description and application

Interference calculations will be necessary in the various phases of the MPM process. An accurate but simple method for estimating mutual interference among the satellite networks would be most useful. Such a method could be used as a means to determine the need to coordinate in the pre-MPM phase and as a means by which coordination could be achieved in the MPM phase. Where coordination could not be achieved, more detailed analyses would need to be made; i.e. such a method could also be a means by which to determine when more detailed coordination is required and the severity of the interference problems.

Such a method is described in the following paragraphs and is referred to as a power density-averaging bandwidth method.

This simple method for estimating the mutual interference levels among satellite networks is based on providing sufficient information to allow computation of the interference power I in any interfered with carrier bandwidth. The interference power I is proportional to the interfering power density P_o times the interfered with bandwidth of interest B_r. The worst case P_o is determined for any transmitting bandwidth B_t by finding the portion of a band having a bandwidth B_t in which the total power P is maximum and thus $P_o(B_t) = P/B_t$.

In order to determine I for any carrier bandwidth B_r it is necessary to have a quantitative power density-averaging bandwidth function over the bandwidth of interest. The total band over which such a function would be provided is the band over which contiguous or potentially contiguous carriers could exist. This would typically be a transponder bandwidth for the fixed-satellite service. It has been demonstrated that only a small number ($\approx$5) of averaging bandwidths with associated power densities are needed to reasonably accurately describe a complete power density-averaging bandwidth function over a transponder bandwidth. Judicious selection of the values of averaging bandwidths can result in small reconstruction errors for the total functions.

The power density values would be provided for up-path (P_e) and down-path (P_s) as now required for the reference bandwidth and used in Appendix 29 calculations. ΔT_e, ΔT_s, and ΔT can be computed for all carrier banwidths of interest using Appendix 29. From these values, $\Delta T/T$ values can be computed: for all carriers and the interference power for all carriers can also be computed; i.e., $I = \Delta T \times K \times B_r$, where K is Boltzman's Constant. Thus the administration with the interfered with network can compute for each carrier: $\Delta T/T$, I, I/N or, knowing the carrier power C, C/I. Worst case aggregation of interference from several networks can be calculated using the sum of ΔT values expressed in degrees, or I as a power summation. From this interference information, an administration can decide if more detailed analyses are required or that the interference levels are acceptable. The information can also be used in an optimization process to determine orbital positions which would minimize the mutual interference using single-entry or aggregate levels and using $\Delta T/T$, I, I/N or C/I (with information on C provided) as criteria.

An important requirement for any interference determination method, is the ability to properly account for multiple interference sources into a wider band carrier; for example: a number of SCPC carriers transmitted from different earth stations and received by different earth stations in one network, which are common sources of interference to a wide bandwidth carrier in an interfered-with network. This method addresses this requirement and accounts for multiple source interference in the determination of the power densities for some of the averaging bandwidths.

3.9.3.4 Approach D: Power density-averaging bandwidth method (section 3.4.1.4)

The current $\Delta T/T$ method provides a conservative estimate of potential interference, because the reference bandwidths used do not correspond to the large variations in carrier bandwidths actually employed. More accurate assessments of interference can be obtained if power densities averaged over desired carrier bandwidths were known. The technique of power density averaged over signal bandwidth, provides a simple method for estimating the mutual interference levels to carriers of satellite networks.

This method is described in section 3.4.1.4. Using this method the administration with an interfered with network can compute for all its carrier bandwidths, i) the $\Delta T/T$ using Appendix 29, ii) the interference level I from the ΔT calculation, iii) the ratio I/N and iv) knowing its carrier levels, the ratio C/I. Threshold values for any or all of those parameters can be used to determine the need to coordinate. Power density values for a number of averaging bandwidths would need to be supplied in Appendix 4, Appendix 3 or both.

4.9 Determination of the need to coordinate satellite networks

Finally, to parallel example approach D of section 3.9.3.4, a method to determine the need to coordinate could be developed based on the power density-averaging bandwidth method. Such a method would be most applicable to unplanned bands and space services since it accounts for carrier types, modulation and multiple access techniques, and regenerative repeaters that are not commonly used in the planned bands in the fixed satellite service.

Section IV
Comparison of Interference from Multiple Satellites Versus a Single Satellite

IV. COMPARISON OF INTERFERENCE FROM MULTIPLE SATELLITES VERSUS A SINGLE SATELLITE

A. Introduction

In the CCIR Report [WARC-ORB(2)], the single entry interference and the multiple entry interference were studied for their impact on allotment planning. In particular, the multiple entry to single entry interference ratio (ME/SE) was computed under various sets of assumptions. The material in this section computes the ME/SE ratio for these and similar conditions and provides additional computations to include the effects of both satellite stationkeeping errors and earth station tracking errors. The paragraphs below summarize the findings to date in this area. More detailed material may be found in Appendix B.

B. Homogeneous Model

The models in this report are defined by uniformly spaced satellites in geostationary orbit sharing the same frequencies. The assumptions which define the homogeneous model include:

- The satellite spacing is consistent with using the 25 log θ sidelobe levels.
- Certain satellites have antenna discrimination while others do not.
- The satellites are arranged in sequences, with the satellites with antenna discrimination intermixed with those without as follows:

  sequence 1: A,B,A,B,A,B...
  2: A,B,C,A,B,C,A...
  3: A,B,C,D,A,B,C,D,...

  where discrimination exists between the different lettered satellites but not between the same lettered satellites.

Equations for computing ME/SE ratios were developed for this set of assumptions as detailed in the Appendix for the ME/SE ratio. Plots of the ME/SE ratio were developed illustrating the ME/SE ratio as a function of the amount of satellite antenna discrimination, the sequence number and the total number of interfering satellites. A typical plot is shown as Figure IV-1. This plot shows the ME/SE ratio as a function of the satellite antenna discrimination in dB. The results for nine different sequences as defined above are shown by the individual lines as marked in the figure. This plot is for a victim satellite with 12 interfering satellites on either side of the victim. The peak ME/SE ratio is about 4.9 (6.9 dB).

A measure of orbit utilization was defined as the increase in the number of satellites per unit angle of the geostationary orbit. Plots of the orbit utilization measure as a function of the antenna discrimination, the sequence number and the total number of interfering satellites were developed. A typical plot is shown as Figure IV-2. This plot shows the orbit utilization factor as a function of the satellite antenna discrimination in dB. The results are for nine different sequences as shown by the individual lines. This plot is for a victim satellite with 12 interfering satellites on either side of the victim.

C. Homogeneous Model with Antenna Patterns

The assumptions which define this model include:

- The earth station antenna discrimination is determined from Annex III of Appendix 29.
- The satellites are arranged in sequences as before.
- The required composite earth station and satellite antenna discrimination is 30 dB.

Equations predicting the ME/SE ratio were developed for this set of assumptions as detailed in the Appendix. Plots of the ME/SE ratio were

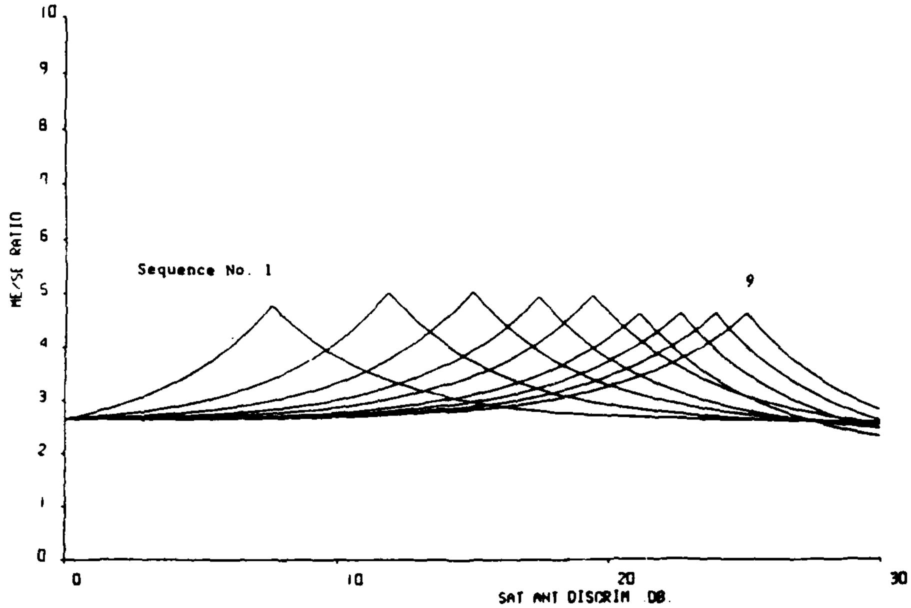

Figure IV-1 Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. There are 12 satellites on either side of the victim.

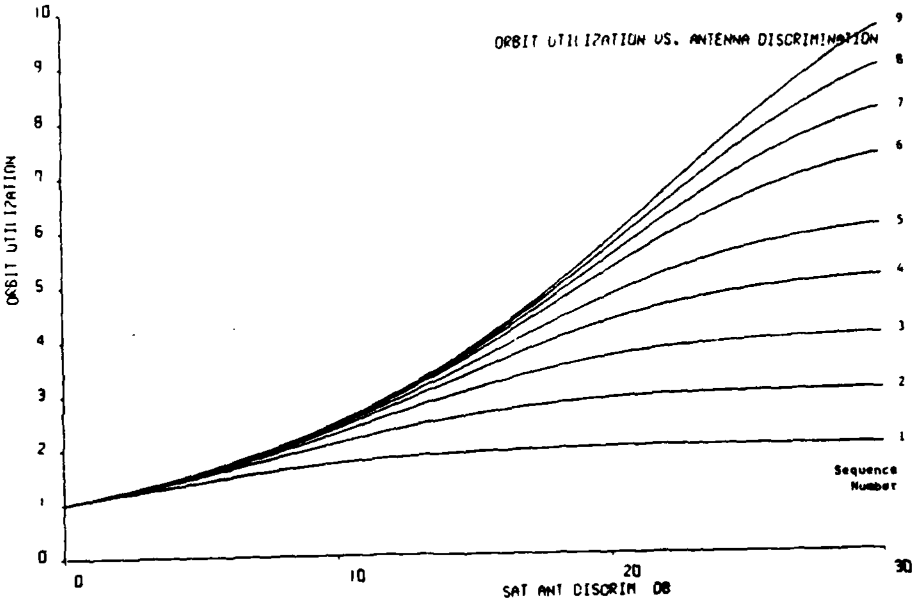

Figure IV-2 Orbit utilization factor as a function of the satellite antenna discrimination for 24 satellites (12 on either side of the victim).

developed as a function of the amount of satellite antenna discrimination, the sequence number, the total number of interfering satellites and the diameter to wavelength ratio of the earth station antenna. A typical plot is shown as Figure IV-3. This plot shows the ME/SE ratio as a function of the satellite antenna discrimination in dB and is for a victim satellite with 12 interfering satellites on either side of the victim. The earth station has a diameter to wavelength ratio of 100. The peak ME/SE ratio is about the same as before.

D. Stationkeeping Error

The assumptions which defined the homogeneous model with antenna patterns were extended to include satellite stationkeeping errors of up to 0.1 degrees of longitude. Various methods of defining the ME/SE ratio were defined, including:

- Worst case ME/SE ratio
- ME/SE ratio based upon the worst case SE, then the worst case ME
- Expected value of the ME/SE ratio
- Expected value of the ME/SE ratio given the worst case SE.

A general model was developed for the ME/SE ratio as detailed in the Appendix. Various specialized cases were identified and studied. Plots of the ME/SE ratio based upon the worst case SE and the worst case ME were developed for the following conditions:

- The victim satellite is at its nominal location.
- All other satellites are moved 0.1 degrees toward the victim.

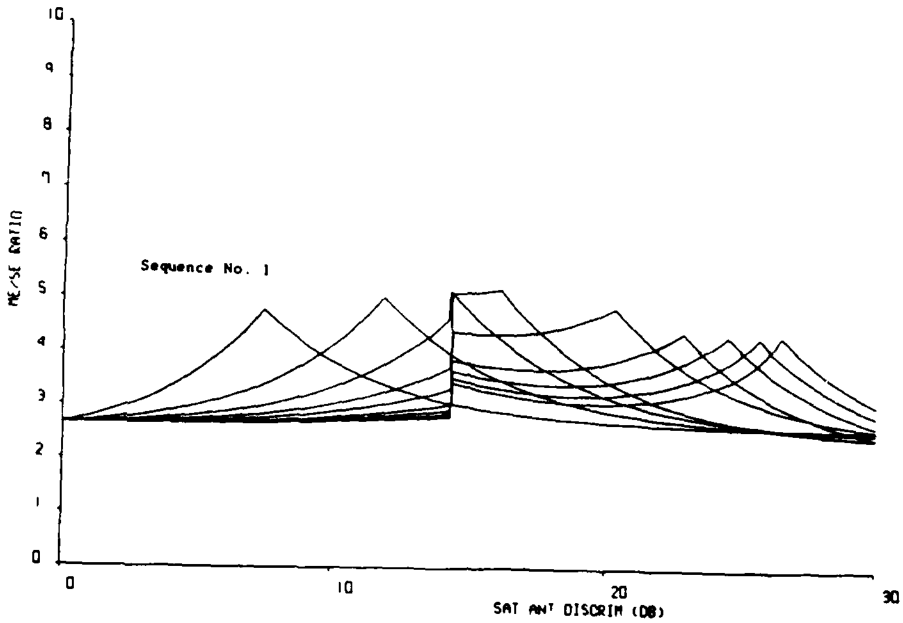

Figure IV-3 Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db.

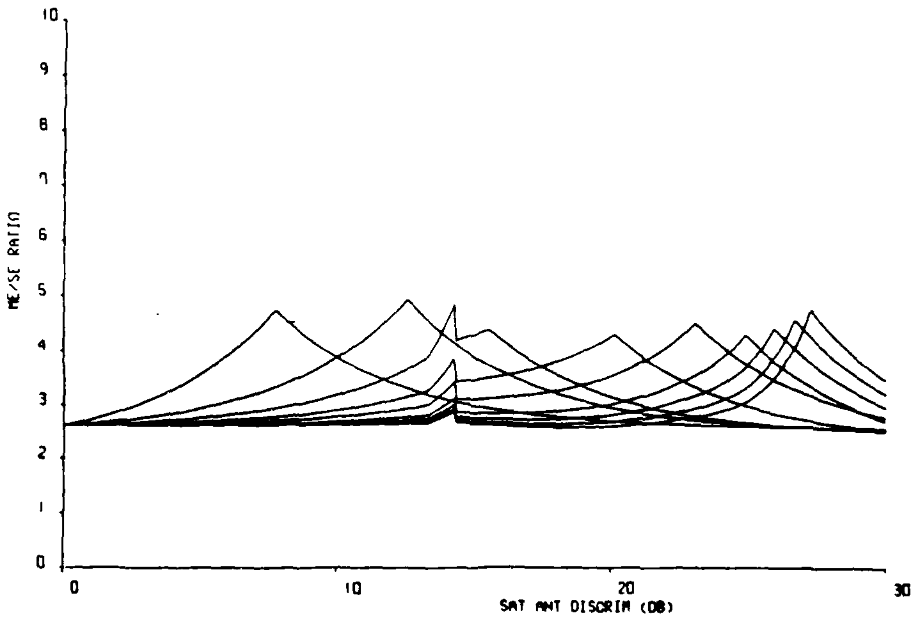

Figure IV-4 Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.

A typical plot is shown in Figure IV-4. Notice the decrease in the peak value of the ME/SE ratio as compared to Figure IV-3. Similar plots were developed for the following conditions:

- The victim satellite is moved toward the worst case interfering satellite to maximize SE.
- All other satellites are moved 0.1 degrees toward the victim.

A typical plot is shown as Figure IV-5. The peak value of the ME/SE ratio has decreased compared to Figure IV-4. Statistical models were also developed to describe the stationkeeping error and its impact on the ME/SE ratio.

E. Earth Station Tracking Error

The assumptions that defined the homogeneous model with antenna patterns were extended to include earth station tracking errors having up to a magnitude of -1 dB relative gain change on the earth station antenna pattern. The angle corresponding to these points detemines the worst case SE and the worst case ME. As before, various specialized cases were identified and studied. Plots of the ME/SE ratio based upon the worst case SE and the worst case ME were developed for the following conditions:

- The victim satellite is moved toward the worst case interfering satellite by the tracking error angle to maximize SE.
- All other satellites are moved by the tracking error angle toward the victim satellite.

A typical plot is shown in Figure IV-6.

F. Combined Stationkeeping and Tracking Errors

Models were developed to determine the effect on the ME/SE ratio of both stationkeeping and tracking errors. The first model estimates the ME/SE ratio

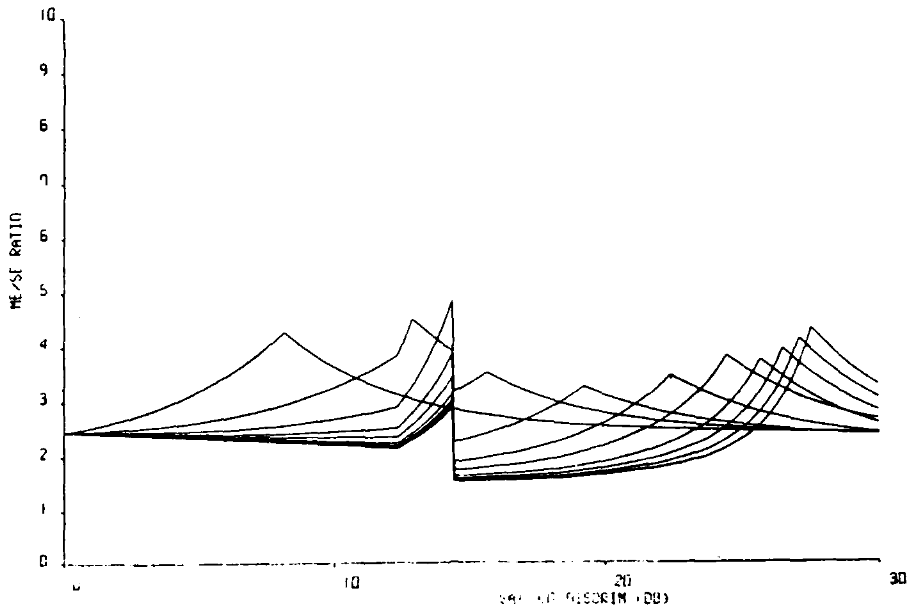

Figure IV-5 Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antennas has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.

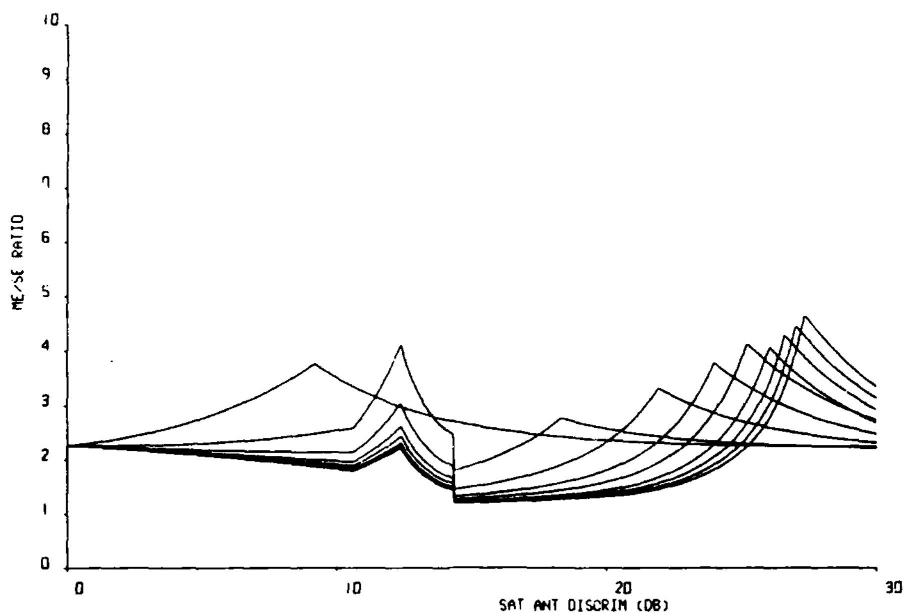

Figure IV-6 Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antennas has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1db.

based upon the worst case SE and the worst case ME under the following circumstances:

- The victim satellite is moved toward the worst case interfering satellite by the combined worst case stationkeeping and tracking error.

- All other satellites are moved toward the victim by the stationkeeping error and all earth station tracking errors are in the direction of the victim.

A typical plot is shown in Figure IV-7. Plots were also developed for other circumstances as defined in the Appendix.

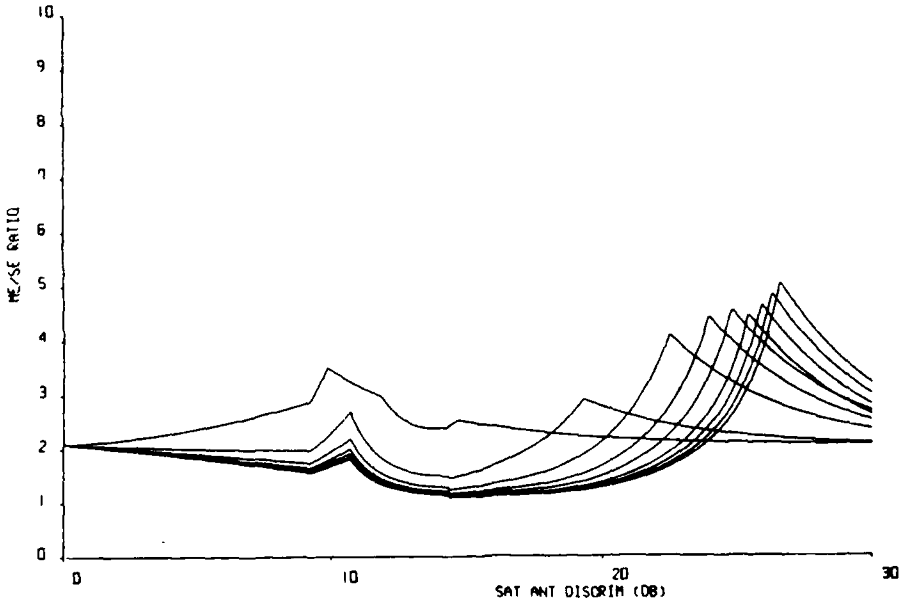

Figure IV-7 Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db, the satellite stationkeeping error is 0.1 degree, and the earth station tracking error is the -1 db relative gain points.

Section V
Multiband Satellite Networks

V. MULTIBAND SATELLITE NETWORKS

A. Introduction

The coordination of GSO satellites having frequencies in multiple regulatory environments could become much more complex and complicated as a result of WARC-ORB-88. As a consequence of WARC '71 and '79, there are 17 satellite services, which have allocations in several bands. Today, with the exception of the Broadcasting Satellite Plans of '77 and '83, and the Feederlink Plan of '83, all satellites in all services using the GSO obtain international recognition for their frequency assignments in accordance with the provisions of Articles 11 and 13 of the Radio Regulations. After WARC-ORB-88, there may be two further methods for achieving such recognition, each applicable to its own set of allocations. This makes six different authorization mechanisms for frequencies on GSO satellites.

The problem is what happens when a satellite platform located at a particular orbit location uses frequencies in more than one band and more than one procedure would be appropriate. This paper analyzes this problem and sets forth an approach on a proposal for WARC-ORB-88.

B. Possible Procedure Approach Combinations

The procedural approaches for authorizing geostationary satellite networks are indicated in Table V-A, along with their distinguishing characteristics.

As suggested in Table V-A, multiple combinations of frequency bands can be put on a single satellite platform. When this occurs, it results in the need to deal with multiple regulating procedures. Some of these situations have already occurred and serve to illustrate the problem:

- o Tele-X is a Scandinavian satellite with transponders on frequencies both in the BSS Plan of WARC '77 and in the 11/14 GHz band allocated to the Fixed Satellite Service. Because of the BSS Plan at 11.7-12.5

Table V-A

Method	Allocations (GHz)	Regulations
BSS '77	BSS 11.7-12.5 (Reg 1) BSS 11.7-12.2 (Reg 3)	Appendix 30,Art.15
BSS '83	BSS 12.2-12.7 (Reg 2) BSS 17.3-17.8 (Reg 2)	Appendix 30,Art.15 Appendix 30A,Art.15A
FSS Allotment	FSS 4.5-4.8/6.725-7-7.025 FSS 10.7-10.95 FSS 11.2-11.45 FSS 12.-75-13.25	Appendix 30 B (WARC-ORB-88)
Improved Procedures (MPM)	Certain FSS bands	Resolution/(?) (WARC-ORB-88)
Simplified Procedures (Unplanned Bands and Services)	Remaining FSS bands and all other Space Services Allocations	Existing Art 11/13 + MODS from WARC-ORB-88
Unplanned BSS	Remaining BSS bands	Resolution 33

GHz, the satellite is obliged to be located at a pre-assigned orbit location. This has created great difficulty in concluding coordination of the Fixed Satellite service transponders, since although they could cause interference to other previously coordinated satellites, its orbital location cannot be adjusted due to the BSS Plan obligations.

- Under the envisioned changes of WARC-ORB-88, INMARSAT satellites will come under two sets of procedures. Their feederlinks in the fixed satellite service will be subject to the improved procedure process, while their mobile satellite service frequencies will continue to be subject to the procedures for the nonplanned bands.

- The "Existing Systems" to be accommodated in the Allotment Plan are another illustration of the complications caused to multiband/ multiservice satellites by multiple procedures. They have allotment band frequencies and will be accommodated in the Allotment Plan. However, these satellites will also be subject to the procedures applicable to other frequencies on the satellite.

As these examples illustrate, there will be a number of situations requiring application of multiple procedures.

As a number of existing and planned satellite systems have this multi-procedural aspect, it is in the interest of WARC-ORB-88 to attempt to provide a basis for dealing with them in as simple a fashion as possible.

C. Analysis

The question becomes how to deal with satellite networks that fall under a multiple set of regulatory procedures. At first glance, there are five different procedure elements to consider when establishing a basis for coordination. These relate generally to the degrees of freedom associated with use of the orbit position. However, they must all be considered as having equal status. Their relation to each other when paired on the same satellite are analyzed below. The elements are:

1. Networks that have completed coordination/registration procedures with their frequency assignments and orbit locations recorded in the IFRB Master Register. These networks generally fall in the MPM or unplanned bands and services, and were coordinated with the least regulatory constraints. However, due to multiple coordination constraints, some of these networks may have little (if any) degrees of freedom remaining.

2. Networks using orbit/spectrum, which are part of BSS Assignment plans (BSS '77, FSS '83, BSS '83). Orbit positions and operating parameters are defined by the Plans and, in practice, there is essentially little flexibility in orbital position short of seeking a formal plan modification; there is only limited flexibility in equipment parameter choice.

3. Those networks using spectrum which are part of the Fixed Satellite Allotment Plan (WARC-ORB-88). The degrees of freedom will be limited by regulation. There may be some orbit position flexibility if a predetermined arc is used. There may be some operating parameter flexibility within regulatory limits.

4. Those networks to which a multilateral improved procedure (WARC-ORB-88) will apply. The regulatory constraints are yet to be determined, but would be between those of elements 3 and 5. However, in congested orbital arcs, there would probably be little degrees of freedom after coordination.

5. Those networks which are in unplanned bands (ART 11/13). These systems would have the largest degree of freedom from a regulatory standpoint.

The extent of complications affected by multiband satellites can be evaluated by examining pairs of the above.

<u>1&5</u> This represents the situation where the networks are in unplanned bands, and it is expected that bilateral (or multilateral) coordination, as appropriate, will continue between the administrations responsible for the networks under the current procedures of Articles 11 and 13.

<u>5&2</u> For an already coordinated satellite network which is
<u>5&3</u> part of a multilateral improved procedures (IP) coordination,
<u>5&4</u> the Allotment Plan or the BSS Feederlink/Assignment Plans, and which also has frequencies which are part of unplanned band allocations, the situation can be particularly difficult, because the networks in the unplanned bands have equal status with the others. Even though the degrees of freedom are more constrained by regulation in planned bands, these constraints cannot be imposed on networks in unplanned bands simply on account of an administration having a satellite using both planned and unplanned bands. Nor should an administration using only unplanned bands be required to participate in coordination meetings mandated by planned band regulations as a consequence of a multiband coordination.

<u>4&3</u> In the case of multilateral coordination (improved procedures) involving a satellite network in the Allotment Plan, there may be some degree of flexibility for the network using allotment frequencies, due to the flexibility hopefully built into the Allotment Plan with the Pre-Determined Arc Concept. The Allotment Plan frequencies should be considered on an equal basis with the multilateral (IP) coordination to the degree permitted by a particular predetermined arc. However, Allotment Plan regulatory constraints cannot be imposed on MPM frequncy bands.

<u>2&4</u> In a multilateral coordination, it may be possible to easily accommodate the consequential effect of Fixed Satellite frequencies on BSS Assignment Plan satellites, since, in an MPM, there would be multiple ways of making adjustments. In addition, the BSS could use its plan modification provisions. BSS Plan regulatory constraints cannot be imposed on MPM frequency bands.

<u>1&4</u> There are many registered and operating networks which are in MPM bands. Since these networks were authorized under "unplanned band" procedures, the associated regulatory constraints would continue to be applicable. However, it appears that administrations with currently registered systems would be required to participate in a multilateral negotiation in order to help accommodate a new system.

2&3 This situation presents no coordination problems. This is due to the fact that either an Administration's assignment in a BSS Plan is in the Pre-Determined Arc of its Allotment or it is not. If it is, then 90% of the coordination with other FSS systems has been accomplished. If it is not, it might not be possible to accomplish except through plan modification. It is simply necessary to make certain that such plan modification is provided.

1&2 The problems posed by this situation should generally be nonexistent, since they are mutually exclusive. If the system is in the Master Register, it has completed the procedures for Coordination/Notification and will have already avoided or cleared coordination with the frequencies of the BSS assignment plans, if necessary. Additional BSS networks would have to be accommodated using a plan modification procedure.

1&3 This situation generally applies during the course of developing the Allotment Plan with regard to accommodating existing systems in the allotment plan. Once this is accomplished, any further satellite network would have to be incorporated through a plan modification procedure.

D. Proposals/Solutions

The CCIR Report JIWP-ORB-88 at Section 3.4.2 states:

"3.4.2 Difficulties regarding multi-purpose, multi-band satellite networks

Problems may arise in cases where it is planned to use a multi-purpose satellite; i.e. for different services, or a multi-band satellite in the fixed-satellite service. In such cases, an administration would be obliged to apply different procedures to access the geostationary-satellite orbit which might result in different and perhaps not always compatible findings for the different bands or services concerned.

As a result, it is possible that in some cases any compulsory procedure involving multilateral meetings for coordination or planning may complicate the process of implementing satellite networks. A combined method based upon an improved Article 11 procedure with the additional possibility of convening multilateral meetings may be useful to resolve these problems in the bands identified by WARC ORB(1) for improved procedures."

It is apparent that there are difficulties posed by multi-procedure satellites. To assist the situation, a resolution could be proposed at WARC-ORB-88 which establishes guidelines for how to handle the coordination of such multiband systems. The resolution woud be referenced to appropriate parts of Articles 11, 15, 15A; Appendices 30, 30A, 30B; and the Improved Procedure Resolution, as appropriate. The resolution would establish certain principles with regard to dealing with the several different multiband situations. A draft of such a resolution is attached. The purpose is to 1) maintain the integrity of the individual coordinating procedures; and 2) provide direction to administrations and the IFRB regarding the coordination of multiband satellites.

Resolution No. J
Relating to the Coordination of Satellite Networks
when the Frequencies to be Used are Subject
to Multiple Procedures

The World Administrative Radio Conference, Geneva 1988

Considering,

- that after the Conference, there may be up to six different regulatory mechanisms for obtaining international recognition for space radiocommunication services using the Geostationary Satellite Orbit;

- that there will be GSO satellites with one or more networks using frequencies subject to more than one regulatory procedure;

- that it is desirable to clarify and simplify the relation of these procedures in obtaining international recognition for the use of frequencies by satellite networks subject to more than one procedure;

- that each procedure has equal status in its own right;

Recognizing,

- that the procedures concerned are those indicated in Table V-A

Resolves,

- that when satellite networks using frequencies which are subject to more than one of the procedures indicated in the ANNEX, the following guidelines should be applied.

1. When satellite networks use frequencies in both unplanned and planned bands, every effort should be made to equally apply the procedural requirements of both. However, the regulatory constraints of the planned bands cannot be imposed on the use of frequencies in the unplanned bands. Administrations using only frequencies in unplanned bands should not be required to participate in planned band coordinations as a consequence of another administration's multiband satellite. Constraints needed to reach a successful accommodation in the unplanned bands and services should be accepted by the multiband system.

2. When planning satellite networks it is desirable to avoid, if possible, using frequency bands to which more than one regulatory procedure applies. In any event an administration seeking coordination/notification for a satellite network using frequency bands subject to different procedures must assume the risk that the coordination can be more difficult and possibly unsuccessful and cannot expect to meet its particular requirements by using as a factor in coordinating with other networks that its satellite is subject to a different procedure.

3. When an administration has a coordinated/notified satellite network using frequencies in unplanned bands that administration should not be required to recoordinate as a consequence of constraints on planned band networks, with an administration using an unplanned band frequencies and planned band frequencies.

4. If an administration has a BSS assignment and also has an FSS allotment in approximately the same orbital location, the implemented allotment position should be able to be adjusted to coincide with the BSS assignment location because of the flexibility provided by the Pre-Determined ARC.

5. Coordinations carried out under improved procedures need to take account of the orbital locations of systems having frequencies in Assignment and Allotment Plans, as well as improved procedure bands. In this connection plan modification procedures should be utilized if possible in the coordination process.

Appendix A
Increase in Interference Due to Inclination Angle

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Section 1

INTRODUCTION

This report examines the consequences of broadening the definition of geostationary orbit (GSO) to include orbital inclination angles of up to a fixed number of degrees.

As is well known, the allowance of inclination angles in geostationary orbits can increase the in-orbit life of geostationary satellites by requiring less fuel for north-south station keeping. Additionally, the use of inclined GSO to allow multiple satellites at a single orbital position has been studied (Report 453-4). Mitigating against the use of inclined GSO is the apparent decrease in longitudinal separation caused by the inclined orbit (Report 556-3) and the resulting increase in potential interference from the smaller separation angle.

This report is concerned with the technical factors impacting the separation angle between two satellites due to the inclined GSO of both satellites. The increase in potential interference is investigated by computing both the total separation angle between the satellites and the percent of time resulting in a decrease in the separation angle below that defined in the equatorial plane. Using the methods of Appendix 29 of the Radio Regulations, the change in total separation angle is converted to an increase in the equivalent noise temperature. Recommendations for broadening the definition of GSO are given and discussed in detail.

1.1 Background

The inclination of an orbit is the angle determined by the plane containing the orbit and the plane of the Earth's equator. The orbital plane carries the satellite above and below the equatorial plane for non-zero inclination angles. At the same time, the Earth is rotating so that a geosynchronous satellite appears to be stable over a single point (the sub-satellite point) when the inclination angle is zero. With an inclination angle, the satellite traces a figure eight pattern about the sub-satellite point.

Figure 1-1 illustrates the figure eight pattern for two adjacent satellites. The horizontal scale is longitude and the vertical scale is latitude. The center of the figure eight patern is the sub-satellite point. The equatorial plane separation angle is the longitudinal difference between the sub-satellite points. The nodal phase angle is the angular

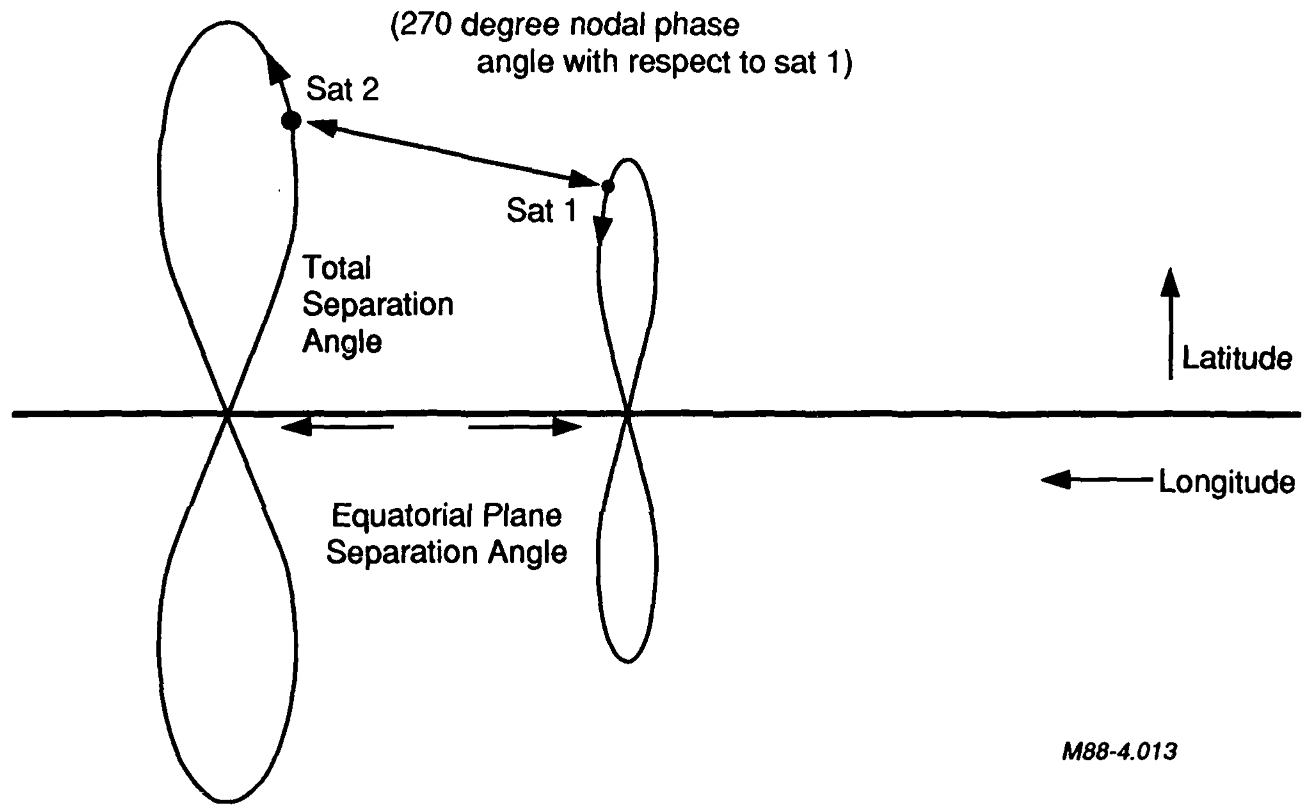


Figure1-1: Latitude and longitude motion of satellites due to inclination angles

difference between the satellites along the figure eight pattern. The nodal phase angle is also the angular difference in ascending nodes. Finally, the total separation angle is the total angle between the two satellites measured in a geocentric coordinate system.

1.2 Results

This study of the impact of inclination angles on the need for coordination yielded the following results:

- Various values were considered as maximum inclination angles not requiring consideration in determining the need for coordination. Additionally, the case against any maximum inclination angle was detailed.

- The details of the impact of inclination angles on the need for coordination were quantified and discussed.

- A procedure was developed which can be used to modify the procedure for computing Delta T/T in accordance with Appendix 29 of the Radio Regulations.

- A revised definition of geostationary orbit is proposed which broadens the current definition to include inclination angles.

- Methods for using inclination angles to reduce the potential interference are identified and discussed.

Detailed conclusions and recommendations can be found in Section 4.

1.3 Synopsis

This section presents the background of the study including the definition of the coordinate systems and angles. Section 2 provides the calculation of the change in total separation angle due to the inclination angles of two adjacent satellites. Additionally, Section 2 presents the percent of time the total separation angle is less than the equatorial plane value as well as statistical estimates of changes in total separation angle due to inclination angles. Section 3 presents the calculation of the increase in equivalent noise temperature due to the inclination angles using the Delta T/T method of Appendix 29 of the Radio Regulations. Broadened definitions and detailed discussions of geostationary orbits are presented in Section 4 which also provides the conclusions and recommendations derived as a result of this study. Appendix A presents the mathematical derivations supporting the other sections.

Section 2

CALCULATION OF THE CHANGE IN SEPARATION ANGLE DUE TO INCLINATION ANGLE

The principal effect of the gravitational fields of the Sun and the Moon on a geostationary satellite is to change the angle of inclination of the orbital plane. For satellites in the equatorial plane, the initial rate of change of inclination is between 0.75 and 0.95 degrees (currently about 0.86 degrees per year) (Report 556-3). As inclination increases, the rate of change tends to decline. Inclination of the orbital plane results in the figure eight locus about the sub-satellite point.

In this section, the figure eight locus is derived as it impacts the total separation angle between two nearby satellites in GSO. Prior to considering the separation angle, definition of terms are presented along with parameter definitions.

2.1 Definitions

TOTAL SEPARATION ANGLE: The angle between the lines drawn from the center of the earth to each satellite.

NOMINAL SEPARATION ANGLE: The longitudinal angle between the satellites measured between the equatorial plane crossings.

INCLINATION ANGLE: The angle between the normal to the equatorial plane and the momentum vector of the satellite.

The parameters are defined as follows:

I = inclination angle

θ_s = equatorial plane separation angle between satellite 1 and 2

t = time in hours from the ascending node

ϕ = latitude

θ = longitude

λ = total separation angle

$\Delta\lambda$ = change in total separation angle

γ_0 = nodal phasing angle of satellite 2 relative to satellite 1, i.e., angular difference between ascending nodes

σ_I = inclination angle deviation

$\sigma_{\Delta\lambda}$ = deviation of change in total separation angle

2.2 Longitude excursions of an inclined geostationary satellite

As is well known, the latitude and longitude excursions due to an inclined orbital plane are given by:

$$\phi = \mathrm{Sin}^{-1} (\mathrm{Sin}\ I\ \mathrm{Sin}\,\gamma) \qquad (2\text{-}1)$$

$$\theta = \mathrm{Tan}^{-1} (\mathrm{Cos}\ I\ \mathrm{Tan}\,\gamma) - \gamma \qquad (2\text{-}2)$$

where: $\gamma = 2\pi t/24$.

Figure 2-1 shows a plot of latitude versus longitude excursions for various inclination angles from 1 to 10 degrees in one degree steps. The resulting figure eight is a function of time from the ascending node. Notice from Figure 2-1 that the longitude scale is greatly expanded and that the maximum longitude excursions are small.

The maximum value of longitude excursion predicted by equation (2-2) is derived in Appendix A-1 and is given by:

$$\theta_{max} = I^2/2 \quad \text{(radians)} \qquad (2\text{-}3)$$

where it has been assumed that the inclination angle is small (<15 degrees) and the maximum longitude excursion is small. The maximum was found over the time parameter. Figure 2-2 shows the maximum longitude excursion as a function of the inclination angle. As an example, for an inclination angle of 10 degrees, the maximum longitude excursion is 0.436 degrees.

2.3 Total separation angle

For purposes of interference calculations, the angle between the vectors from the earth's center to each satellite is of interest. This total separation angle can be found from solid analytic geometry as:

$$\begin{aligned}\mathrm{Cos}\,\lambda = {} & \mathrm{Cos}\phi_1 \mathrm{Cos}\phi_2 \mathrm{Cos}\theta_1 \mathrm{Cos}\theta_2 \\ & + \mathrm{Cos}\phi_1 \mathrm{Cos}\phi_2 \mathrm{Sin}\theta_1 \mathrm{Sin}\theta_2 \\ & + \mathrm{Sin}\phi_1 \mathrm{Sin}\phi_2 \qquad (2\text{-}4)\end{aligned}$$

where the subscripts refer to the satellite and both satellites are assumed to have the same distance from the center of the earth. As is usual, the geocentric angle is being used as opposed to the topocentric angle.

The difference in time from ascending node of satellite 1 to the ascending node of satellite 2 is designated Δt so that:

$$t_2 = t_1 + \Delta t \qquad (2\text{-}5)$$

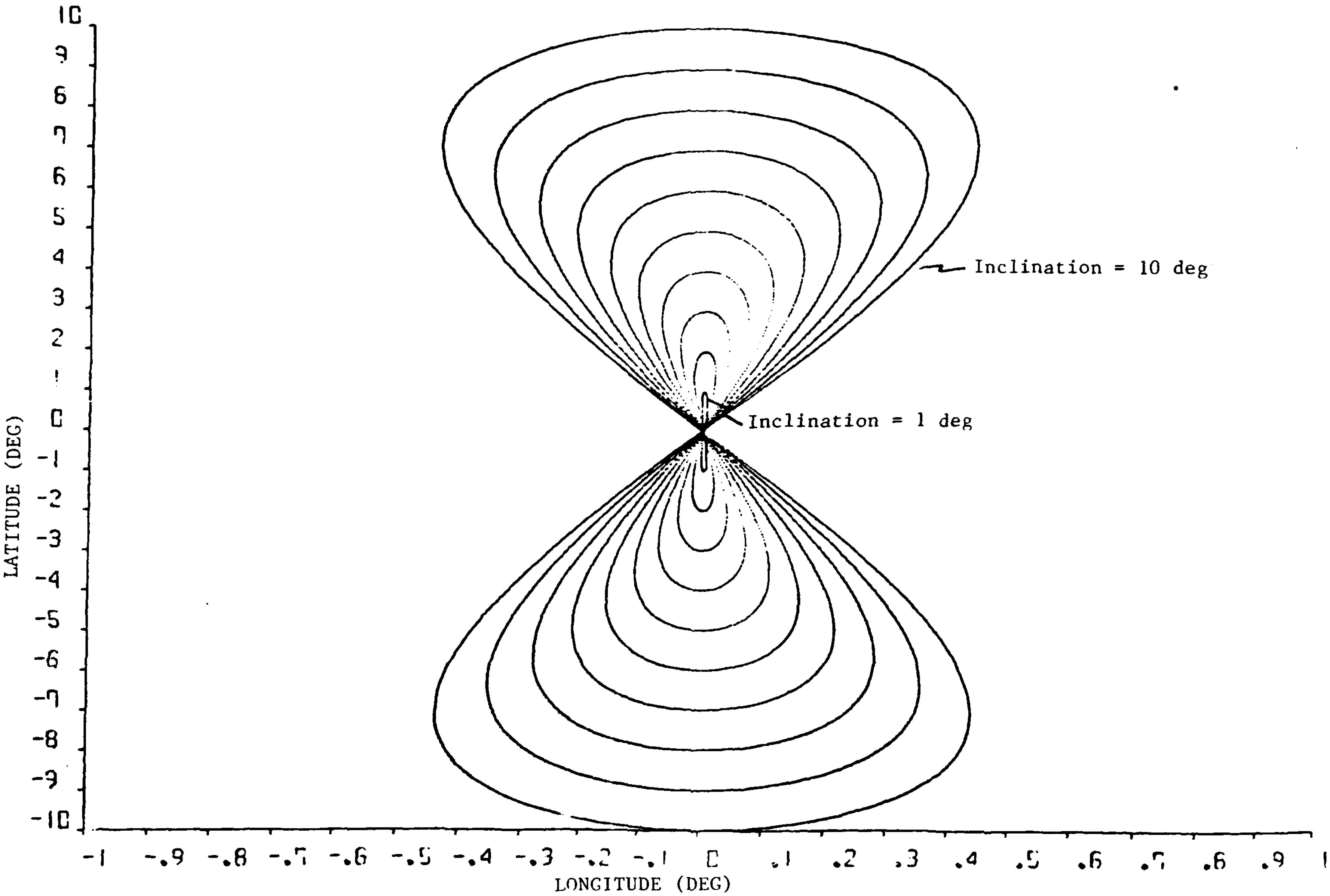


Figure 2-1: Latitude and longitude excursions due to inclination angle.

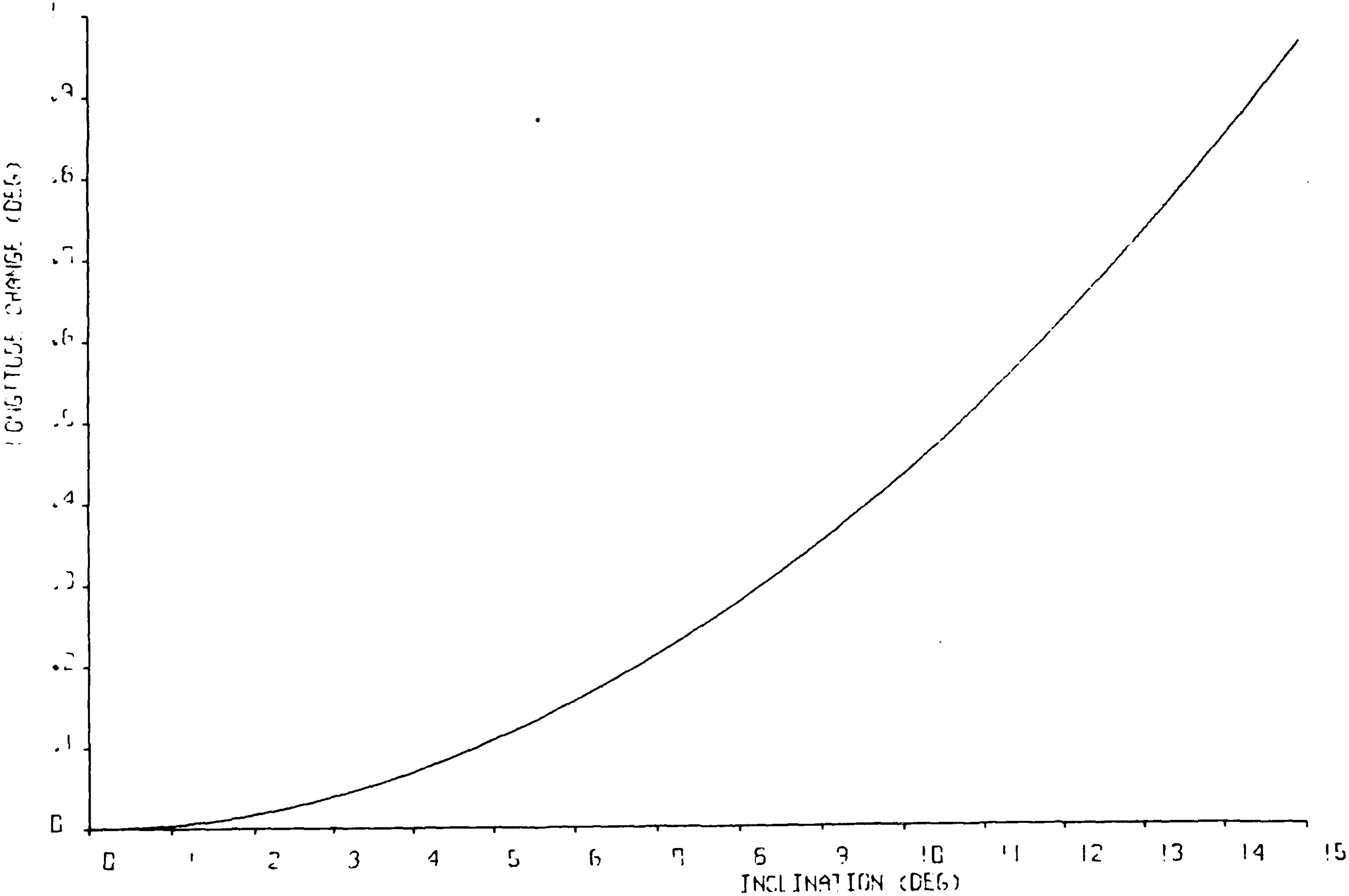


Figure 2-2: Maximum longitude excursions as a function of inclination angle

or in terms of the node phasing angle:

$$\gamma_2 = \gamma_1 + \gamma_0 \qquad (2\text{-}6)$$

where γ_0 lies between 0 and 2π.

The nominal latitude of each satellite is zero, i.e., latitude excursions about zero latitude as given by equation (2-1). The longitude of each satellite is given by:

$$\theta_1 = \theta_0 + \Delta\theta_1 \qquad (2\text{-}7)$$

$$\theta_2 = \theta_0 + \theta_S + \Delta\theta_2 \qquad (2\text{-}8)$$

where $\Delta\theta$ is given by equation (2-2). Using small angle approximations, Appendix A-2 shows the total separation angle is approximately:

$$\lambda^2 = [I_1 \mathrm{Sin}\gamma_1 - I_2 \mathrm{Sin}(\gamma_1 + \gamma_0)]^2 + [-I_1^2/4\ \mathrm{Sin}2\gamma_1 + I_2^2/4\ \mathrm{Sin}2(\gamma_1 + \gamma_0) - \theta_S\ ]^2 \qquad (2\text{-}9)$$

Figures 2-3 through 2-5 are a series of plots, using equation (2-9), of the total separation angle as a function of time from the ascending node of satellite 1. The nominal separation angle is 2 degrees as shown by the dashed line on each plot and satellite 2 has a 5, 10, or 15 degree inclination angle in Figures 2-3, 2-4, and 2-5, respectively. Figures 2-3(a), 2-4(a), and 2-5(a) have a node phase angle of 0 degrees. The node phase angle increases by 45 degree increments to the 315 degrees in Figures 2-3(h), 2-4(h), and 2-5(h). In each plot, the inclination angle of satellite 1 is varied from 0 to 15 degrees. Notice that a node phase angle of 270 degrees results in the smallest total separation angle.

2.4 Worst case change in total separation angle due to inclination angle

Figures 2-3, 2-4, and 2-5 indicate the minimum total separation angle occurs with a 270 degree nodal phase angle. For this condition, the total separation angle becomes:

$$\lambda^2 = [I_1 \mathrm{Sin}\gamma_1 + I_2 \mathrm{Cos}\gamma_1]^2 + [-\mathrm{Sin}2\gamma_1\ (I_1^2/4 + I_2^2/4) - \theta_S\ ]^2 \qquad (2\text{-}10)$$

Notice that the first term is zero when:

$$\mathrm{Tan}\gamma_1 = -I_2/I_1 \qquad (2\text{-}11)$$

Appendix A-3 shows that this yields the approximate minimum separation angle. Substituting equation (2-11) into (2-10) gives the minimum total separation angle as:

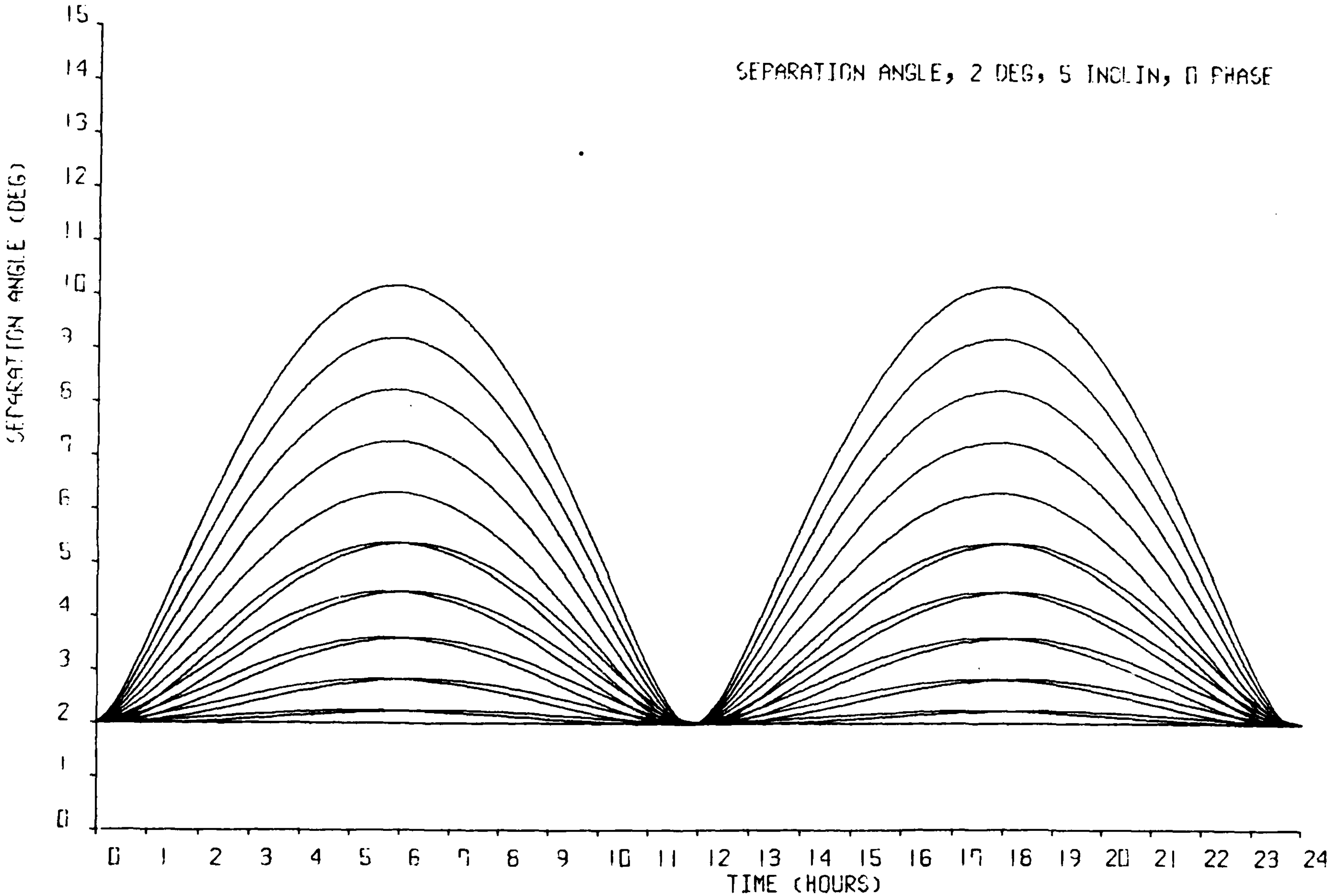


Figure 2-3(a): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 0 degrees.

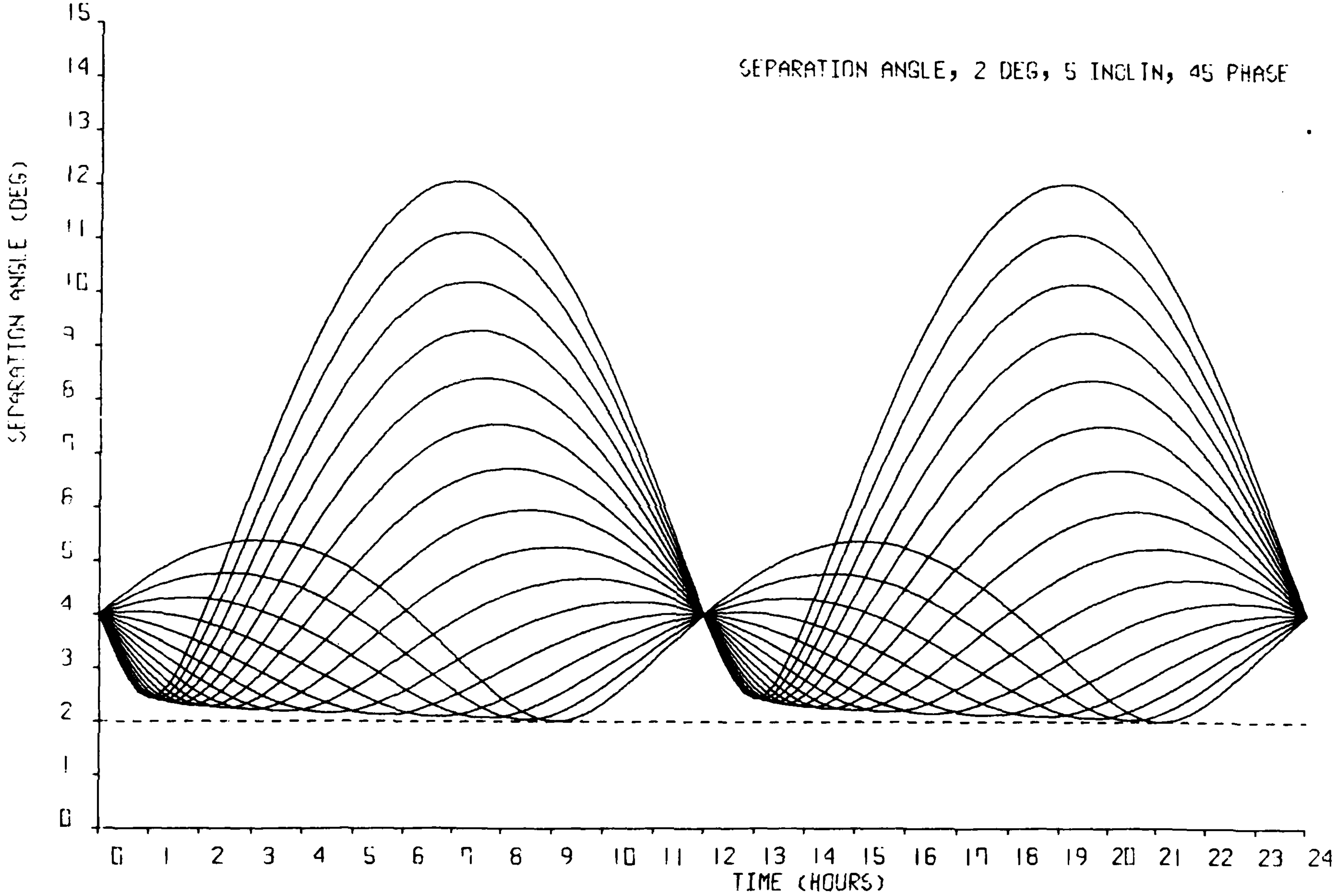


Figure 2-3(b): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 45 degrees.

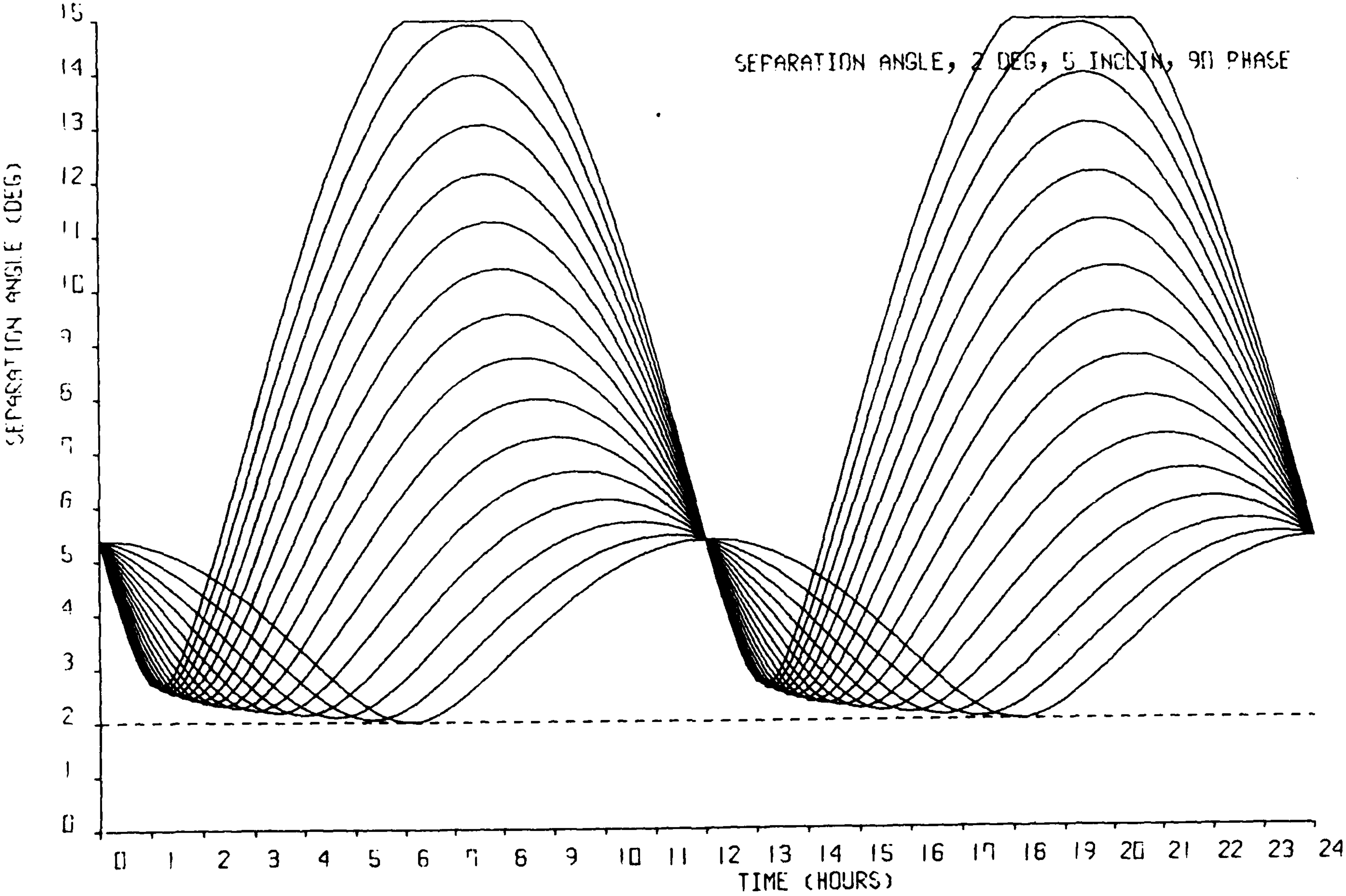


Figure 2-3(c): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 90 degrees.

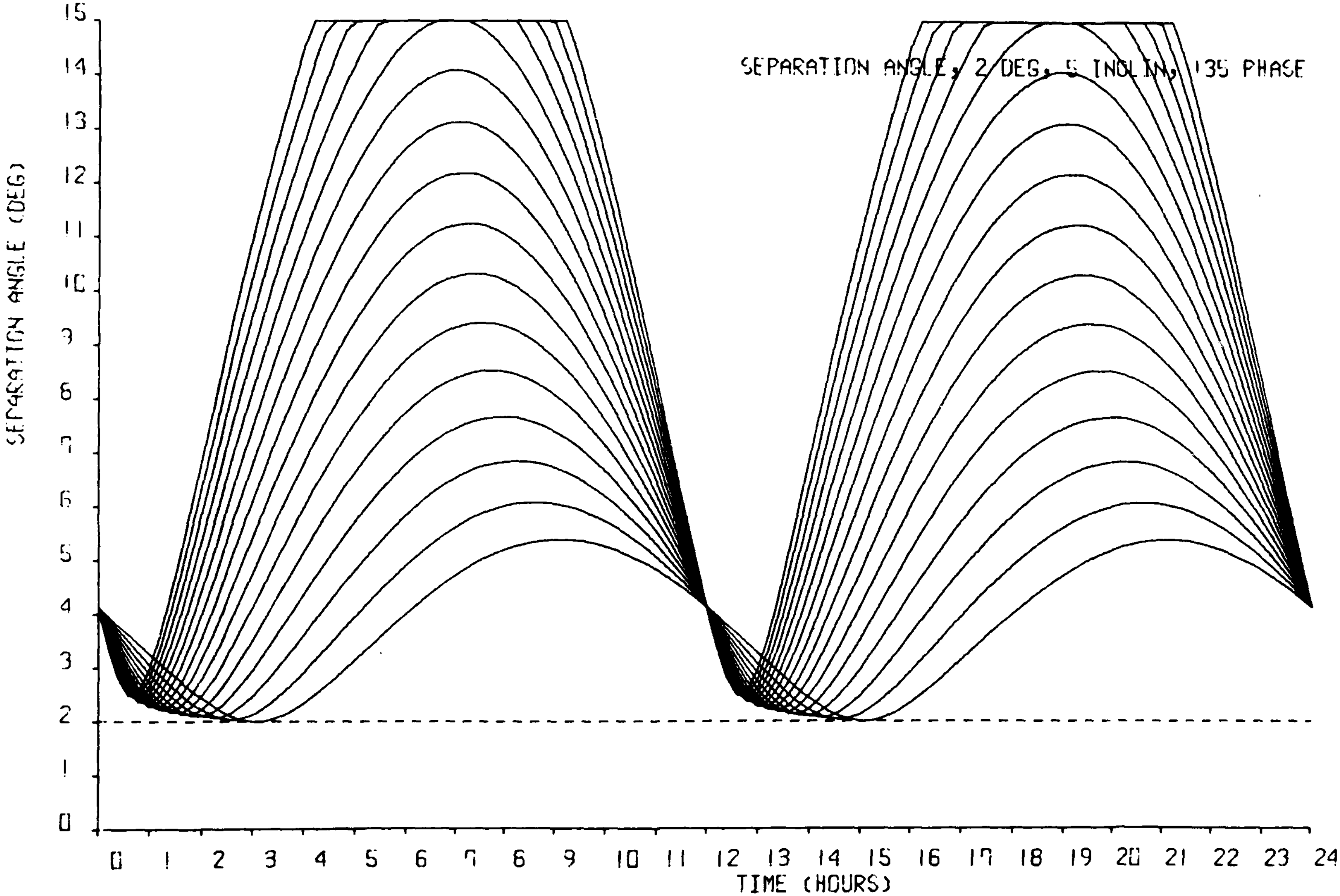


Figure 2-3(d): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 135 degrees.

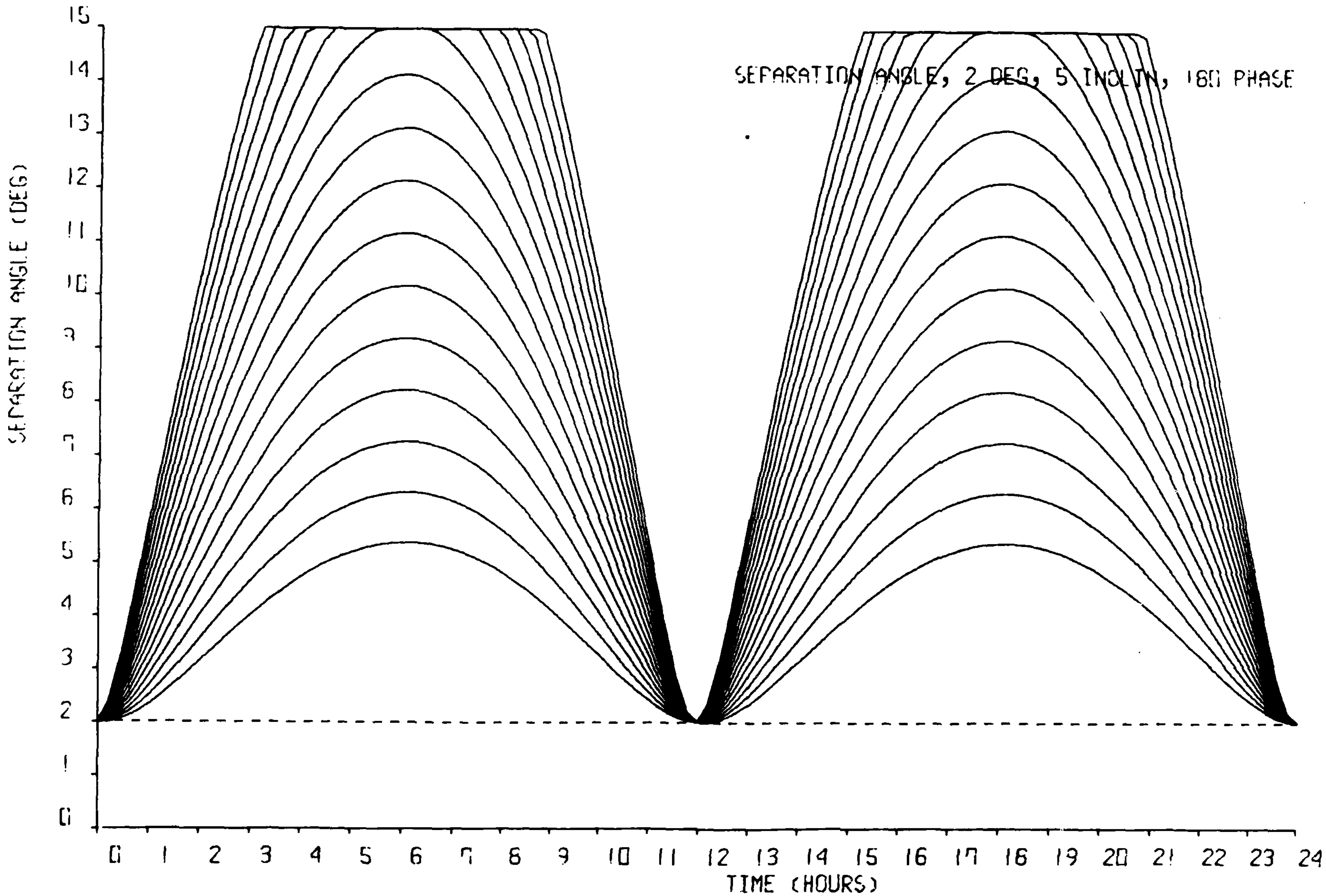


Figure 2-3(e): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 180 degrees.

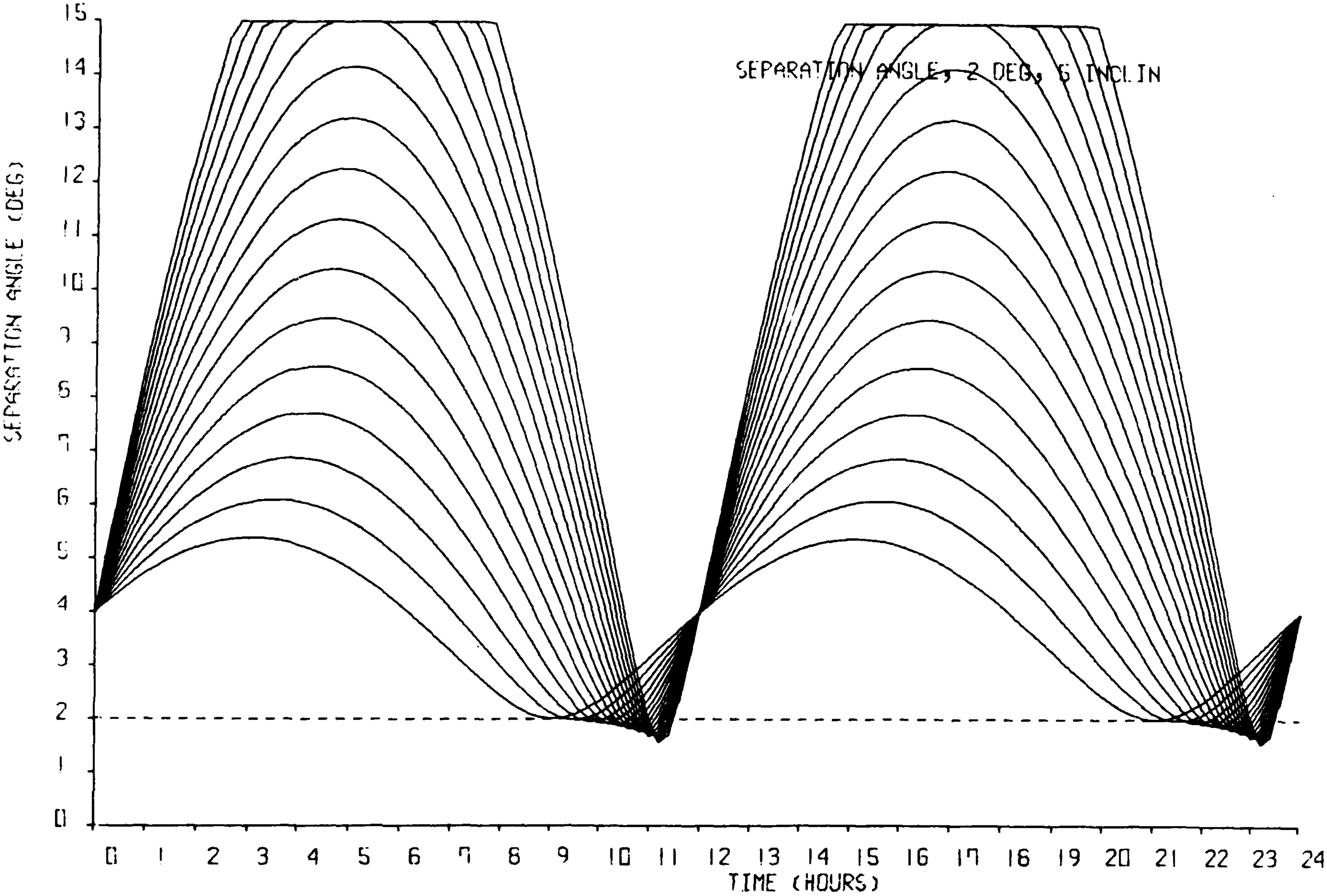


Figure 2-3(f): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 225 degrees.

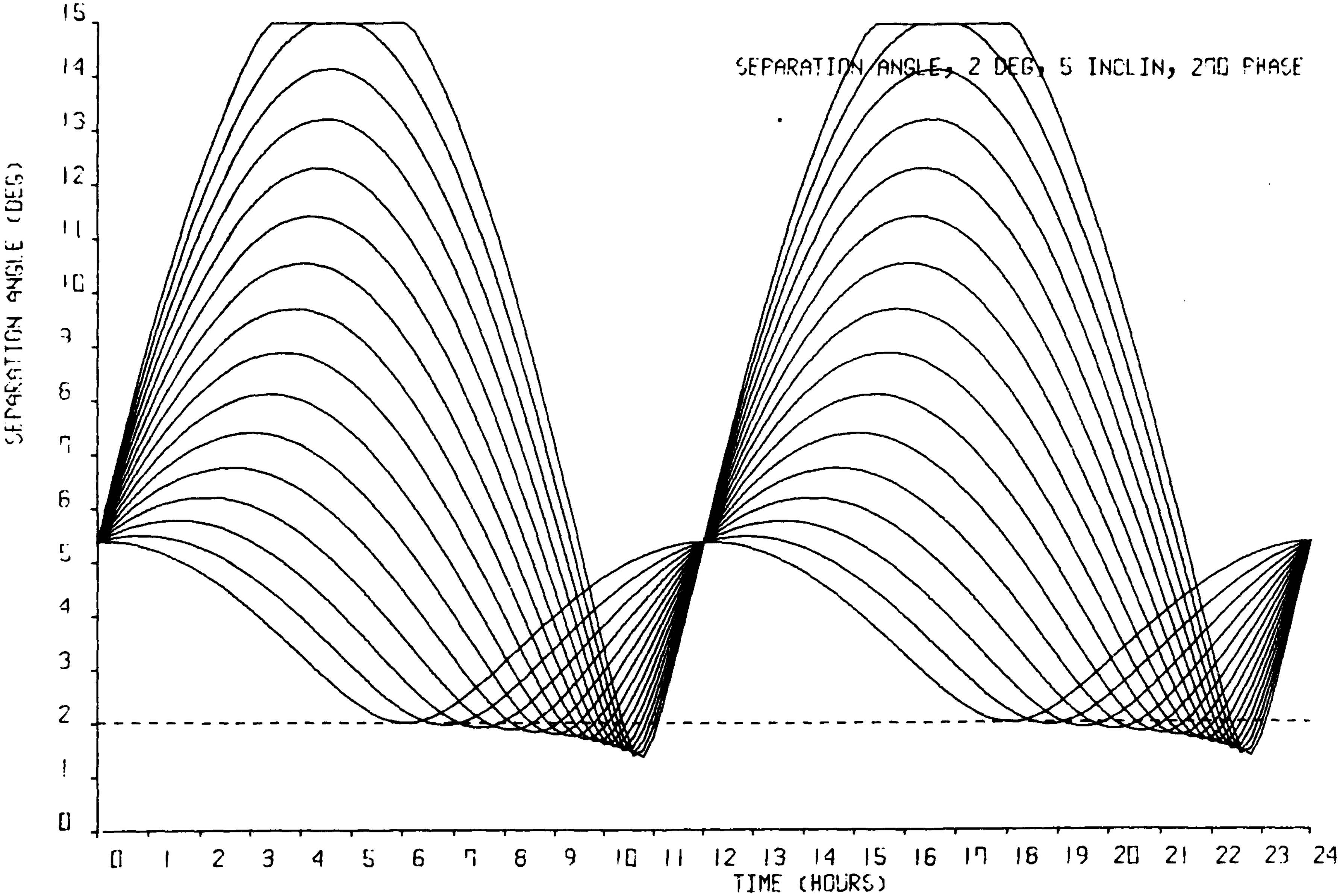


Figure 2-3(g): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 270 degrees.

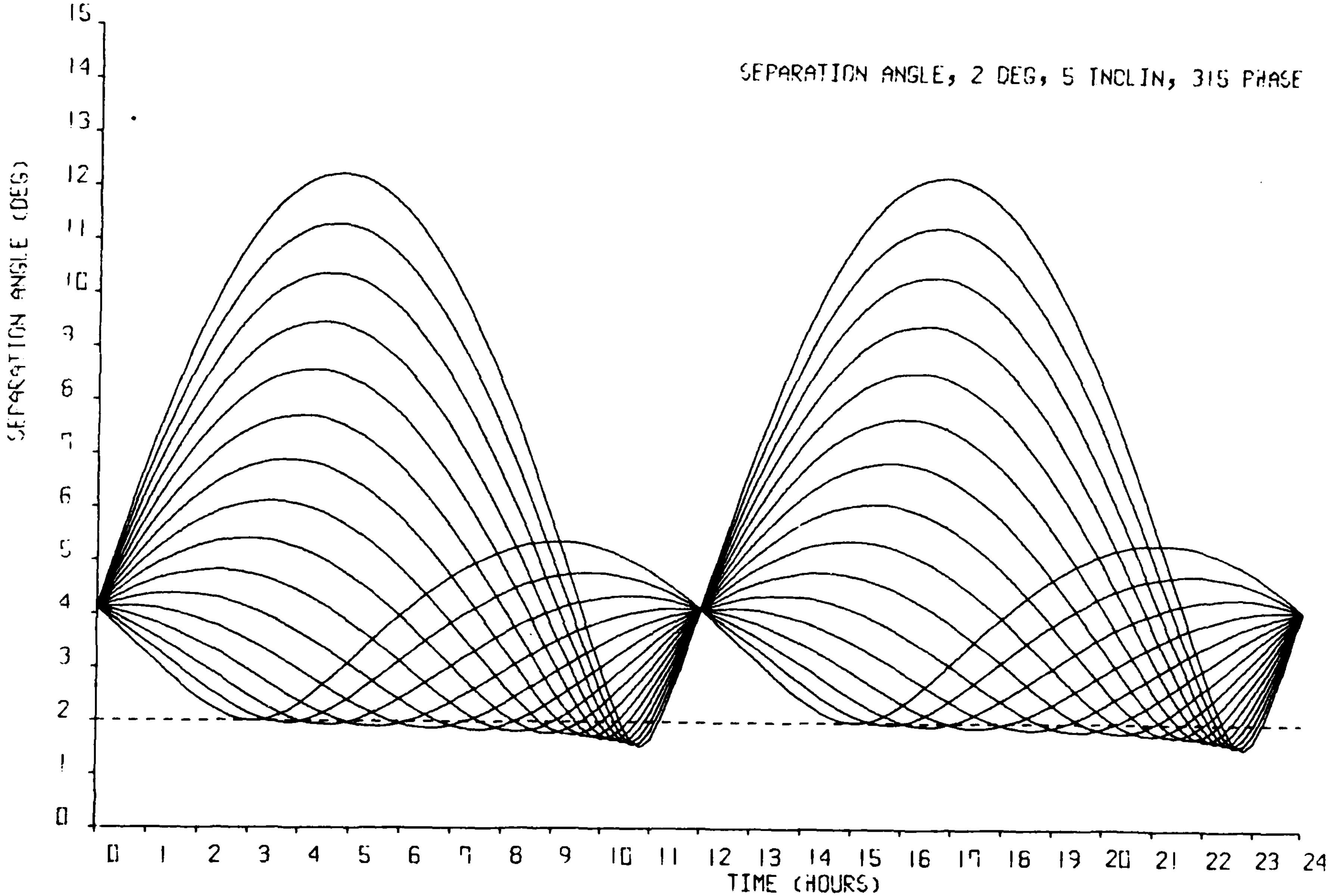


Figure 2-3(h): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 5 degrees and node phase angle is 315 degrees.

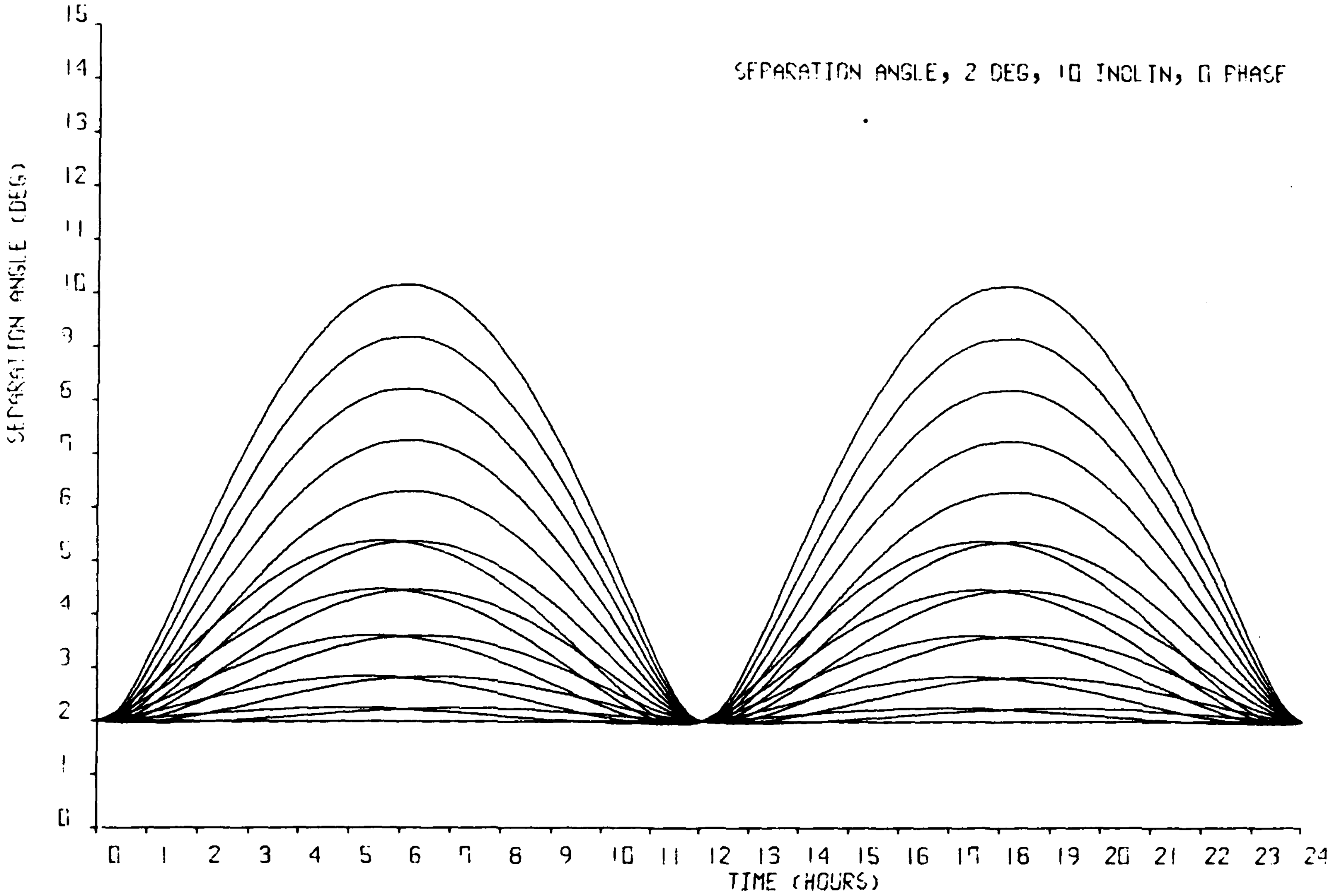


Figure 2-4(a): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 0 degrees.

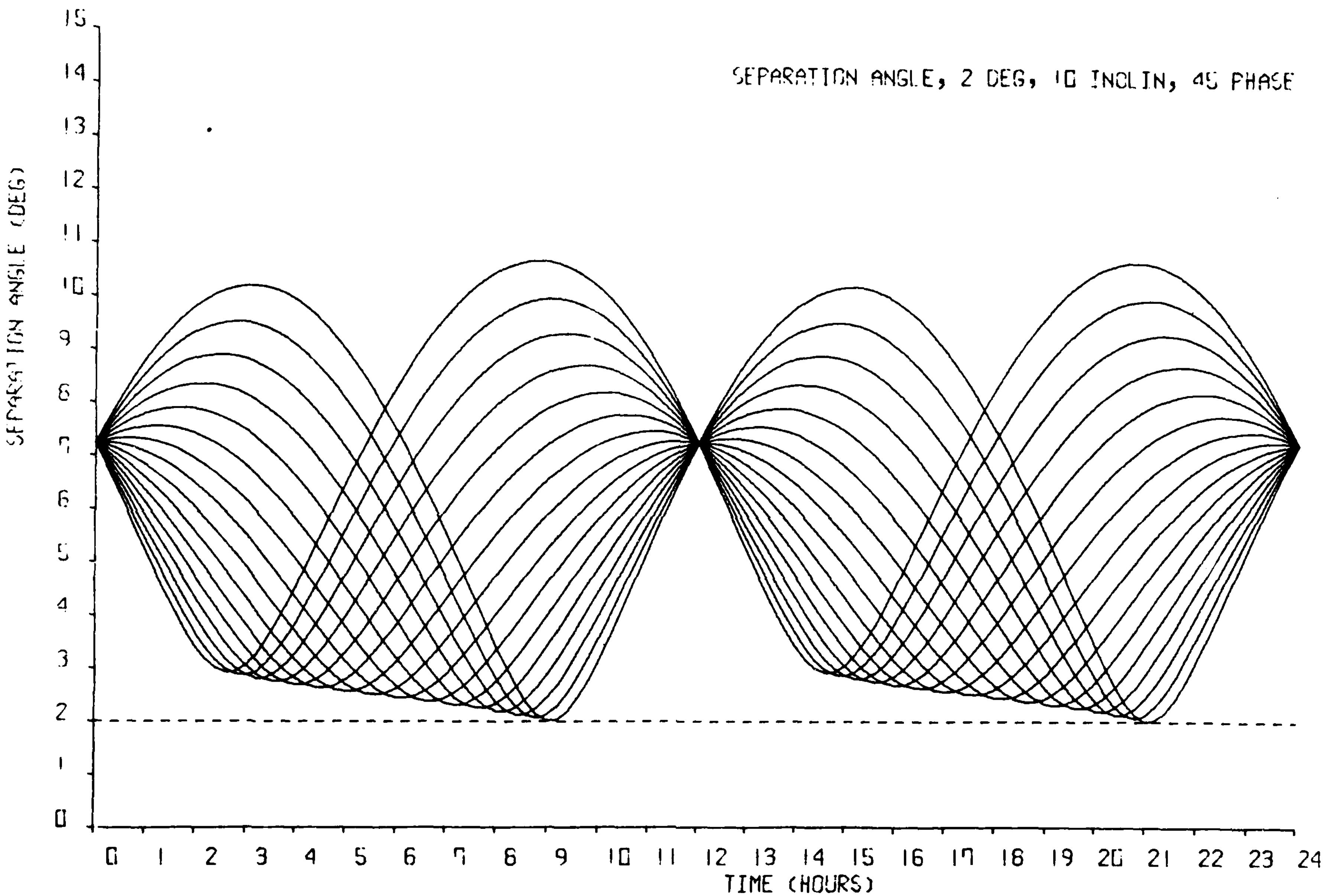


Figure 2-4(b): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 45 degrees.

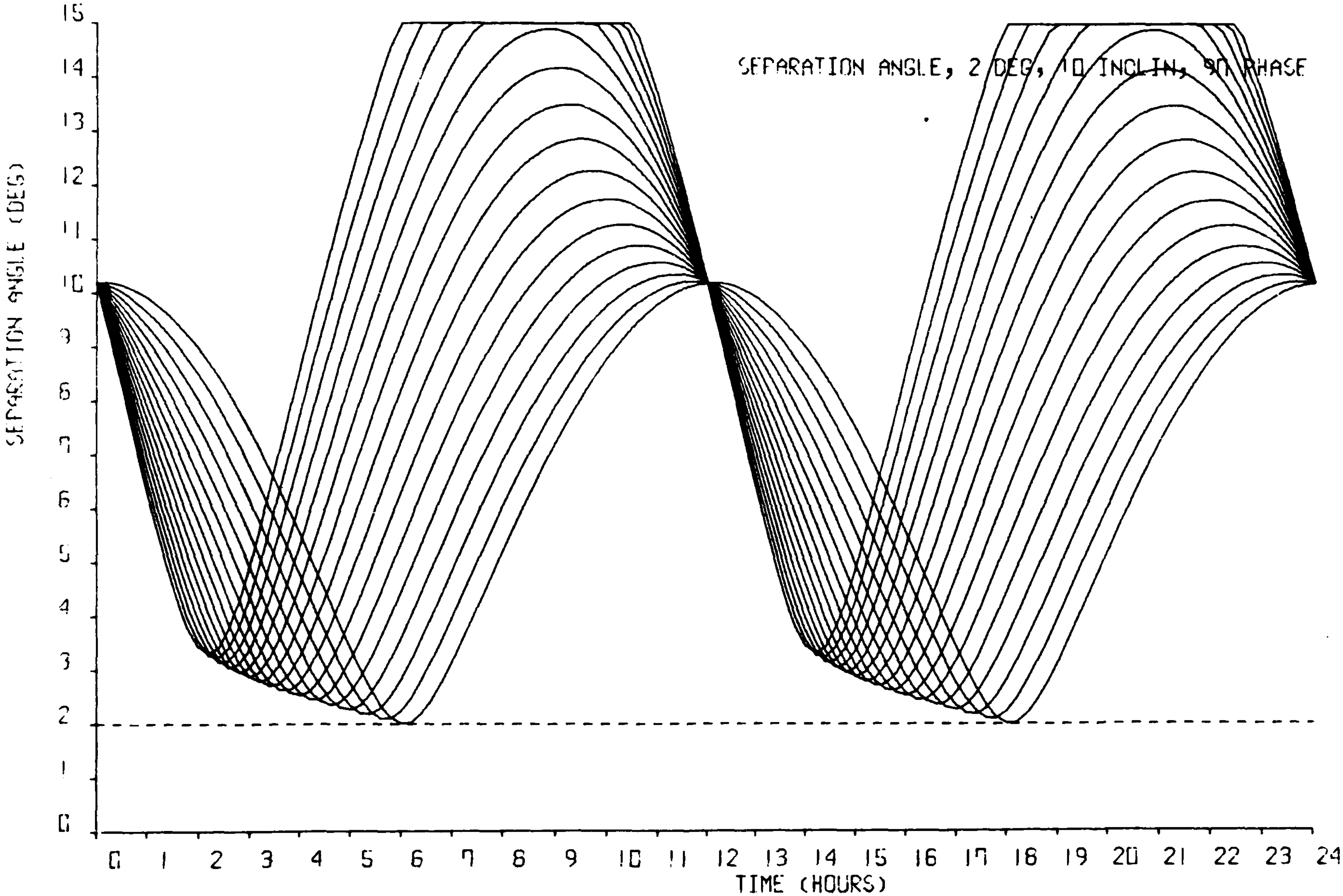


Figure 2-4(c): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 90 degrees.

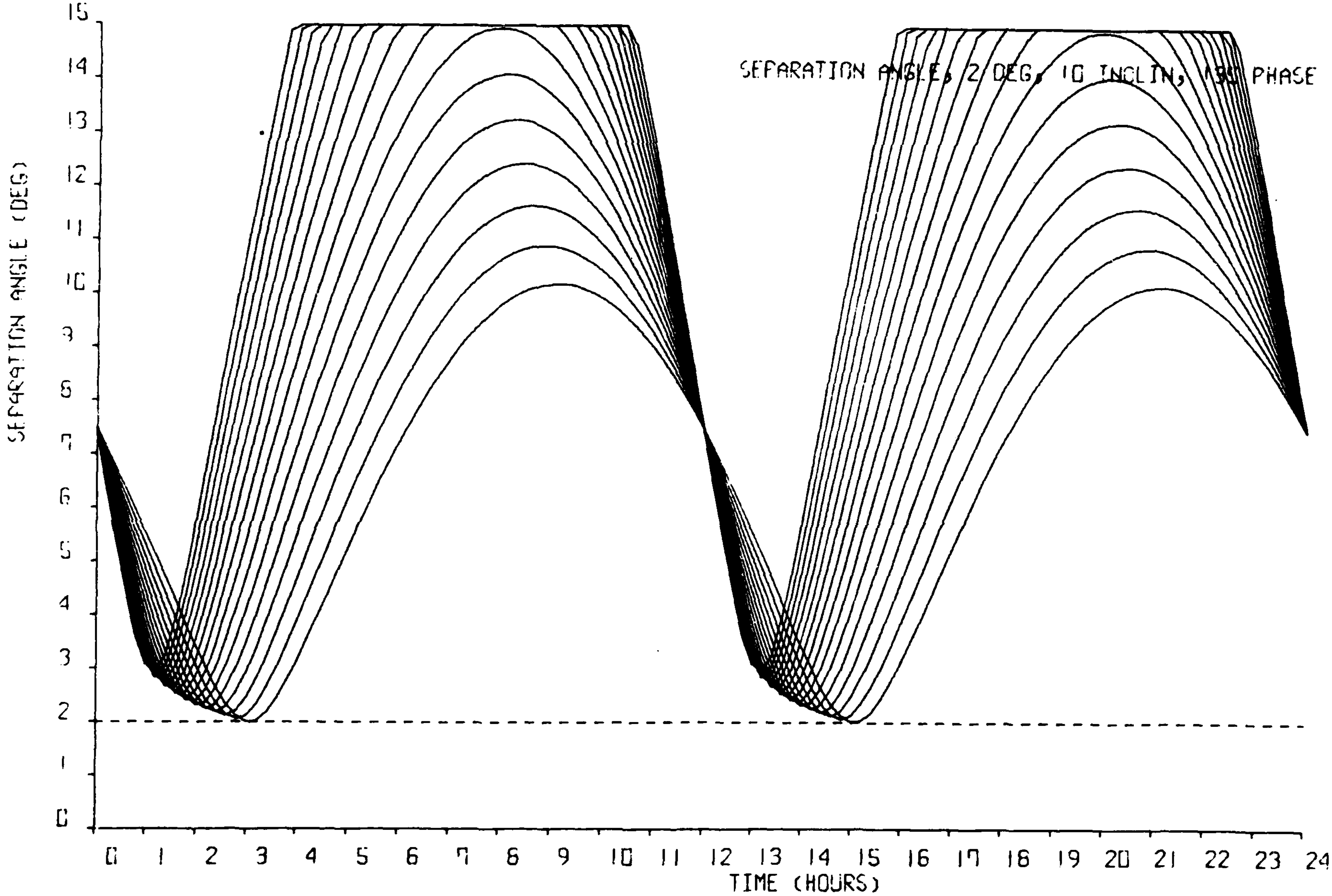


Figure 2-4(d): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 135 degrees.

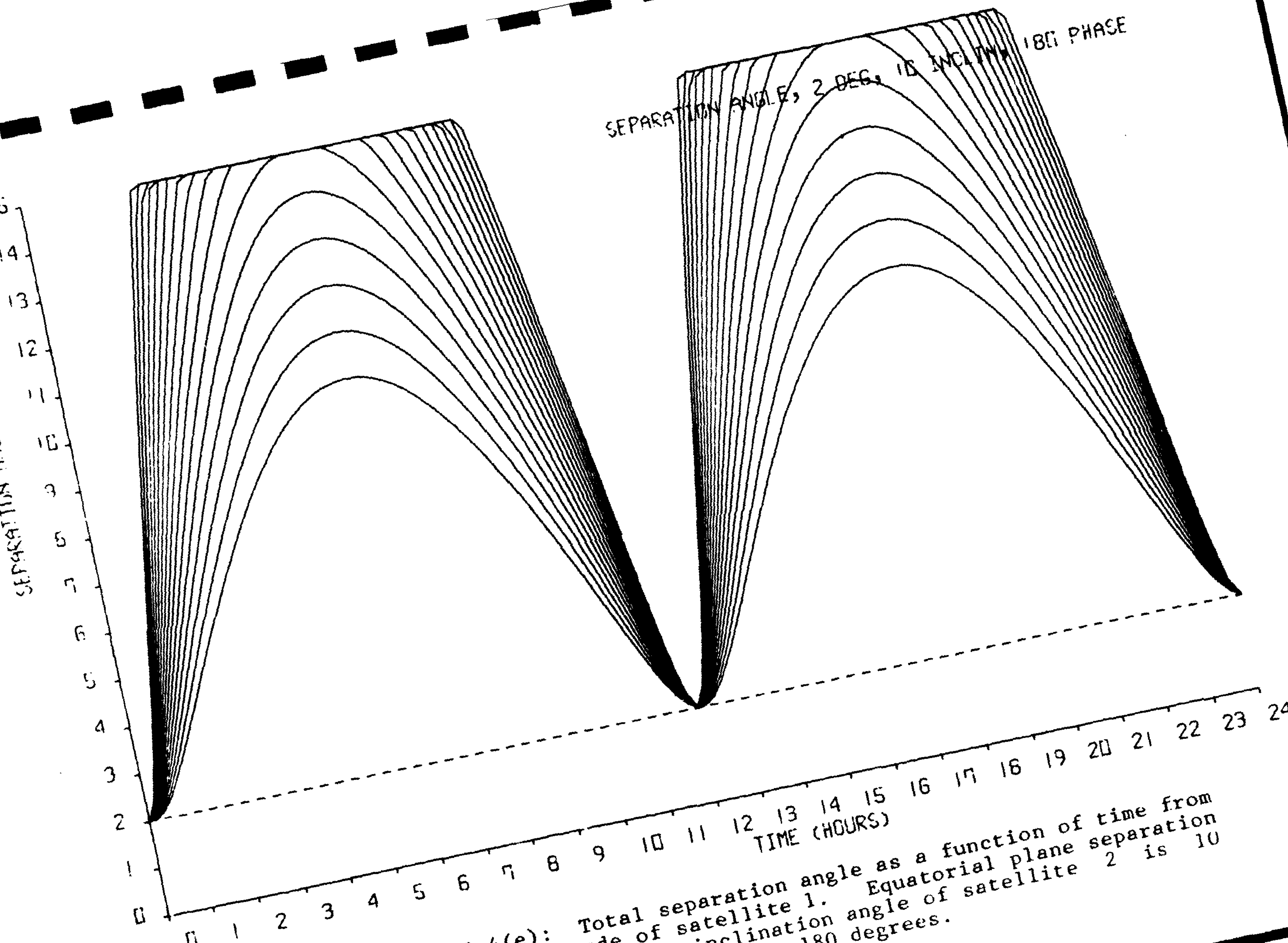


Figure 2-4(e): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 180 degrees.

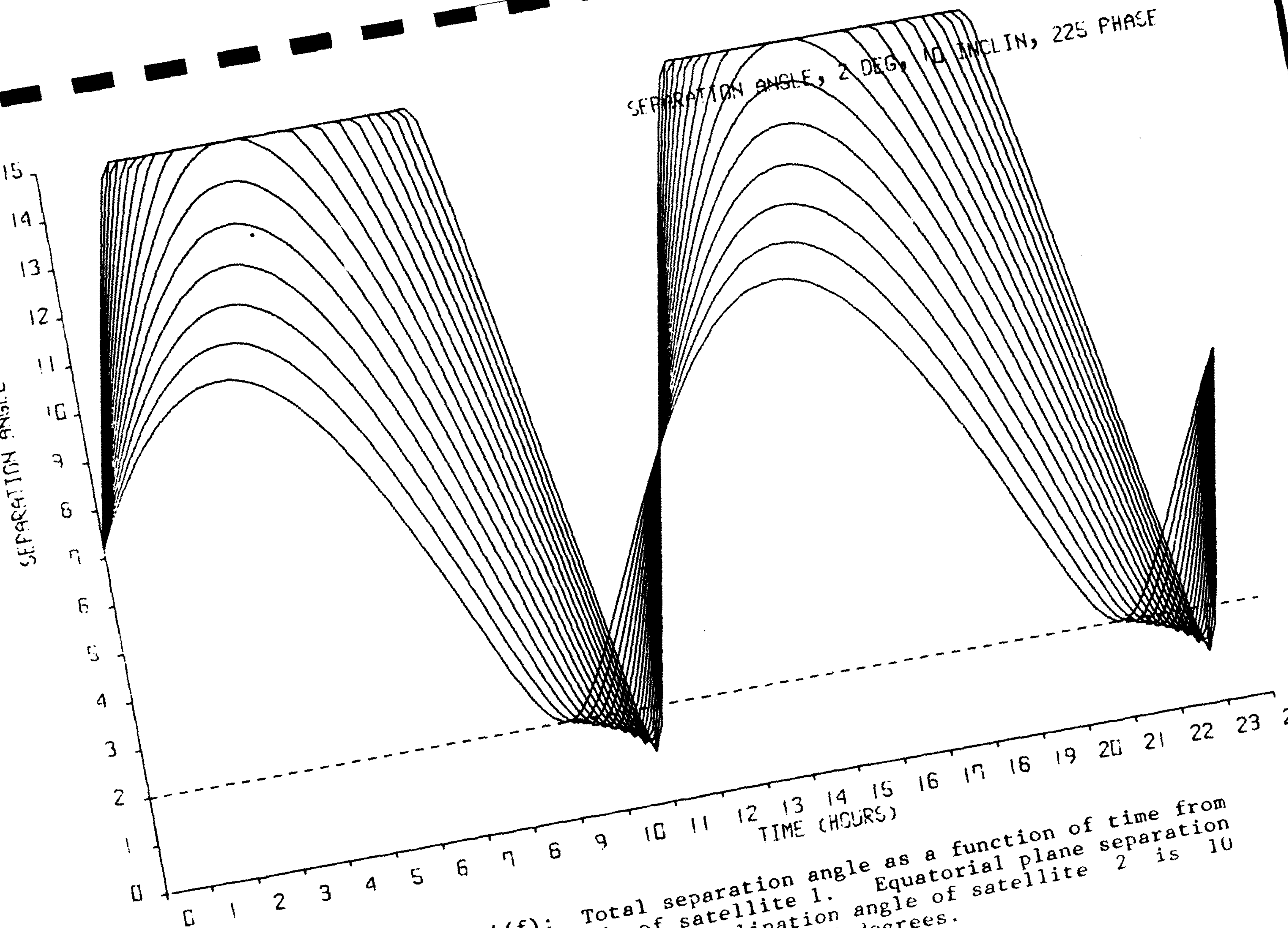


Figure 2-4(f): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 225 degrees.

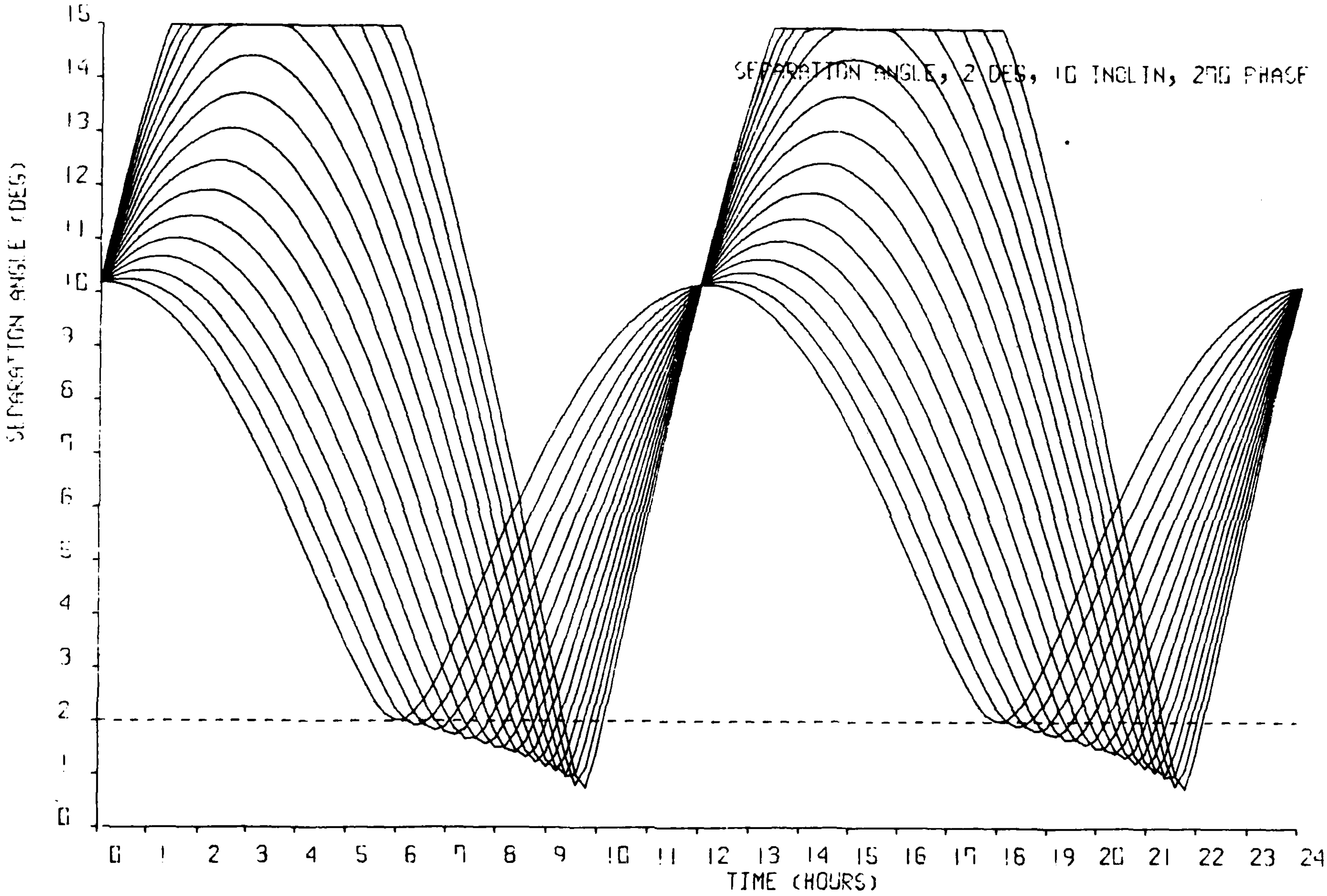


Figure 2-4(g): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 270 degrees.

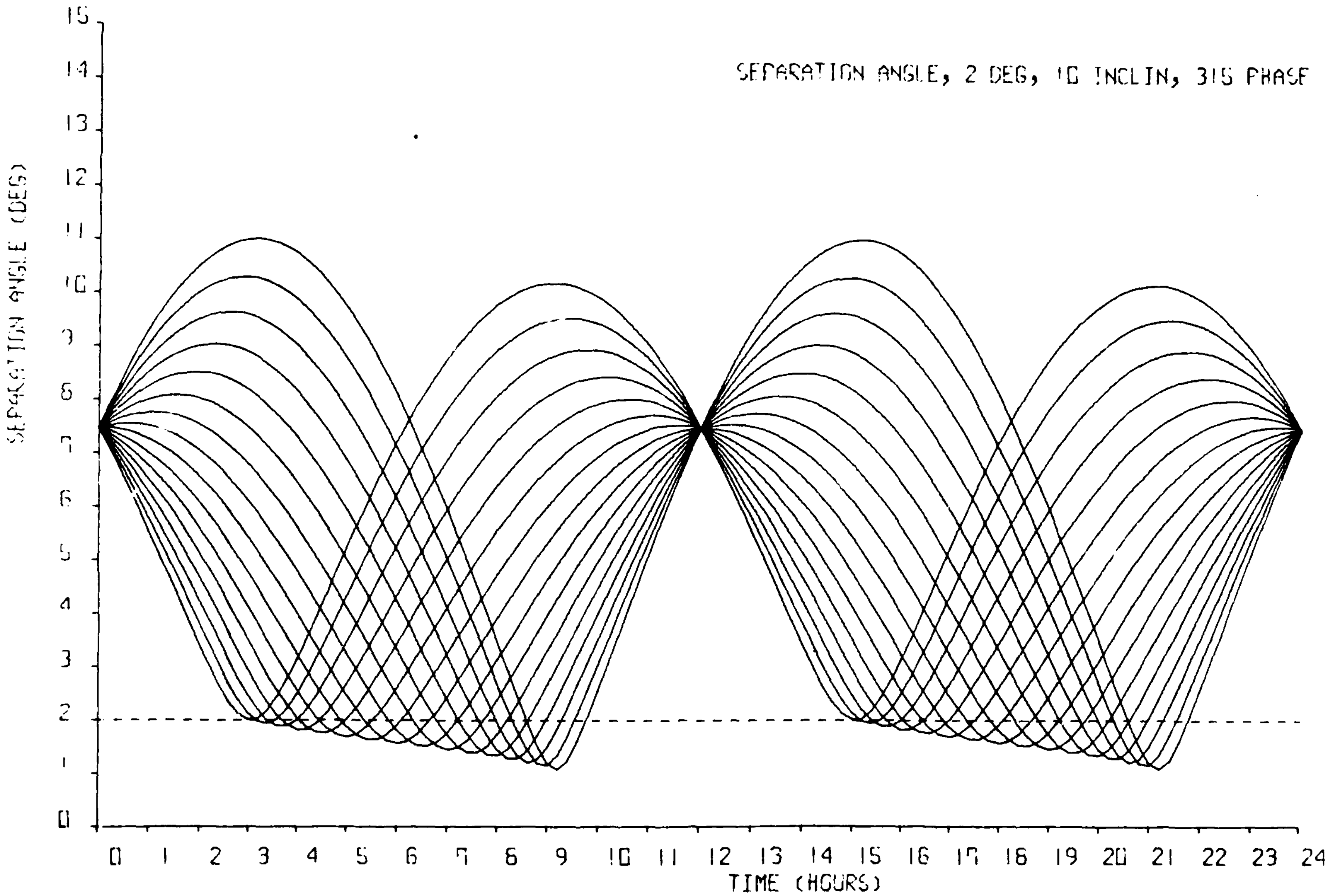


Figure 2-4(h): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 315 degrees.

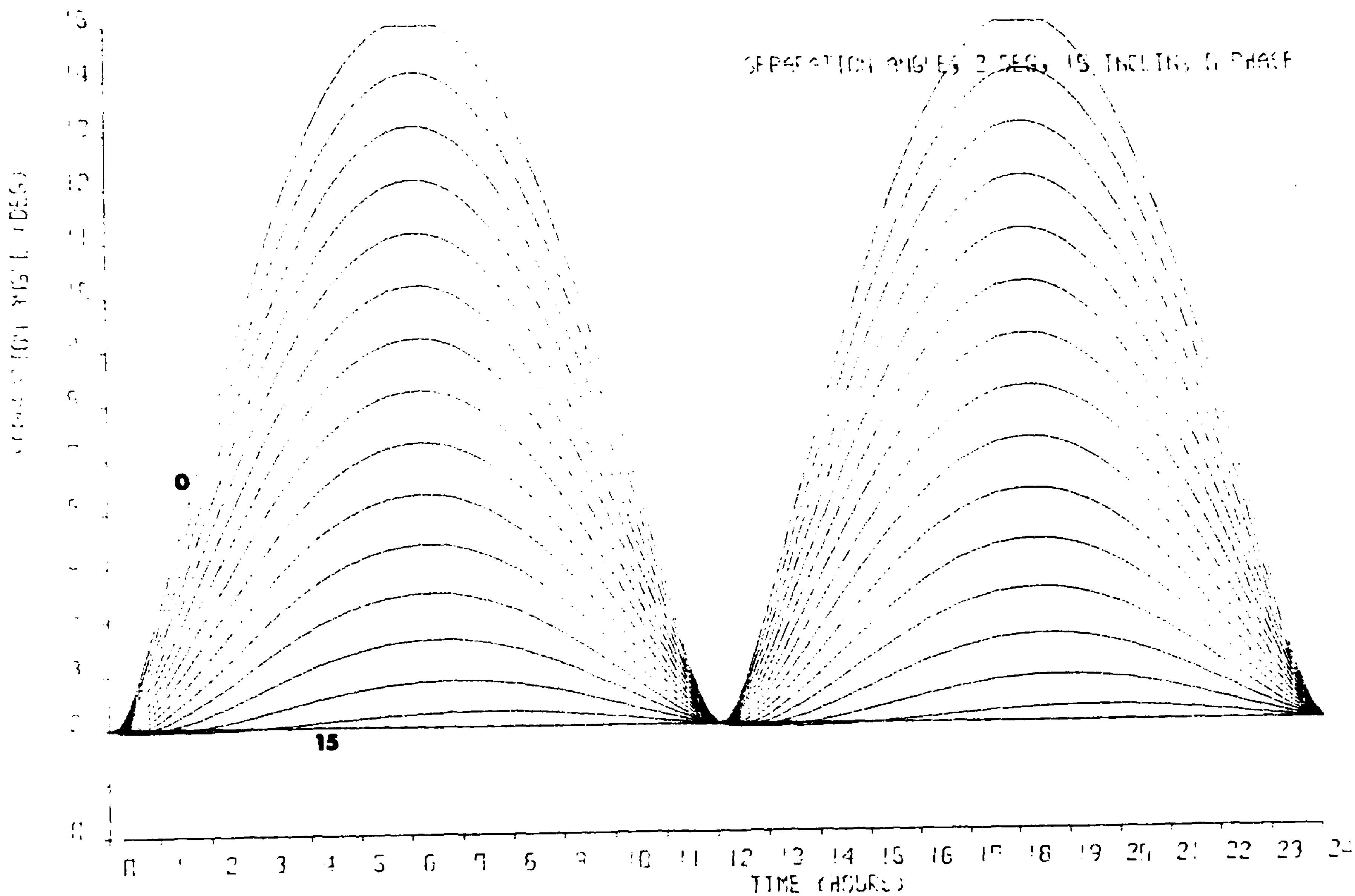


Figure 2-5(a): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 0 degrees.

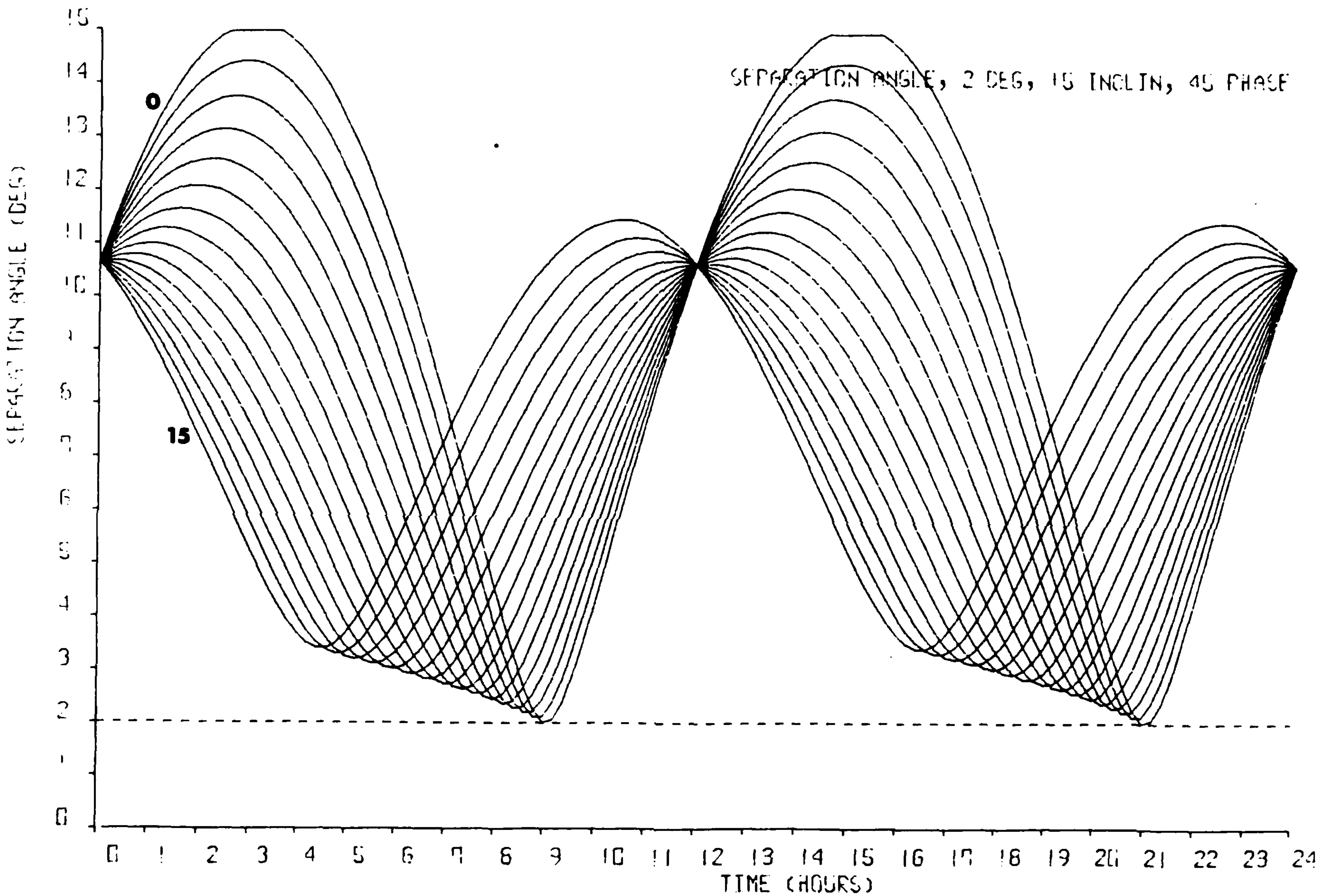


Figure 2-5(b): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 45 degrees.

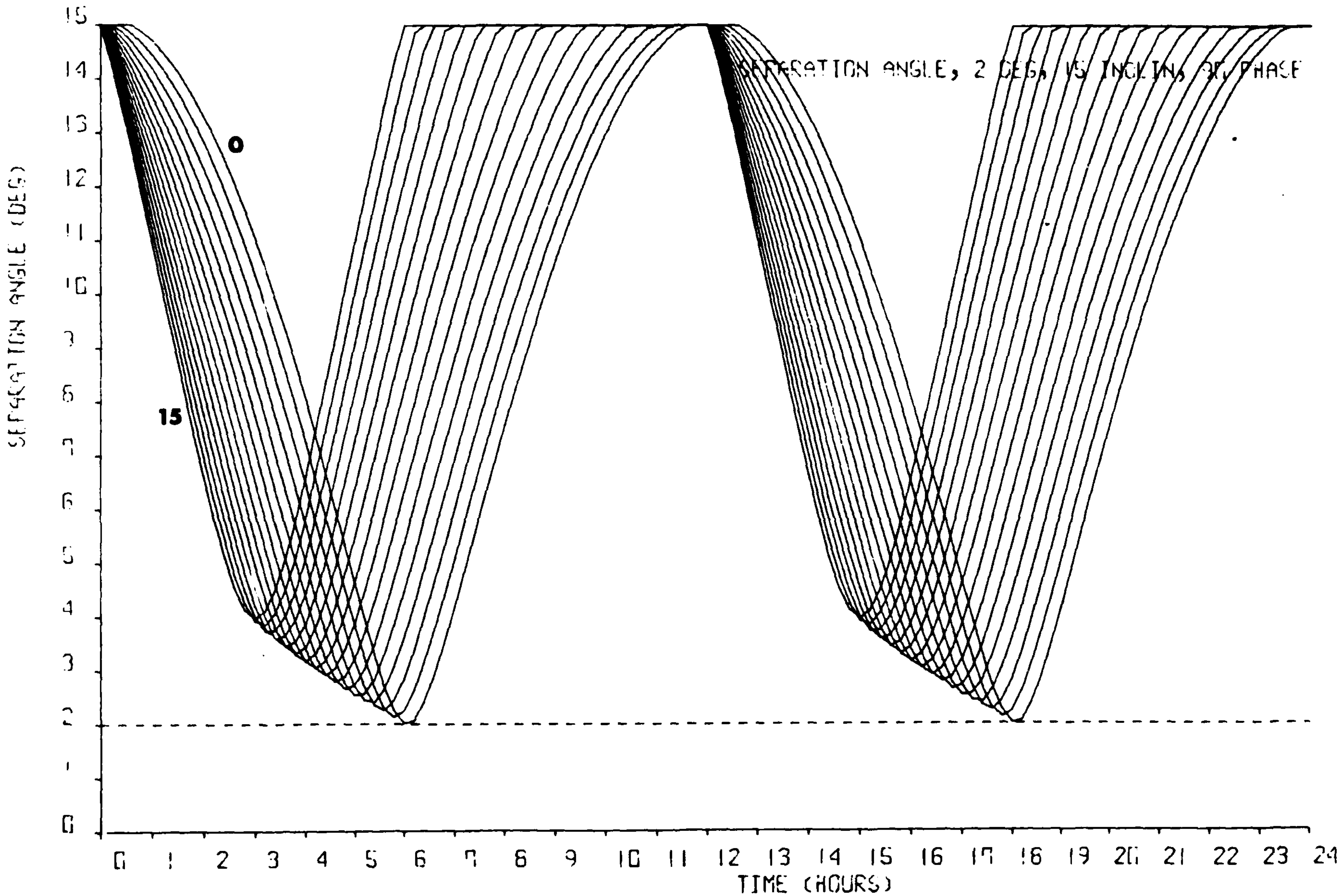


Figure 2-5(c): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 90 degrees.

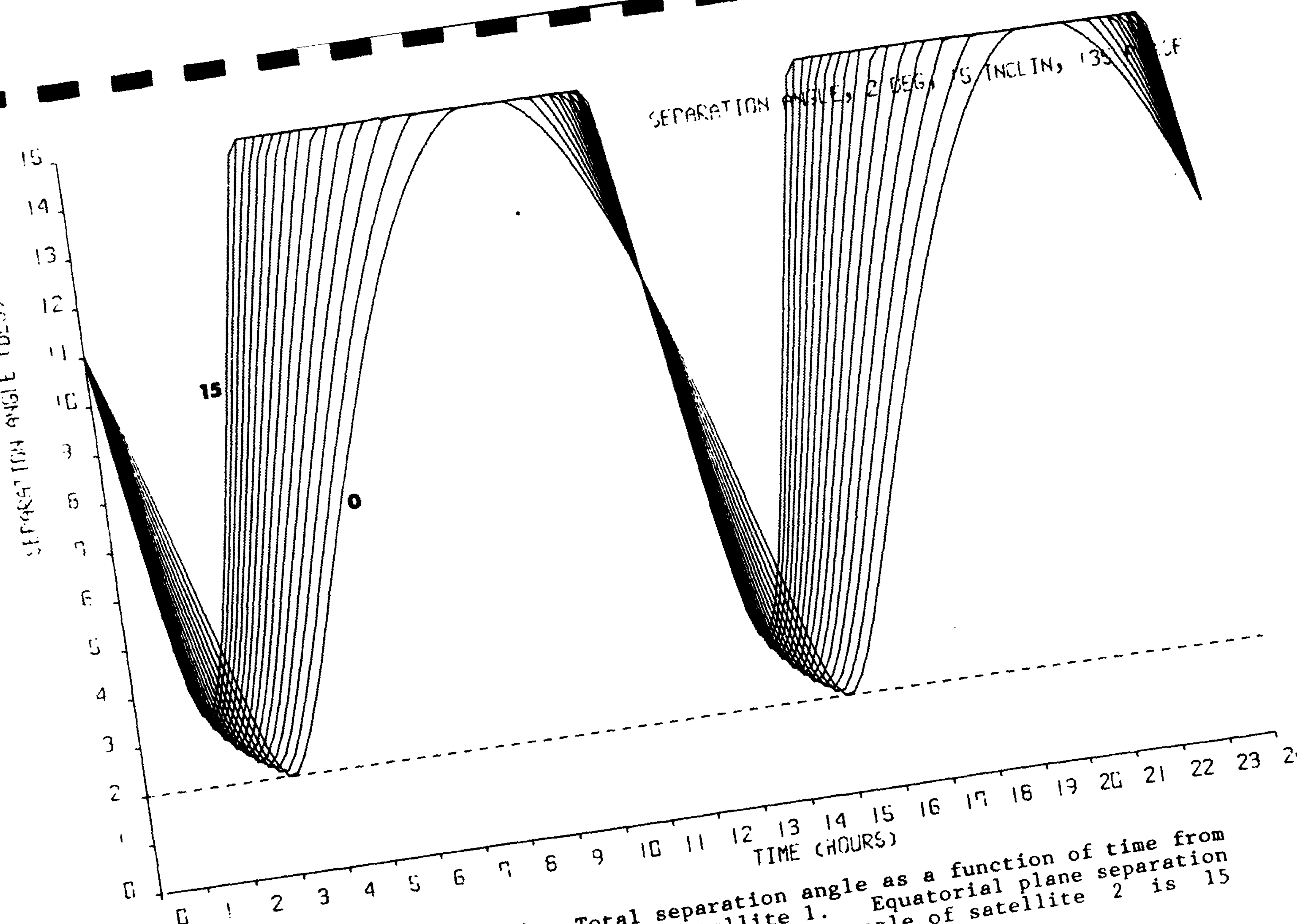


Figure 2-5(d): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 135 degrees.

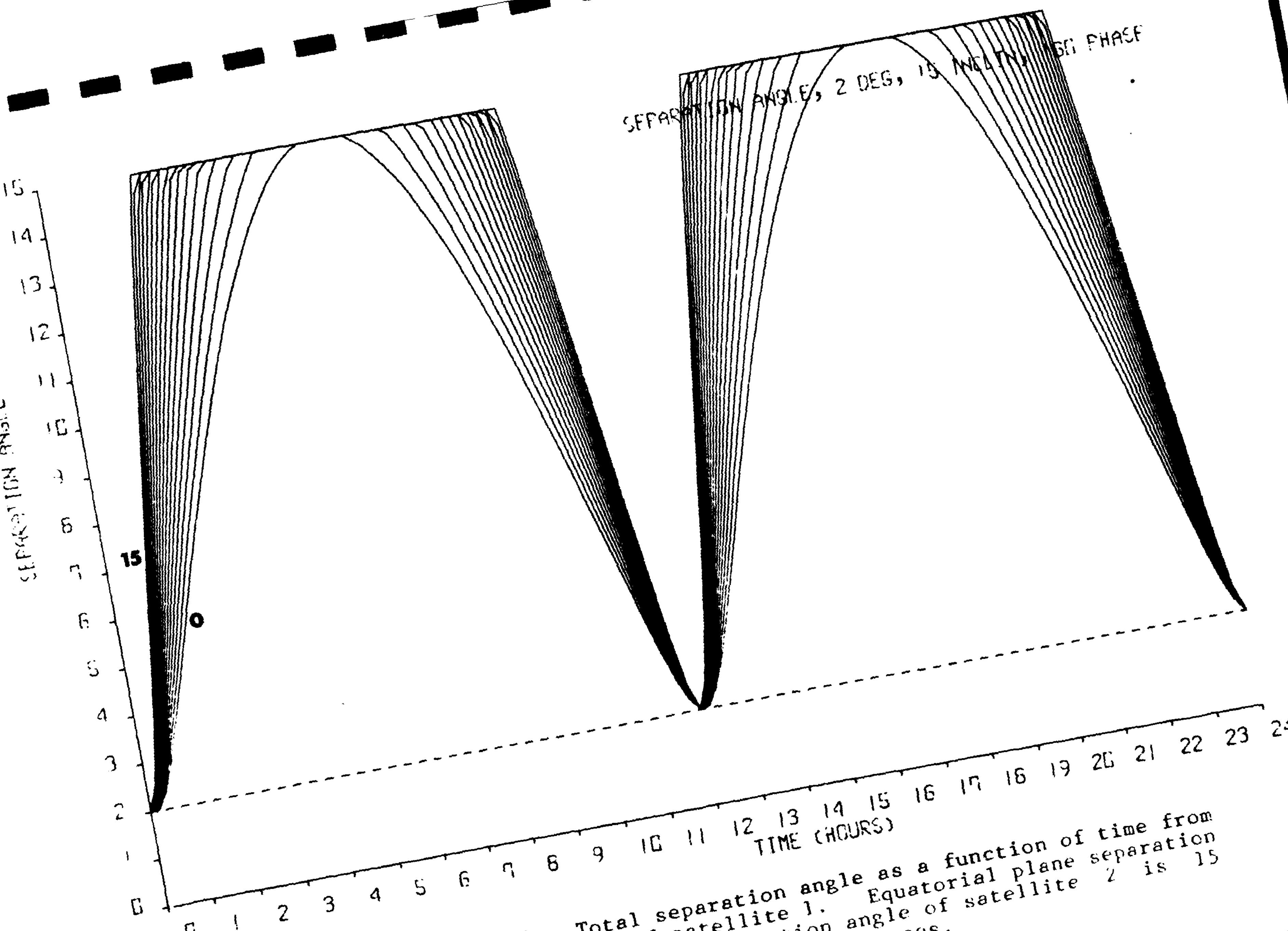


Figure 2-5(e): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 180 degrees.

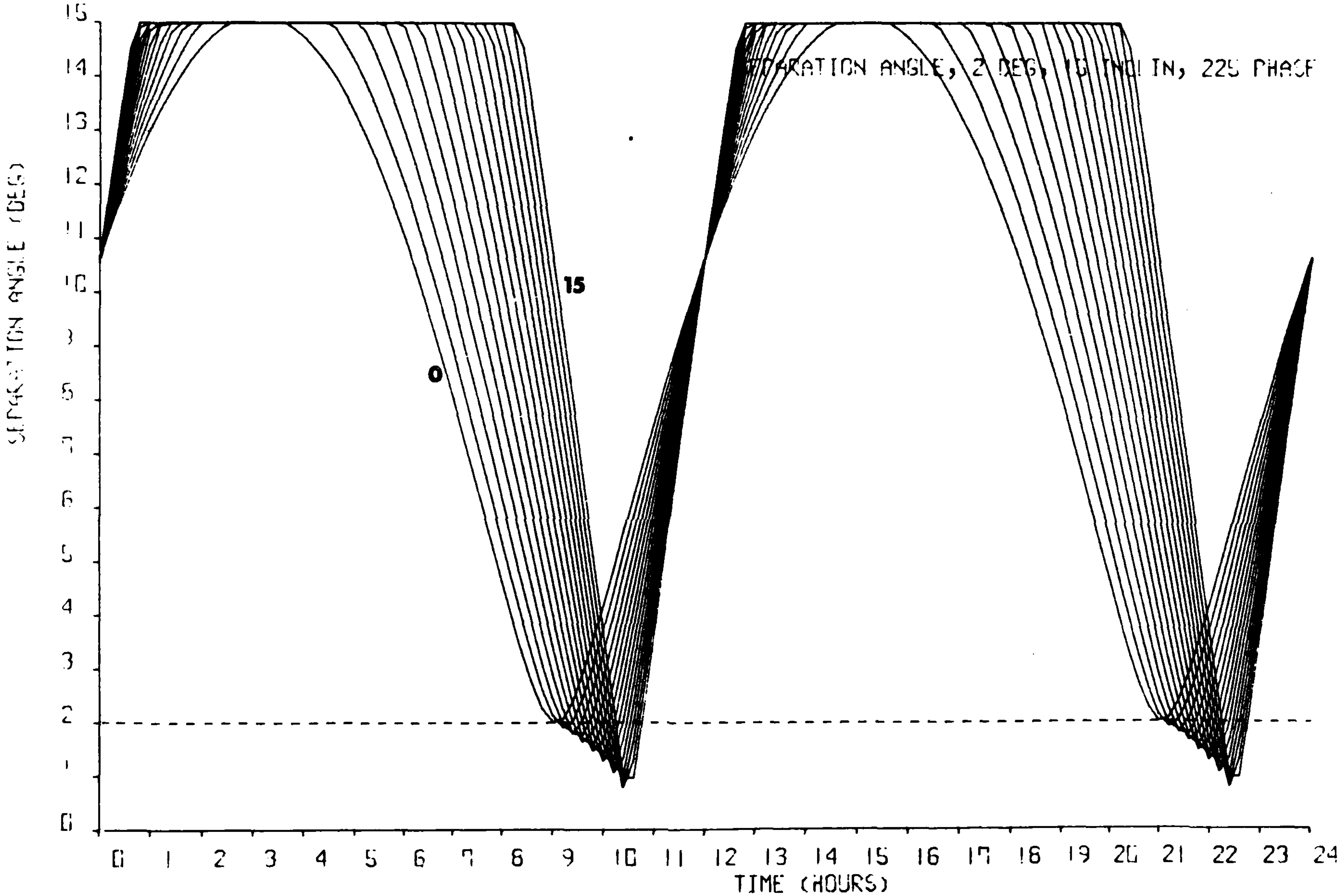


Figure 2-5(f): **Total separation angle as a function of time from the ascending node of satellite 1.** Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 225 degrees.

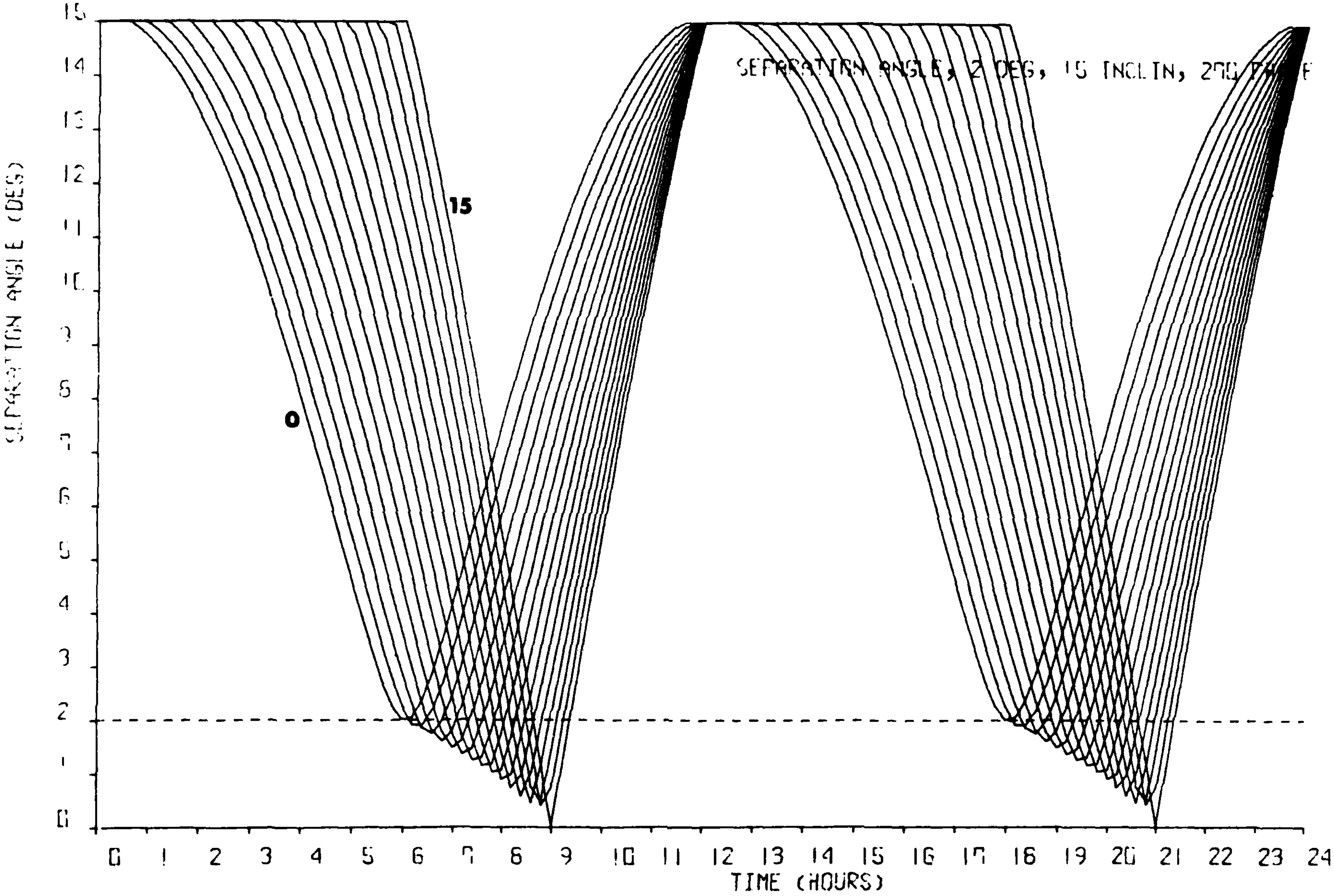


Figure 2-5(g): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 270 degrees.

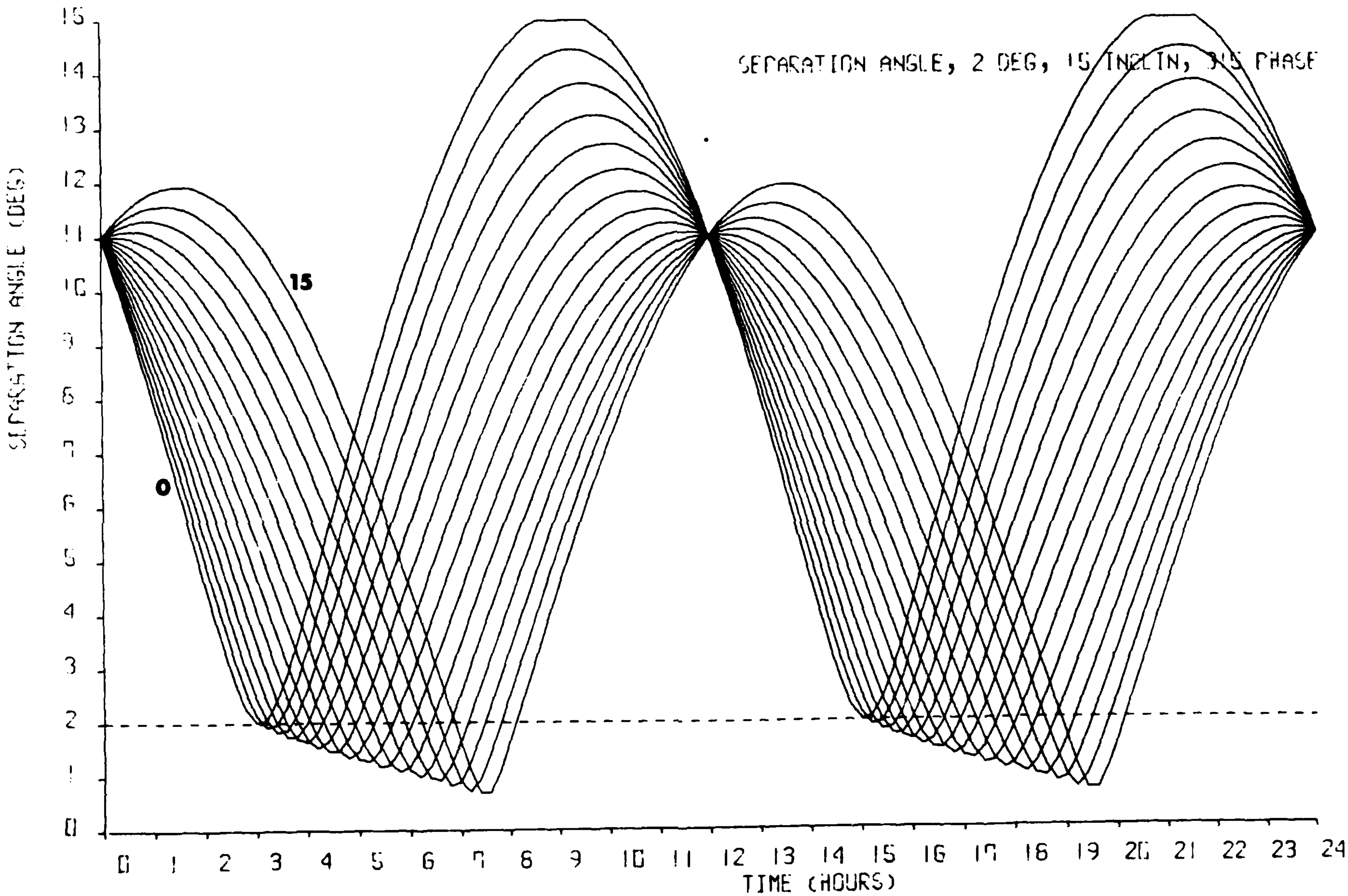


Figure 2-5(h): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 2 degrees, inclination angle of satellite 2 is 15 degrees and node phase angle is 315 degrees.

$$\lambda^2 = [I_1 I_2/2 - \theta_S]^2 \qquad (2\text{-}12)$$

The change in the total separation angle is:

$$\Delta\lambda = \theta_S - \lambda = I_1 I_2/2 \quad \text{(radians)}$$

$$\Delta\lambda^0 = \pi/360\ I_1^0 I_2^0 \quad \text{(degrees)} \qquad (2\text{-}13)$$

where the superscript o indicates the value in degrees. Equation (2-13) predicts the worst case total separation angles in Figures 2-3, 2-4 and 2-5.

Figure 2-6 shows contours of constant change in total separation angle as a function of the inclination angles of both satellites for the worst nodal phase angle. For example, for inclination angles of 10 degrees for both satellites, the change in total separation angle can be read as about 0.9 degrees. For a nominal separation angle of 2 degrees, the minimum total separation angle is then 1.1 degrees.

It is possible to solve equation (2-9) for an arbitrary nodal phase angle by noting that the first term of equation (2-9) is zero when:

$$\mathrm{Tan}\gamma_1 = I_1 \mathrm{Sin}\gamma_0 / (I_1 - I_2 \mathrm{Cos}\gamma_0) \qquad (2\text{-}14)$$

Appendix A-4 shows that this yields the approximate minimum total separation angle. Substituting equation (2-14) into (2-9) and simplifying gives:

$$\lambda^2 = [-I_1 I_2/2\ \mathrm{Sin}\gamma_0 - \theta_S]^2 \qquad (2\text{-}15)$$

and gives the change in total separation angle as:

$$\Delta\lambda = I_1 I_2/2\ \mathrm{Sin}\gamma_0 \quad \text{(radians)} \qquad (2\text{-}16)$$

Figures 2-7(a) through (c) present plots of the minimum total separation angle as a function of the node phase angle for satellite 2 inclination angles of 5, 10, and 15 degrees. Each plot shows the variation with the inclination angle of satellite 1 from 0 to 15 degrees. Notice the worst case total separation angle occurs at a nodal phase angle of 270 degrees while a nodal phase angle of 90 degrees provides an increase in total separation angle.

2.5 Percent of time separation angles is less than equatorial plane value

Figures 2-3, 2-4 and 2-5 show that the time during which the total separation angle is smaller than the equatorial plane value is small and is near the time when the separation angle is minimum. Then let:

$$\gamma_1 = \gamma_{min} + \Delta\gamma \qquad (2\text{-}17)$$

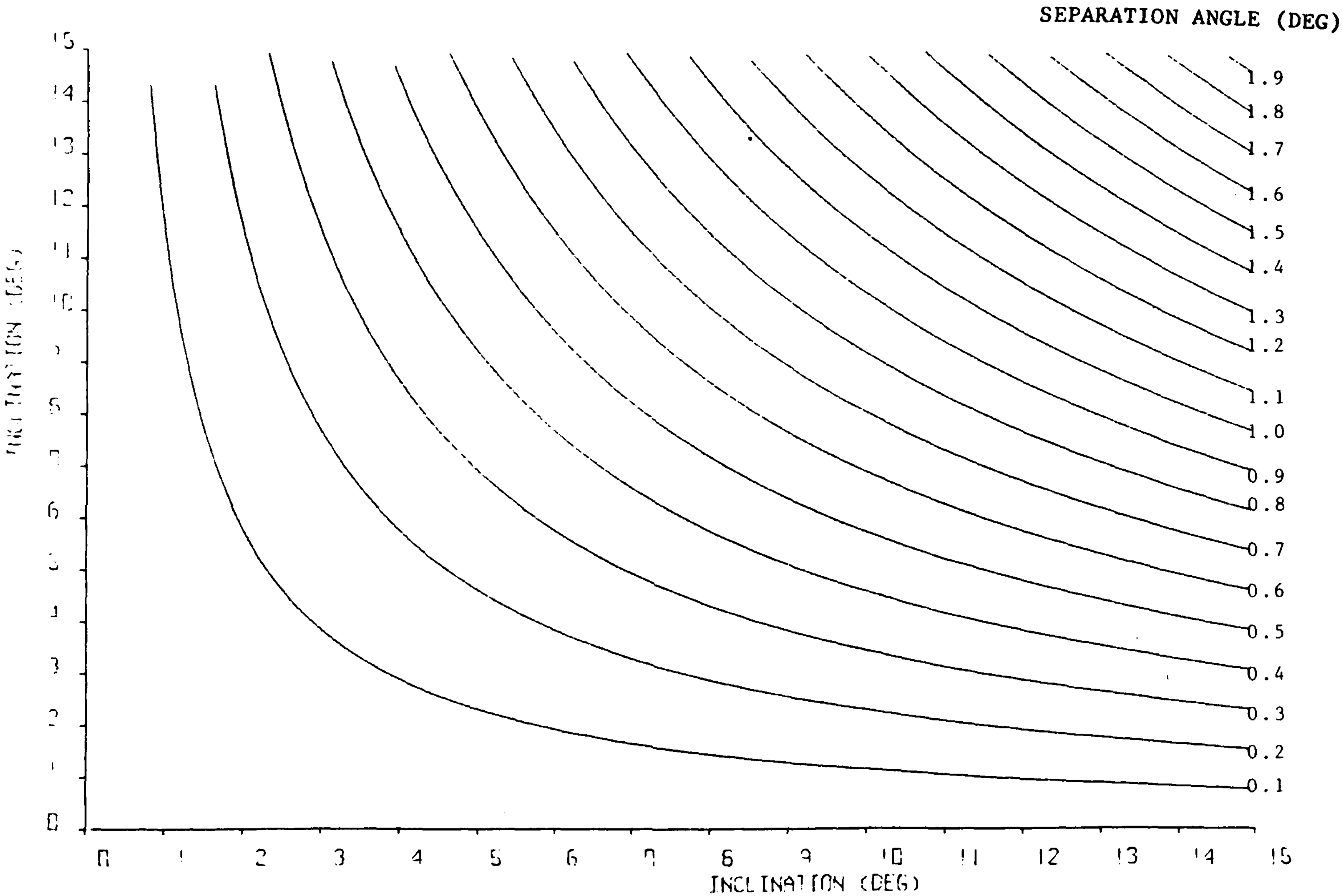


Figure 2-6: Total separation angle as a function of inclination angles for the worst nodal phase angle.

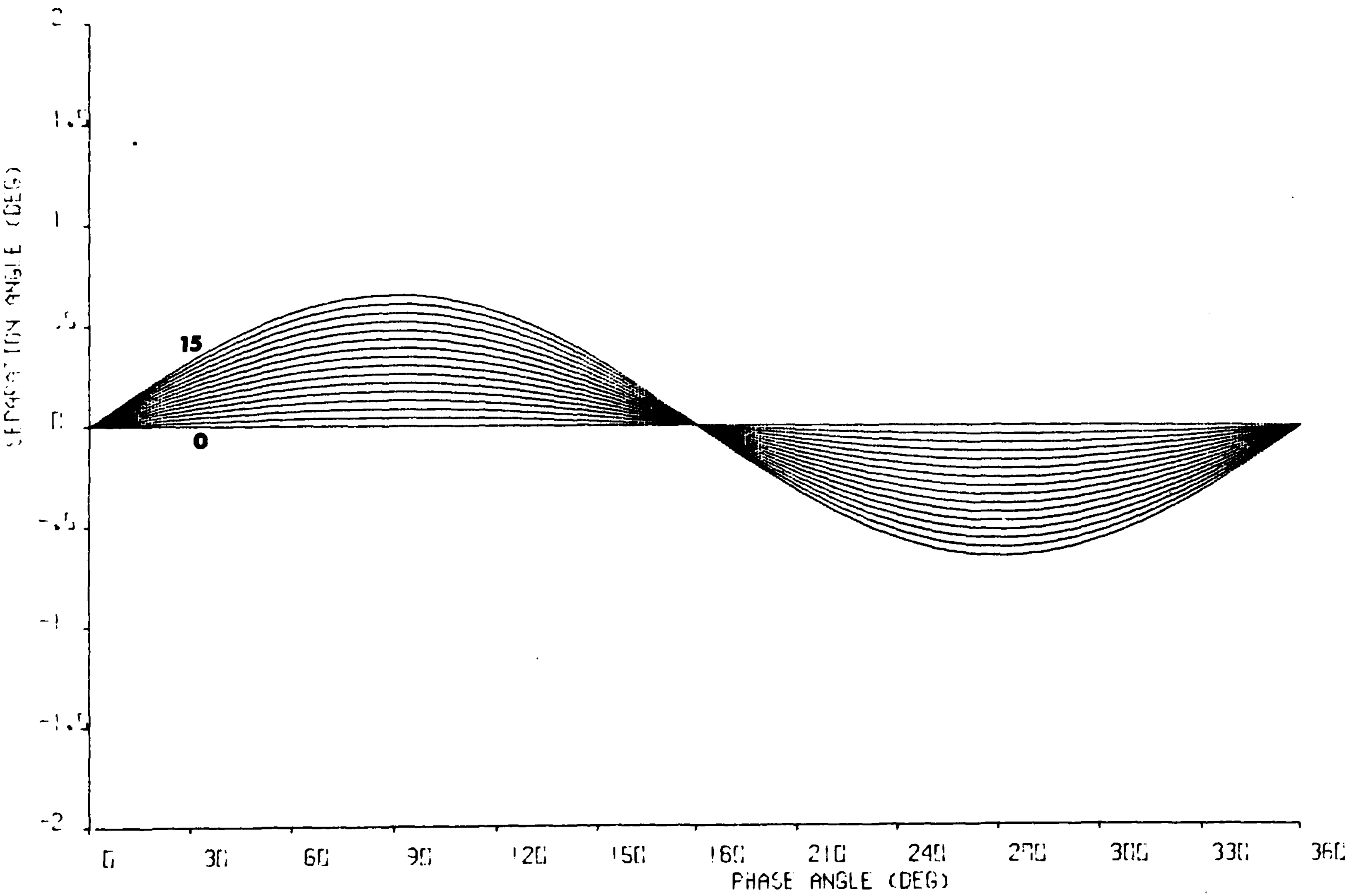


Figure 2-7(a): Minimum total separation angle as a function of nodal phase angle. Satellite 2 inclination angle is 5 degrees.

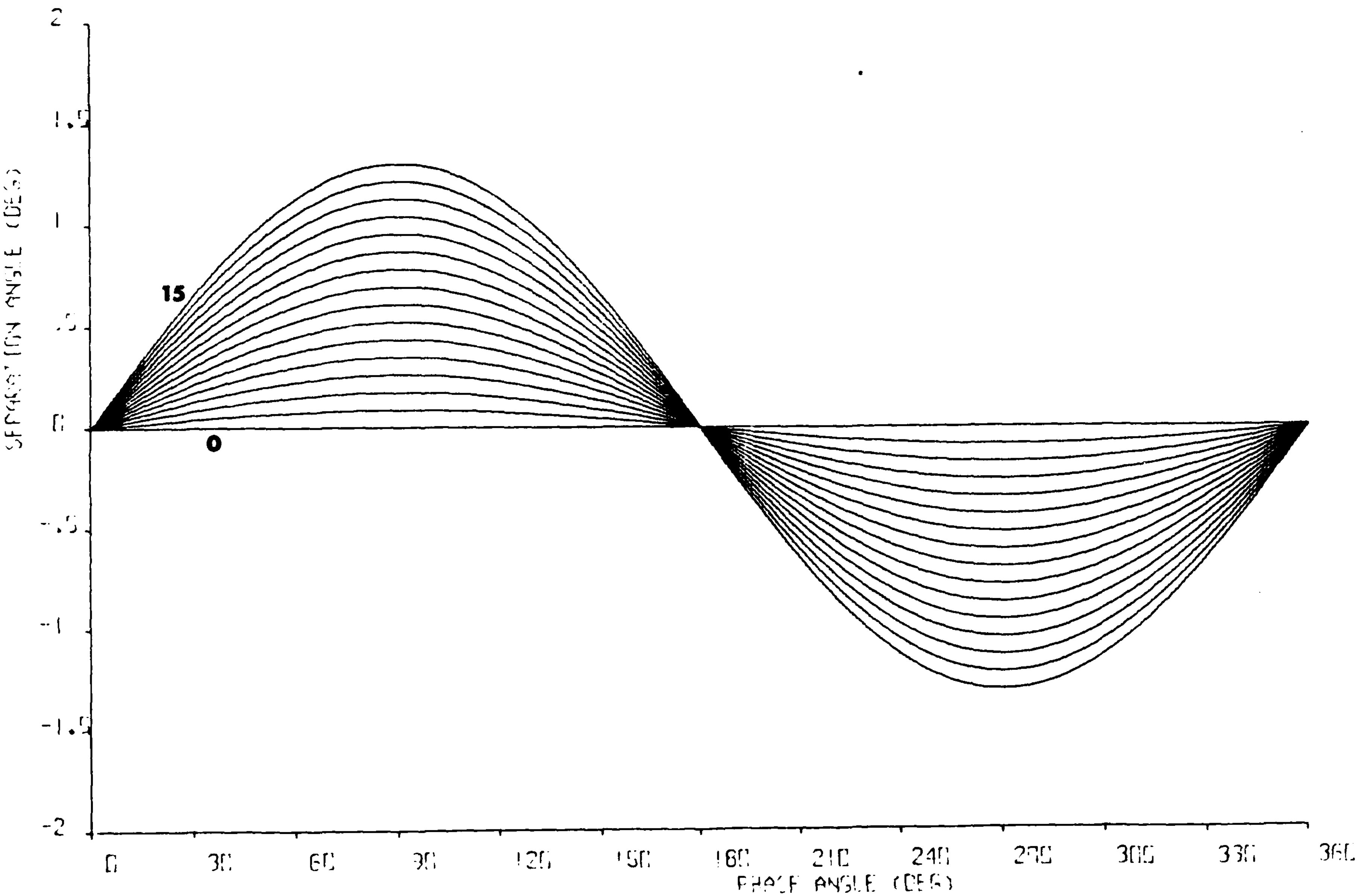


Figure 2-7(b): Minimum total separation angle as a function of nodal phase angle. Satellite 2 inclination angle is 10 degrees.

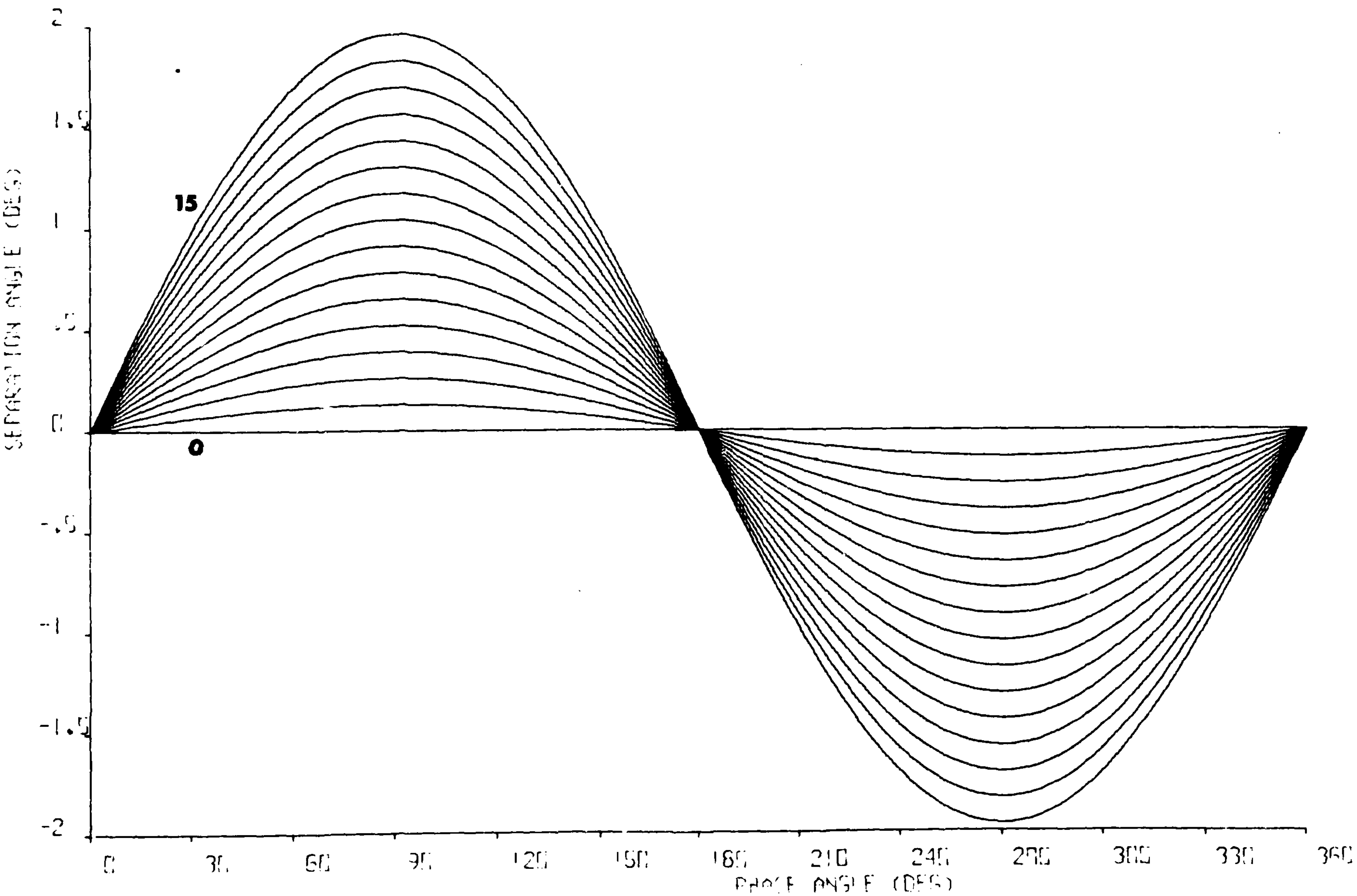


Figure 2-7(c): Minimum total separation angle as a function of nodal phase angle. Satellite 2 inclination angle is 15 degrees.

where $\Delta\gamma$ is a small angle around the minimum. Using the small angle approximations and the worst case nodal phase angle (note that this is not the worst percent of time) yields solutions for $\Delta\gamma$ where the total separation angle is equal to the equatorial plane value. As shown in Appendix A-5, the solutions are:

$$\Delta\gamma_{1,2} = [-\theta_S(I_1^2 - I_2^2)/2 \pm \sqrt{(I_1^2 + I_2^2)\ I_1 I_2 \theta_S}\,] \quad /(I_1^2 + I_2^2) \qquad (2\text{-}18)$$

The angle over which the total separation angle is less than the equatorial plane value is:

$$\Delta\gamma_1 - \Delta\gamma_2 = 2\sqrt{I_1 I_2 \theta_S/(I_1^2 + I_2^2)} \qquad (2\text{-}19)$$

Over one orbit, this angle occurs twice as shown in Figures 2-3, 2-4, and 2-5. The total time over which the total separation angle is less than the equatorial plane value is:

$$\text{Time} = 48/\pi \sqrt{I_1 I_2 \theta_S/(I_1^2 + I_2^2)} \qquad (2\text{-}20)$$

The percent of time over which the total separation angle is less than the equatorial plane value can be written as:

$$\%\text{Time} = 200/\pi \sqrt{I_1 I_2 \theta_S/(I_1^2 + I_2^2)} \qquad (2\text{-}21)$$

Notice this equation has a maximum when the inclination angles are equal. Notice also that the total time is dependent upon the nominal separation angle. Figures 2-8(a) through 2-8(d) illustrate this dependency of total time on the nominal separation angle. Comparison of the %Time in these figures with that computed from equation (2-21) for equal inclination angles of 10 degrees is as follows:

Separation Angle (deg)	%Time (plots)	%Time (equation)
2	7.3%	8.1%
3	8.8	9.9
4	10.6	11.4
5	12.0	12.8
6	13.1	14.0

Figures 2-9(a) through (e) show plots of the percent of time the total separation angle is smaller than the equatorial plane value as a function of the inclination angles of both satellites. The equatorial plane separation angle ranges from 2 degrees in Figure 2-9(a) to 6 degrees in Figure 2-9(e).

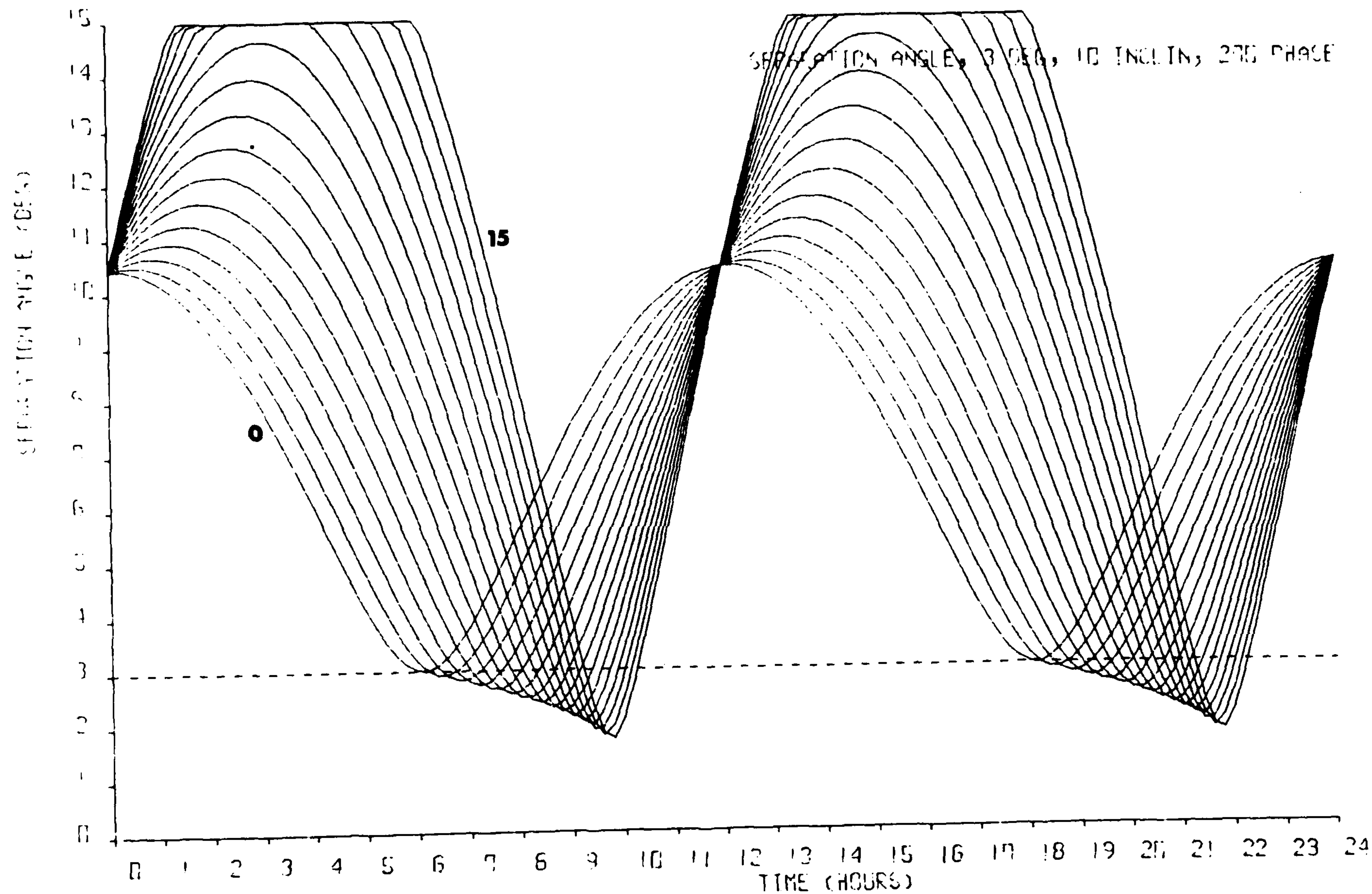


Figure 2-8(a): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 3 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 270 degrees.

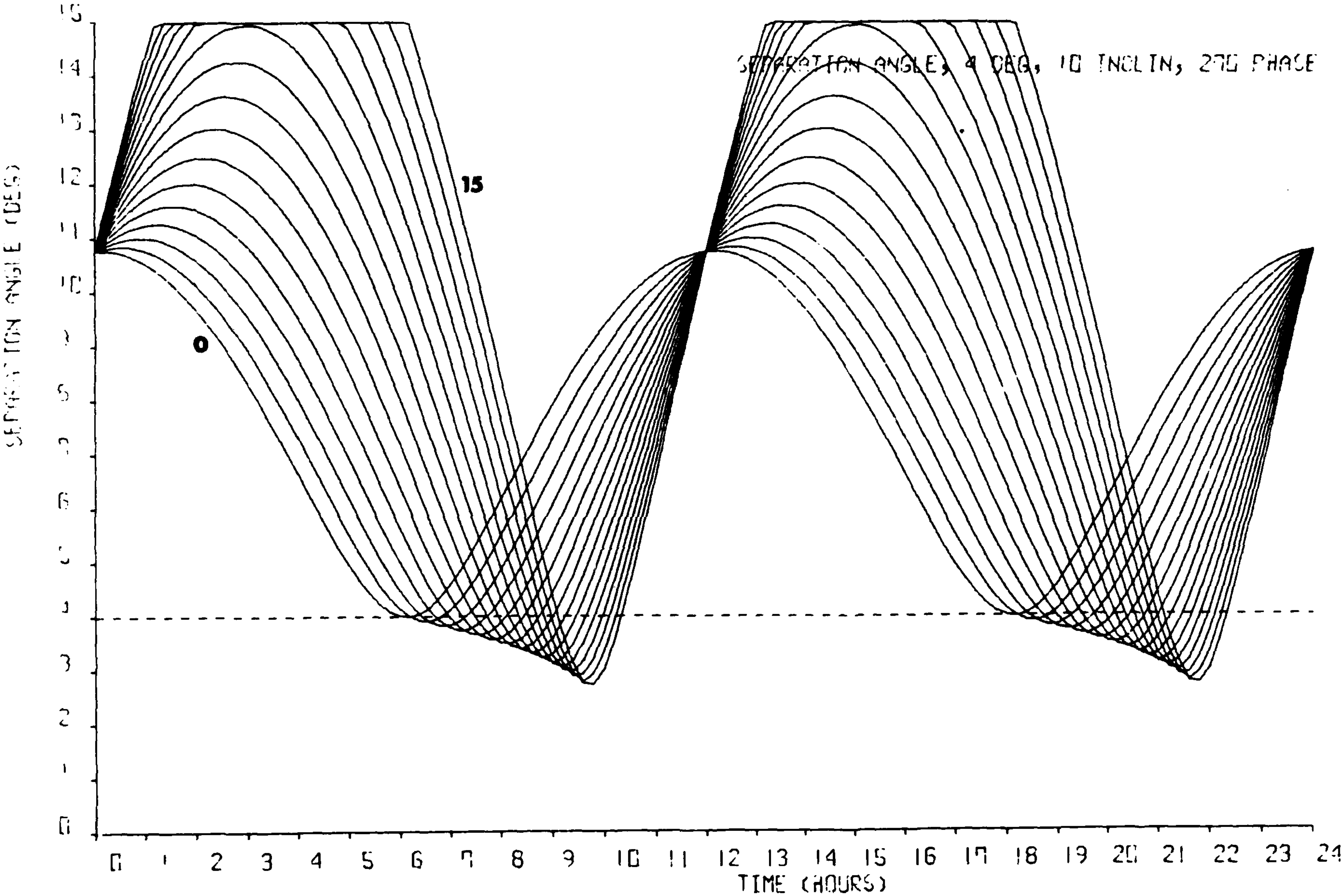


Figure 2-8(b): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 4 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 270 degrees.

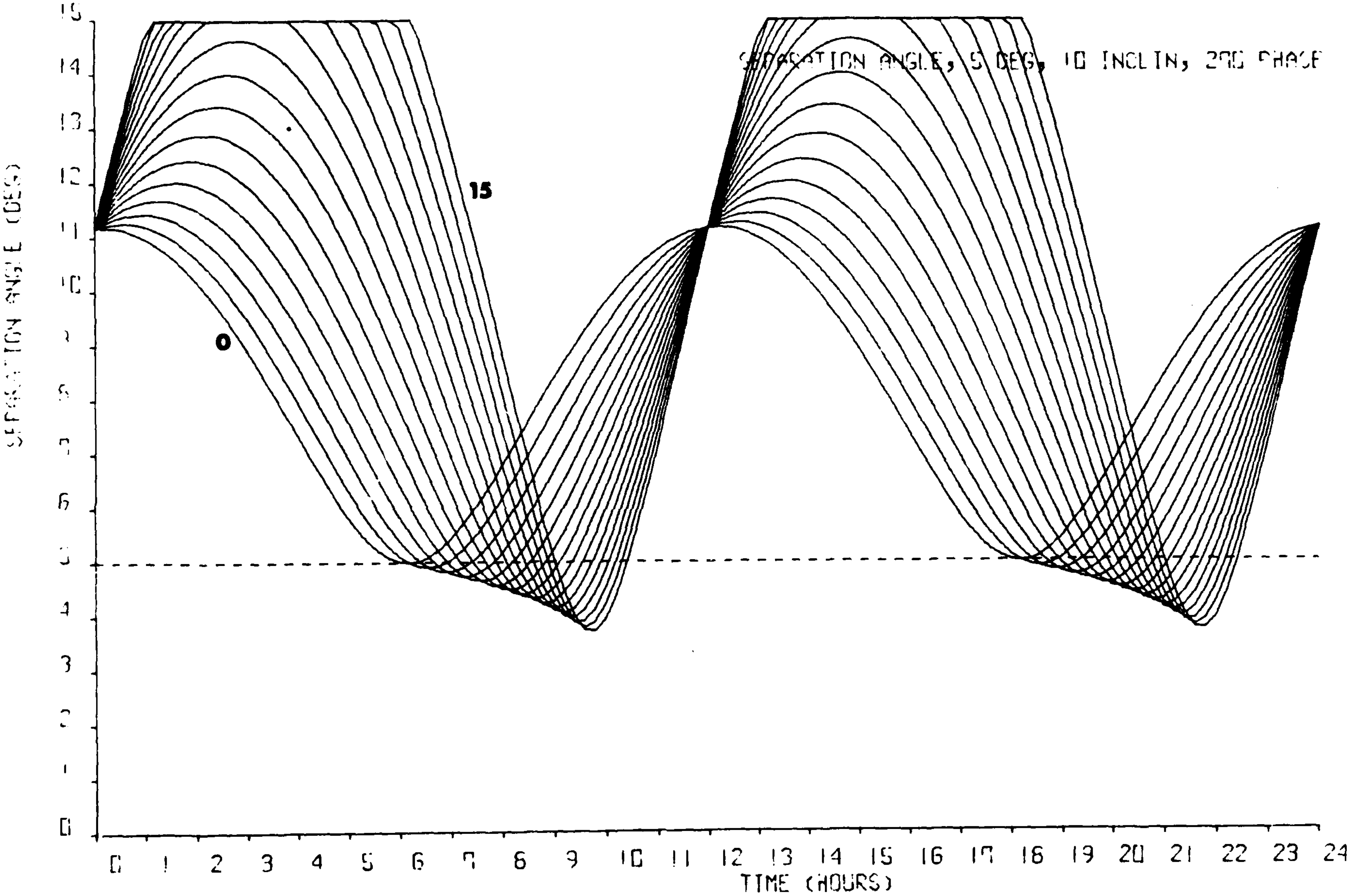


Figure 2-8(c): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 5 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 270 degrees.

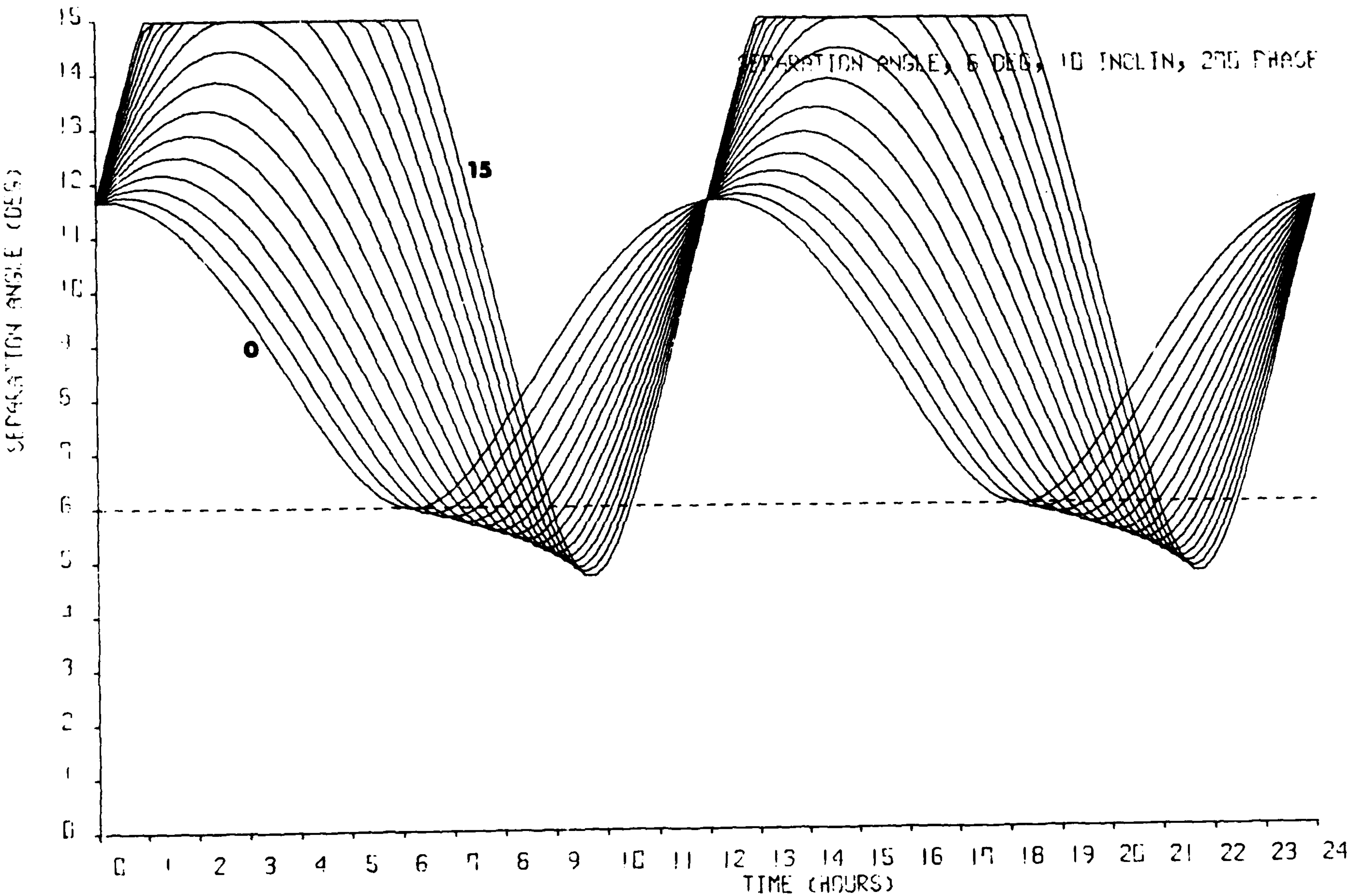


Figure 2-8(d): Total separation angle as a function of time from the ascending node of satellite 1. Equatorial plane separation angle is 6 degrees, inclination angle of satellite 2 is 10 degrees and node phase angle is 270 degrees.

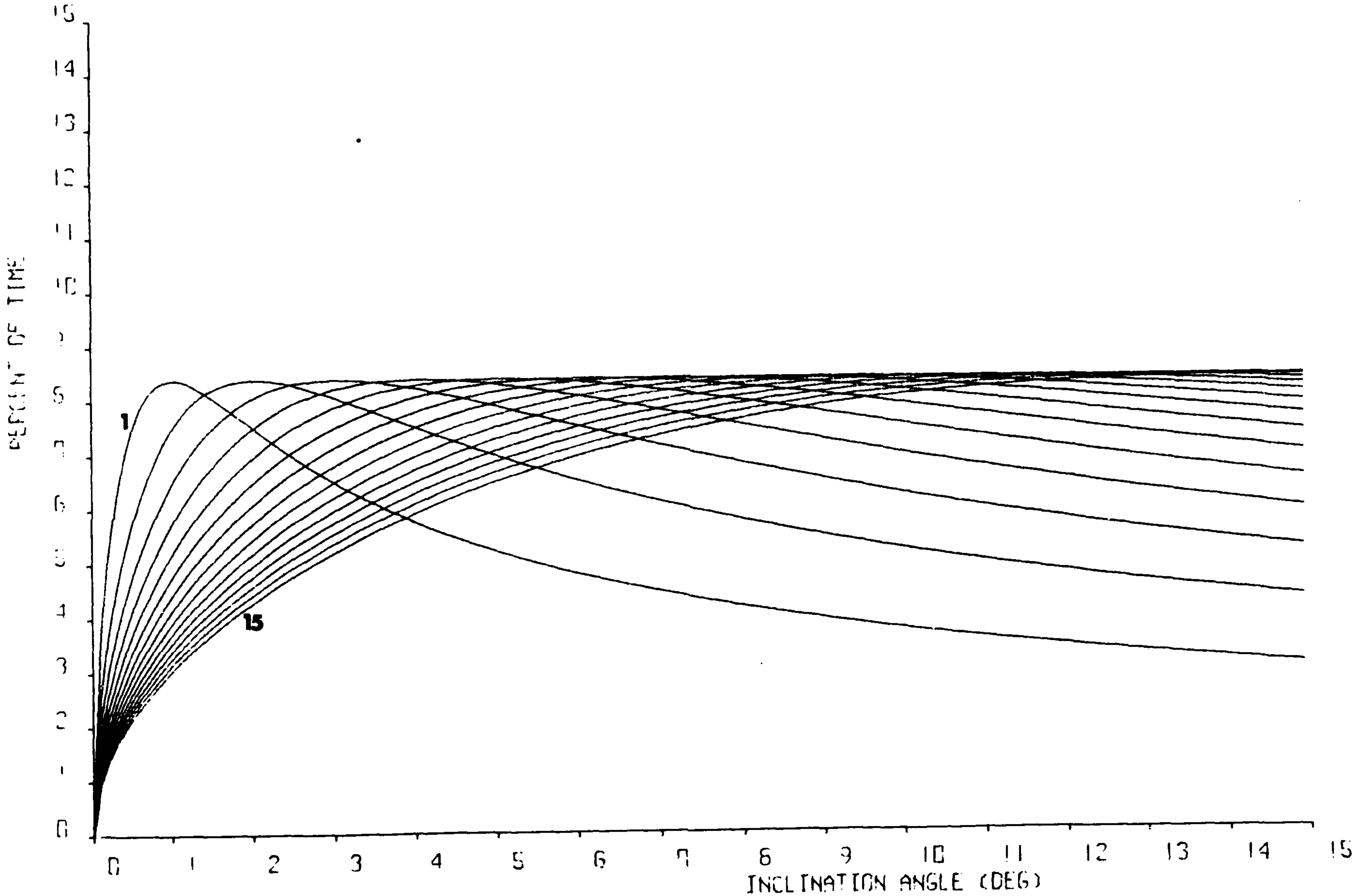


Figure 2-9(a): Percent of time the total separation angle is smaller than the equatorial plane value as a function of inclination angle. Nominal separation angle is 2 degrees.

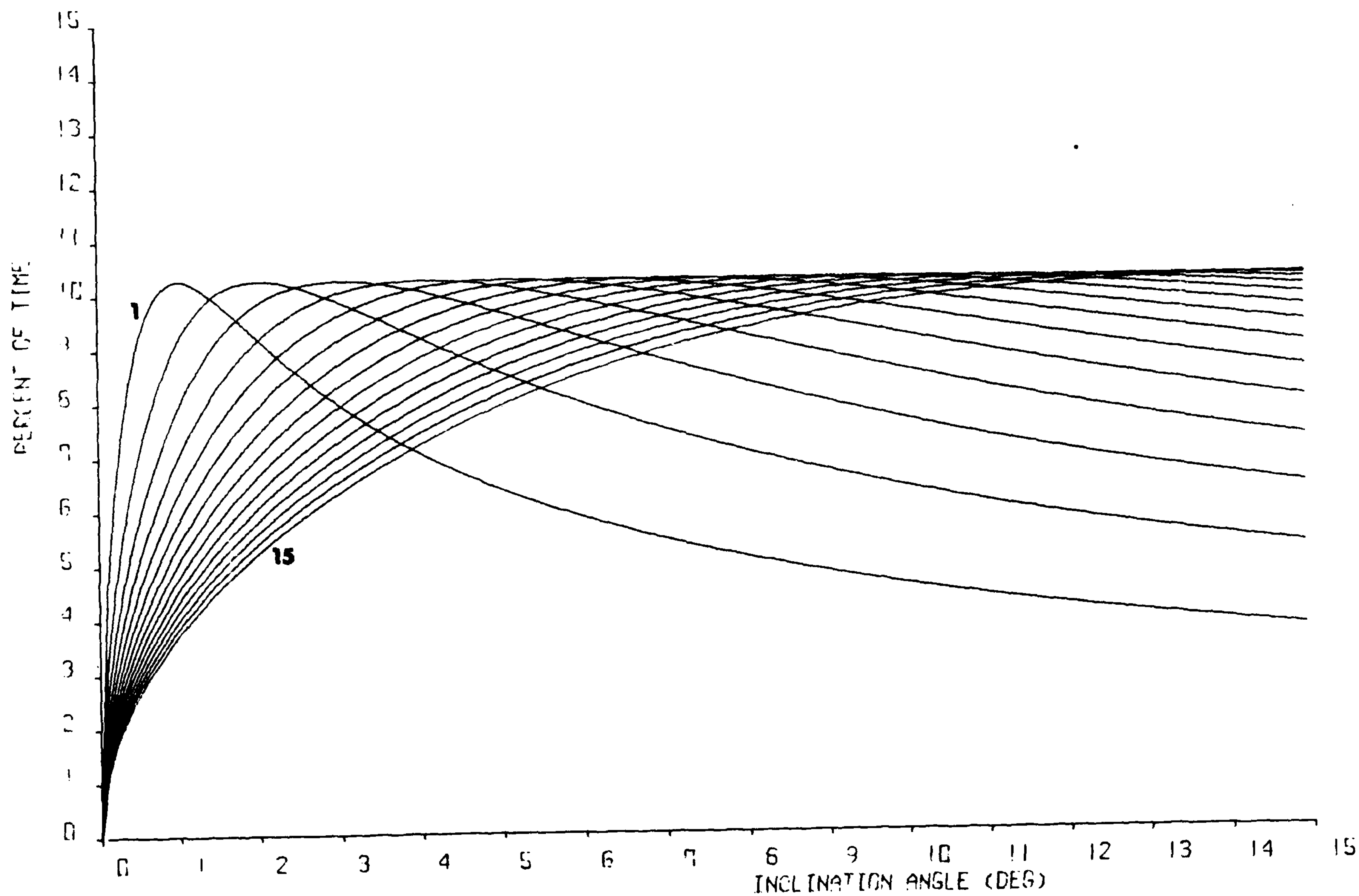


Figure 2-9(b): Percent of time the total separation angle is smaller than the equatorial plane value as a function of inclination angle. Nominal separation angle is 3 degrees.

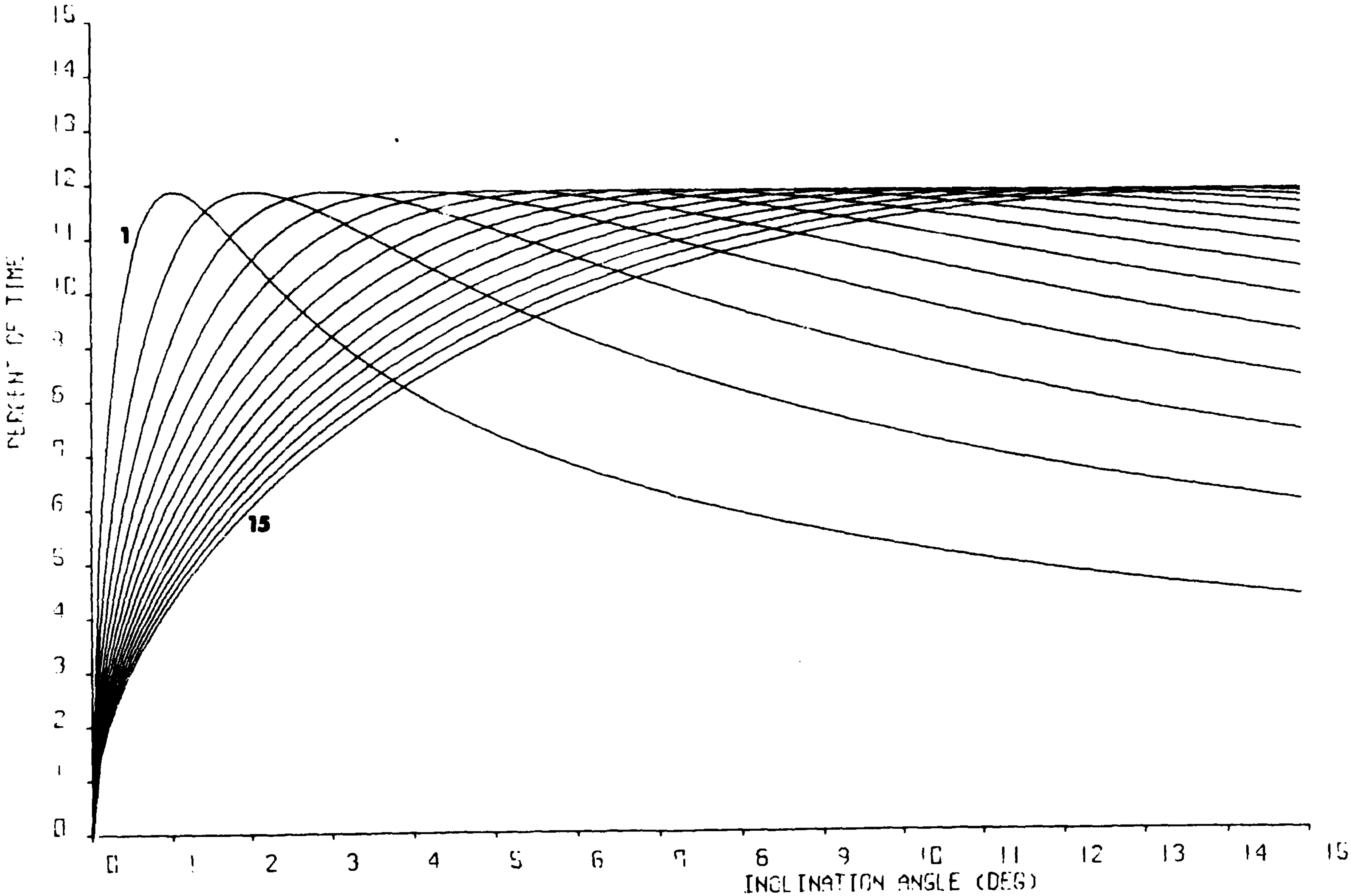


Figure 2-9(c): Percent of time the total separation angle is smaller than the equatorial plane value as a function of inclination angle. Nominal separation angle is 4 degrees.

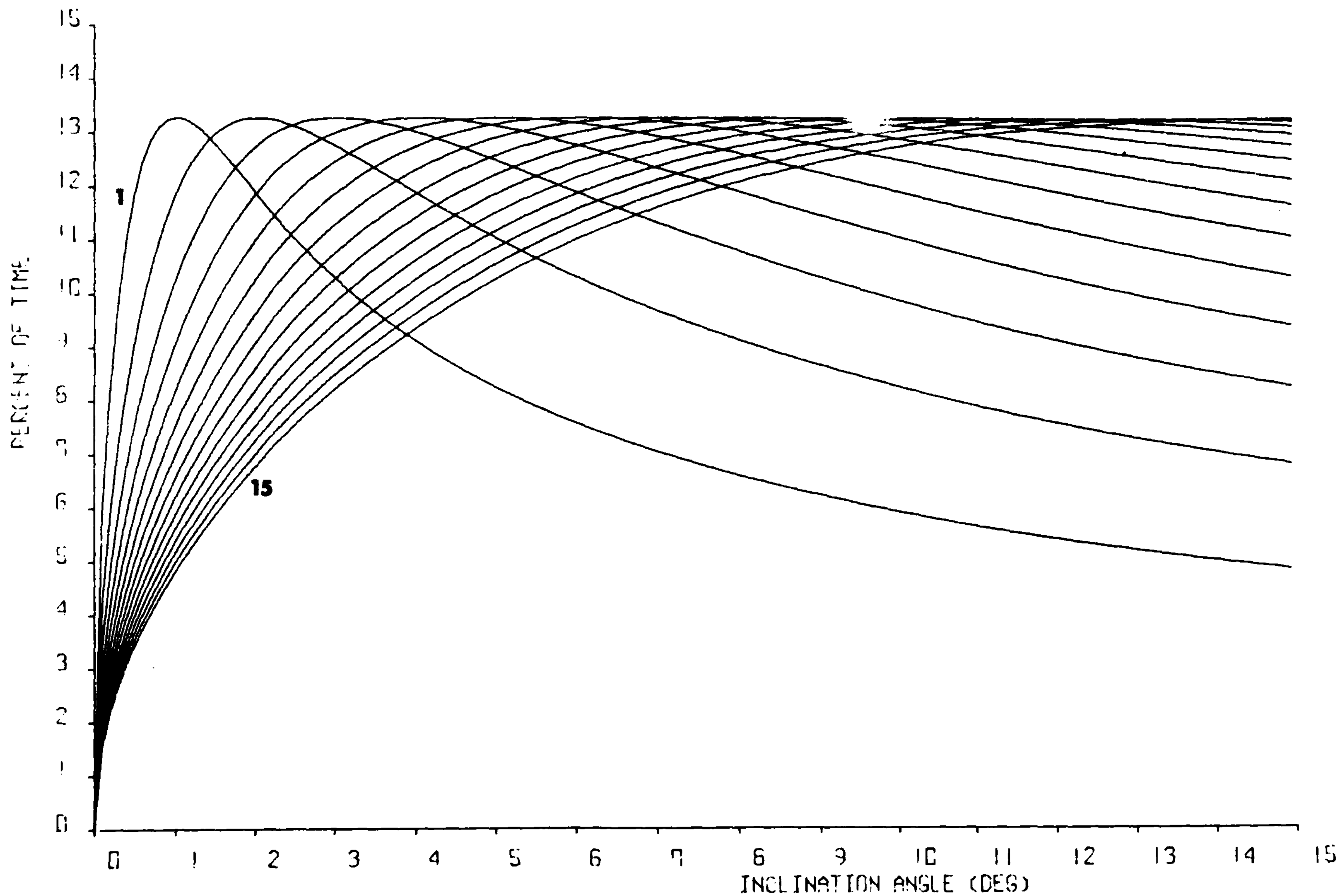


Figure 2-9(d): Percent of time the total separation angle is smaller than the equatorial plane value as a function of inclination angle. Nominal separation angle is 5 degrees.

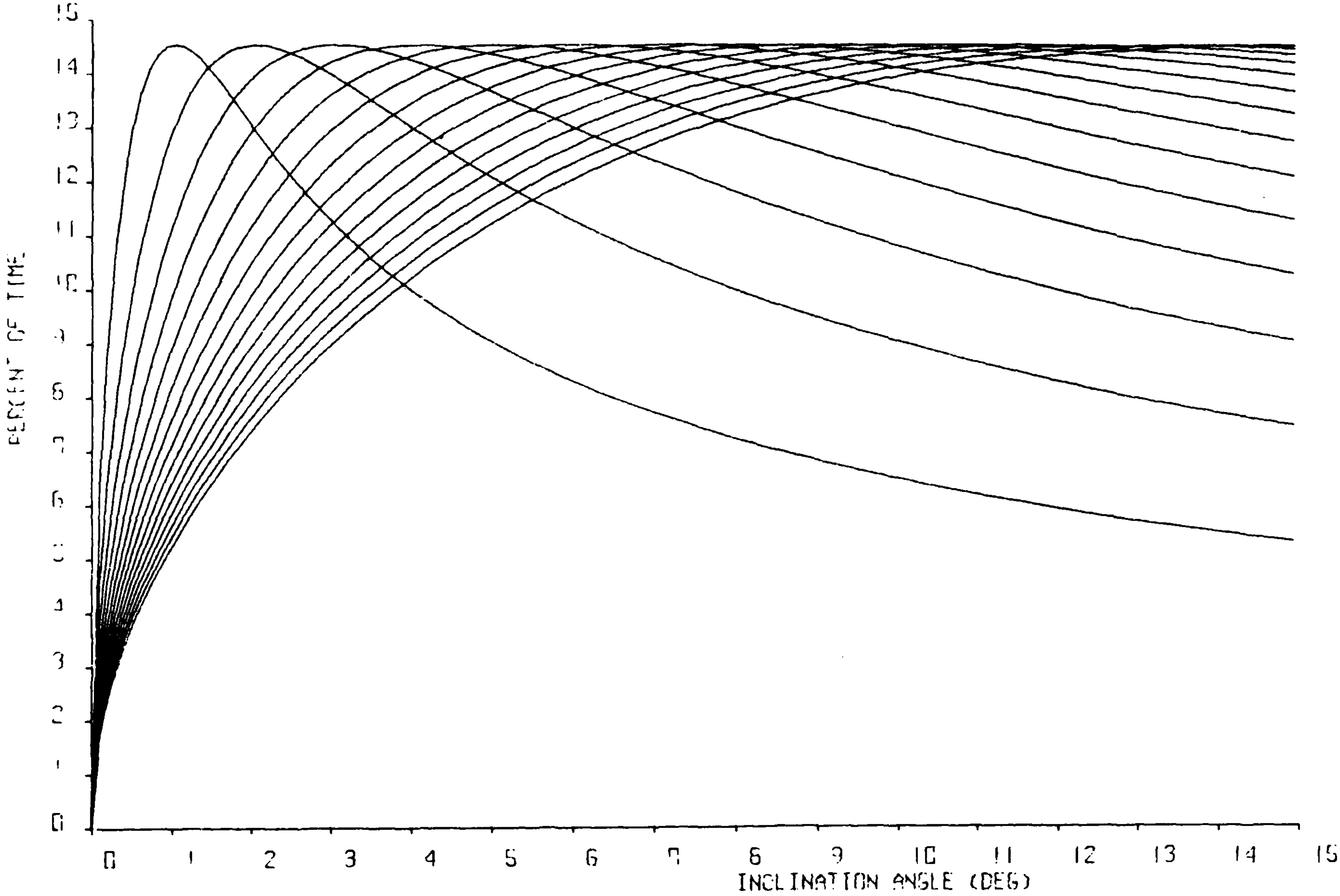


Figure 2-9(e): Percent of time the total separation angle is smaller than the equatorial plane value as a function of inclination angle. Nominal separation angle is 6 degrees.

2.6 Statistical estimates of changes in separation angle

The key to developing statistical estimates of changes in the total separation angle is to establish statistics of the nodal phase angle as well as the inclination angles of both satellites. Previously, it was shown that the change in total separation angle for any nodal phase angle and various inclination angles is given by:

$$\Delta\lambda = I_1 I_2/2 \text{ Sin}\gamma_0 \qquad (2\text{-}22)$$

It is not unreasonable to assume that the nodal phase angle and inclination angles are statistically independent. Using this assumption, the expected value of the change in total separation angle is:

$$\begin{aligned} E[\Delta\lambda\ ] &= E[\ I_1 I_2/2 \text{ Sin}\gamma_0\ ] \\ &= 1/2\ E[I_1]E[I_2]E[\text{Sin}\gamma_0\ ]. \end{aligned} \qquad (2\text{-}23)$$

Assuming that the nodal phase angle has a zero expected value gives the expected value of the change in total separation angle as:

$$E[\Delta\lambda\ ] = 0. \qquad (2\text{-}24)$$

Then the variance of the change in total separation angle is given by:

$$\begin{aligned} E[\ \Delta\lambda^2] &= E[\ I_1^2 I_2^2/4 \text{ Sin}^2\gamma_0\ ] \\ &= 1/4\ E[I_1^2\ ]E[I_2^2\ ]E[\text{Sin}^2\gamma_0\ ]. \end{aligned} \qquad (2\text{-}25)$$

Various assumptions will be applied to equation (2-25) to derive estimates of the deviation of the change in total separation angle.

The first set of assumptions is that the inclination angles are not random variables and that the nodal phase angle is uniformly distributed between $-\pi$ and π radians. Then:

$$E[\text{Sin}^2\gamma_0\ ] = 1/2$$

and:

$$E[\ \Delta\lambda^2\ ] = \sigma^2_{\Delta\lambda} = I_1^2 I_2^2/8 \qquad (2\text{-}26)$$

or in terms of deviation:

$$\sigma_{\Delta\lambda} = I_1 I_2 / \sqrt{8} \qquad (2\text{-}27)$$

The second set of assumptions is that the inclination angles are zero mean random variables and the nodal phase angle is

uniformly distributed between $-\pi$ and π radians. In this case, the deviation is given by:

$$\sigma_{\Delta\lambda} = \sigma_{I1}\sigma_{I2} / \sqrt{8} \qquad (2\text{-}28)$$

This equation is plotted in Figure 2-10 as contours of the deviation of the change in the total separation angle as a function of the deviations of the inclination angles.

As an example, if the inclination angles have guassian distributions with a deviations of 5 degrees, the deviation of the change in total separation angle can be read from Figure 2-10 as 0.15 degrees.

If the inclination angles have uniform distributions between -Im and Im, the deviation of the change in total separation angle is given by:

$$\sigma_{\Delta\lambda} = I_m^2/3\sqrt{8} \qquad (2\text{-}29)$$

Currently, allowable longitude station keeping error is defined as 0.1 degree for each satellite giving a total allowable error of 0.2 degrees. Solving equation (2-29) for the maximum inclination angle given a deviation of 0.2 degrees gives:

$$I = 9.9 \text{ degrees}$$

or a maximum inclination of 10 degrees provides a deviation in separation angle of no more than 0.2 degrees which is equivalent to the allowable station keeping error.

2.7 Summary

Figures 2-11(a) through 2-11(e) show the percent of time that the total separation angle is less than the equatorial plane value versus the worst case change in total separation angle (i.e., a nodal phase angle of 270 degrees). In Figure 2-11(a), the equatorial plane separation angle is 2 degrees increasing to 6 degrees in Figure 2-11(e). The inclination angle of satellite 2 varies from 1 to 15 degrees in one degree steps as shown by the 15 lines in the figure. For each of these 15 lines, the inclination of satellite 1 varies from 0 to 15 degrees continuously. The zero inclination angle value of satellite 1 is at the origin while the 15 degree inclination angle of satellite 1 lies on the curve labeled 15 which is the inclination angle of satellite 2.

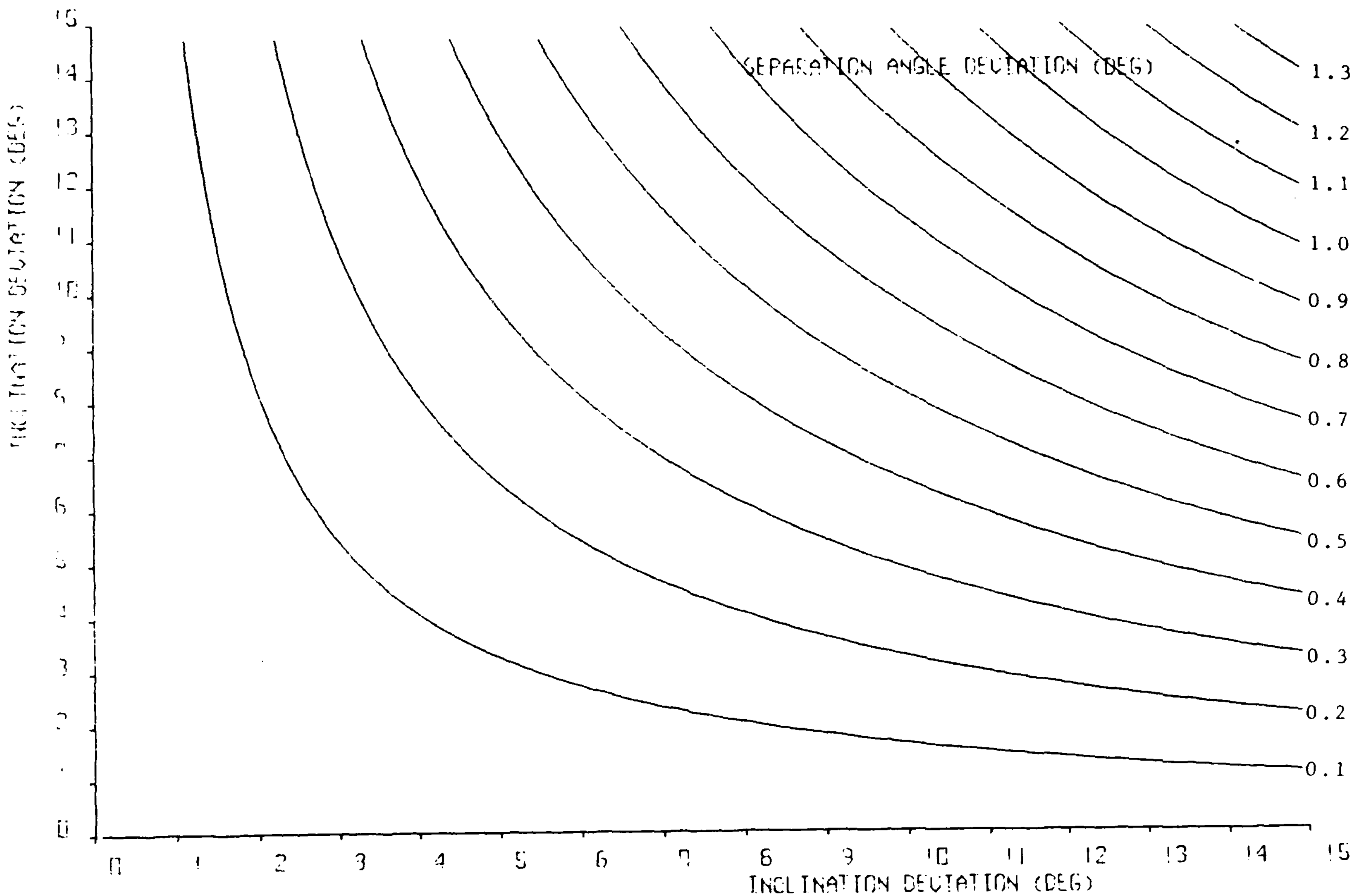


Figure 2-10: Deviation of the change in total separation angle from the equatorial value as a function of the deviations of the inclination angles. *The nodal phase* angle has a uniform distribution from -180 to 180 degrees.

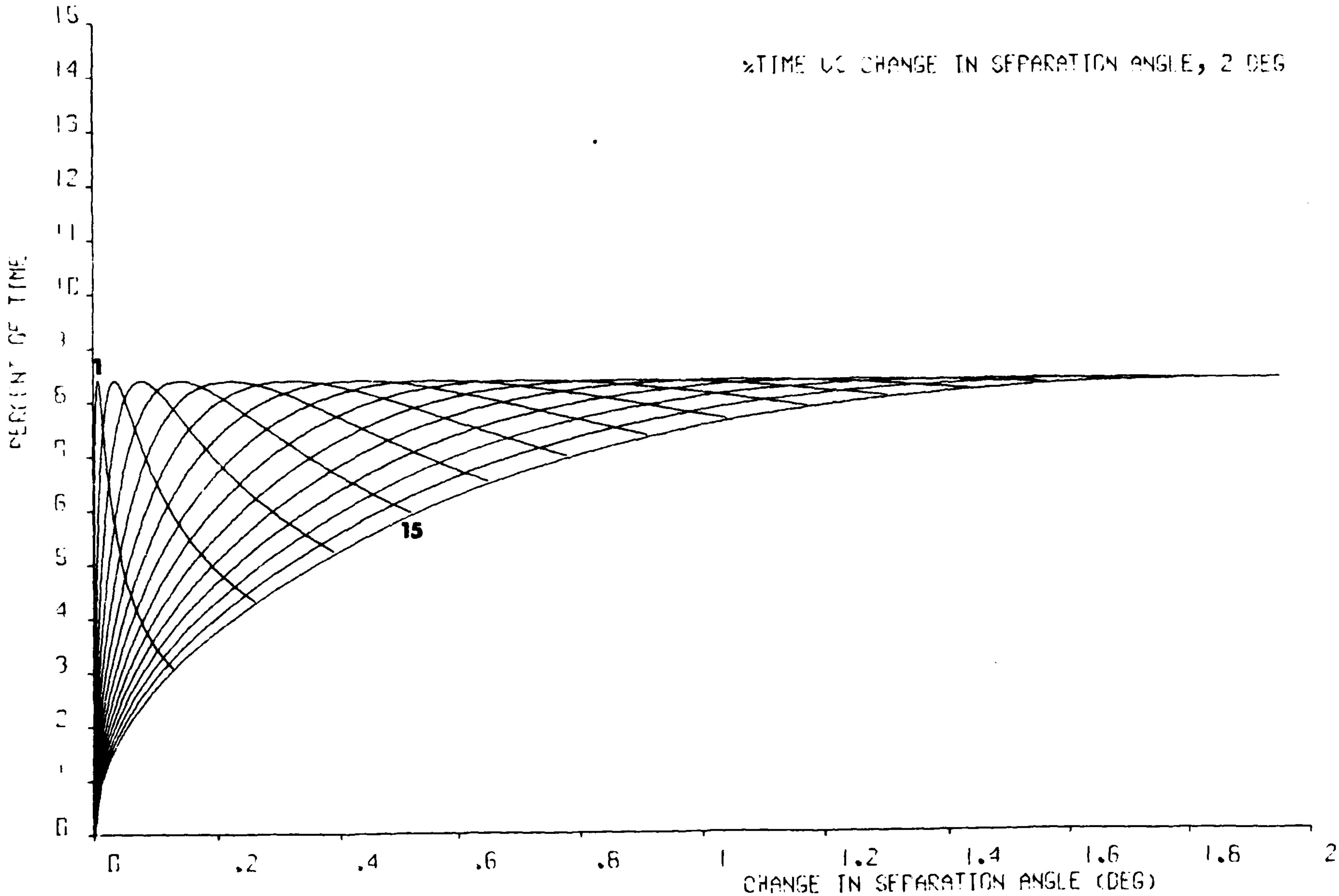


Figure 2-11(a): Percent of time that the total separation angle is less than nominal versus the worst case change in total separation angle as a function of inclination angles. Equatorial plane separation angle is 2 degrees.

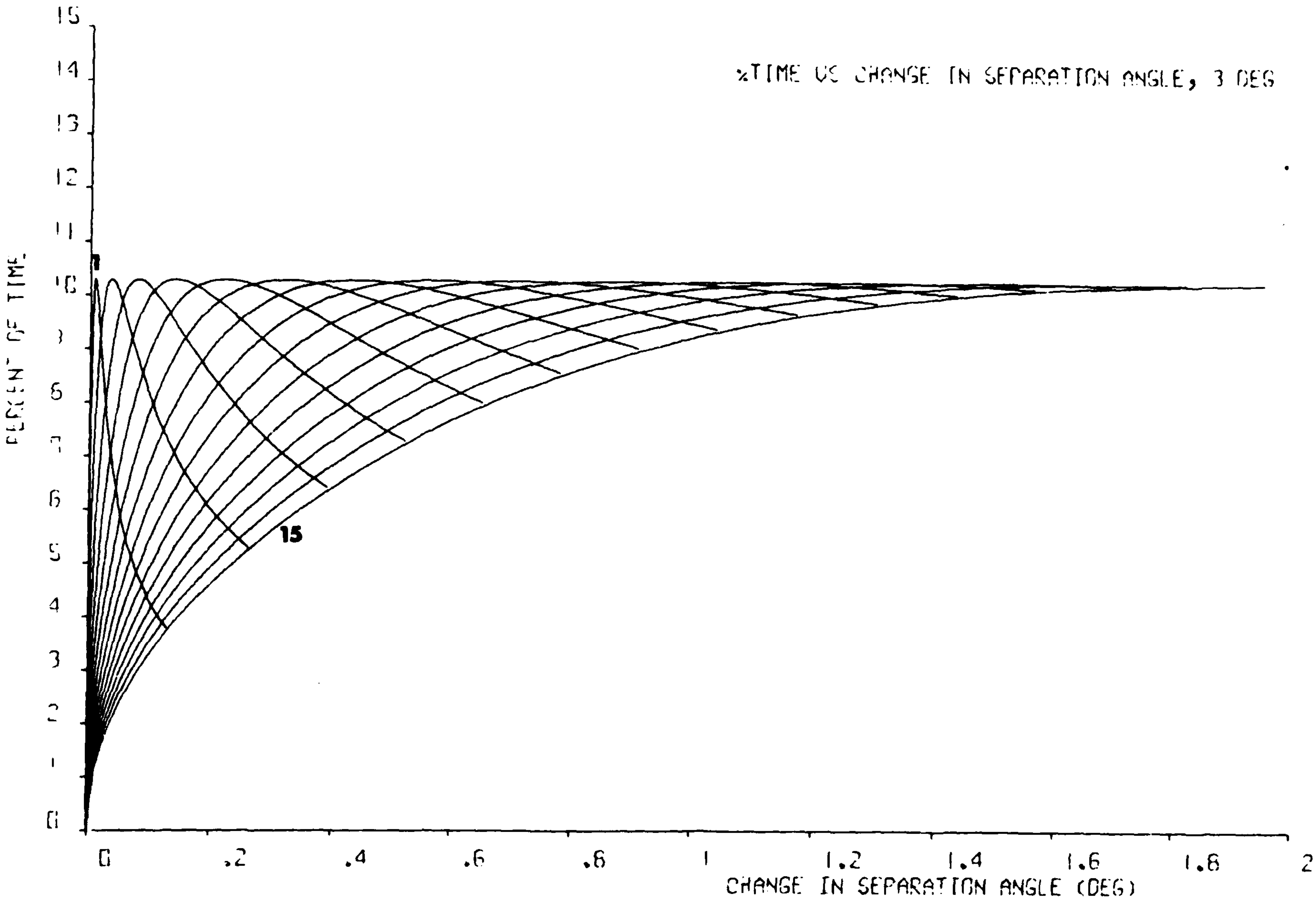


Figure 2-11(b): Percent of time that the total separation angle is less than nominal versus the worst case change in total separation angle as a function of inclination angles. Equatorial plane separation angle is 3 degrees.

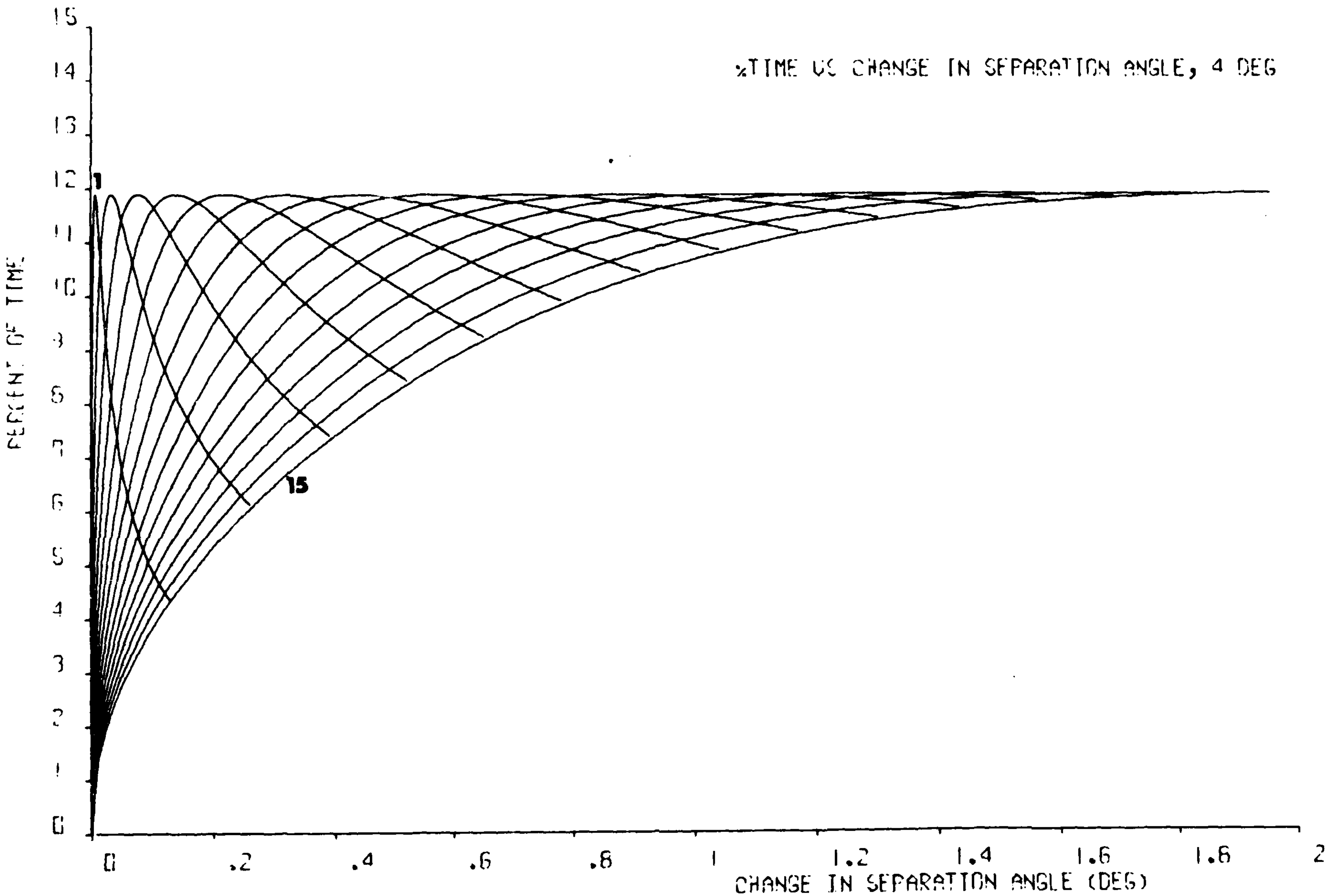


Figure 2-11(c): Percent of time that the total separation angle is less than nominal versus the worst case change in total separation angle as a function of inclination angles. Equatorial plane separation angle is 4 degrees.

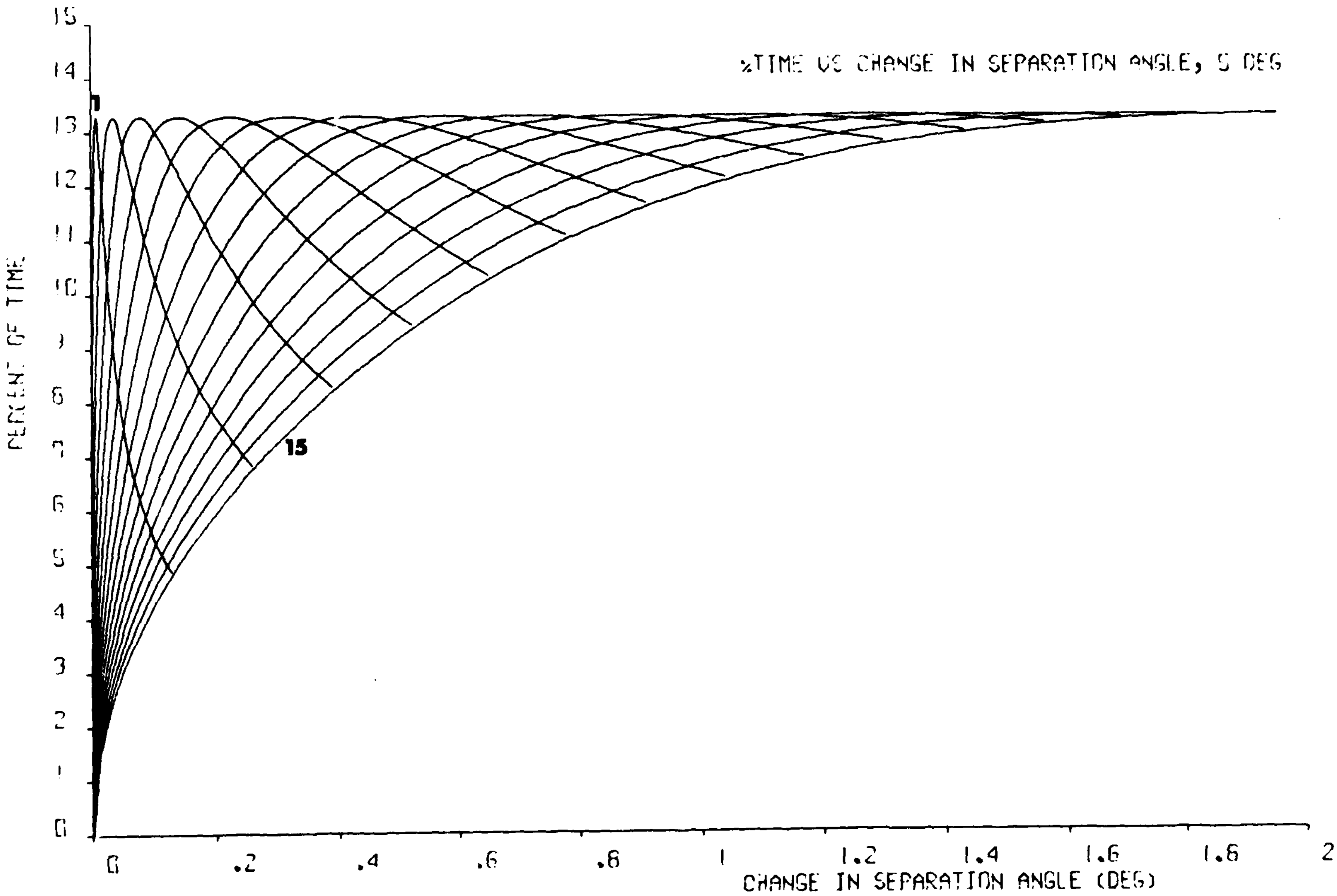


Figure 2-11(d): Percent of time that the total separation angle is less than nominal versus the worst case change in total separation angle as a function of inclination angles. Equatorial plane separation angle is 5 degrees.

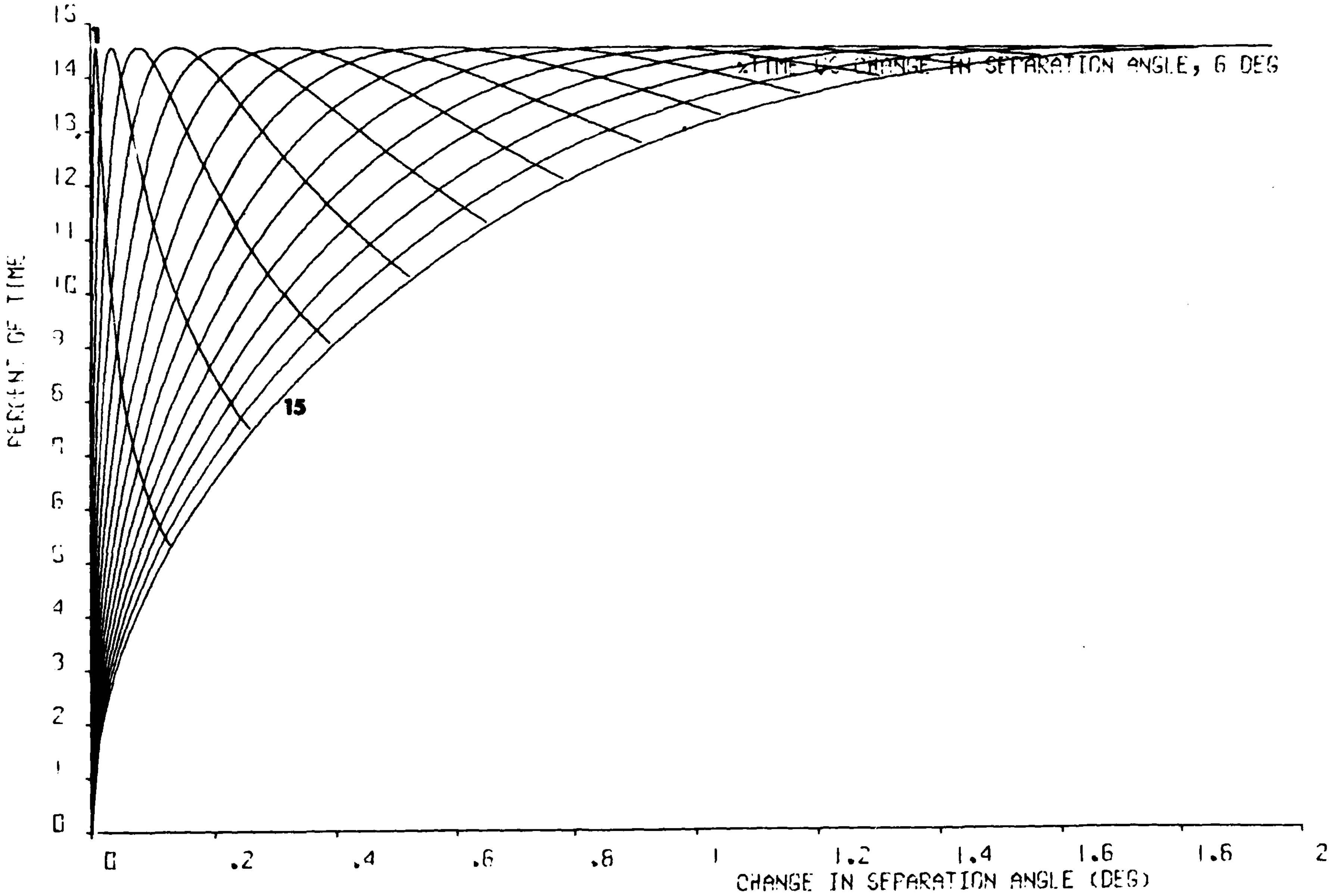


Figure 2-11(e): Percent of time that the total separation angle is less than nominal versus the worst case change in total separation angle as a function of inclination angles. Equatorial plane separation angle is 6 degrees.

Section 3

CALCULATION OF THE INCREASE IN EQUIVALENT NOISE TEMPERATURE DUE TO INCLINATION ANGLES

The change in total separation angle derived in Section 2 is converted to an increase in equivalent noise temperature using the Delta T/T method of Appendix 29 of the Radio Regulations. After a brief review of Delta T/T calculations, the increase in equivalent noise temperature as a result of the worst case change in total separation angle is derived. Various additional plots are provided to summarize the increase in equivalent noise temperature due to inclination angles. This section concludes with statistical estimates of the increase in equivalent noise temperature.

3.1 Delta T/T

An approximate method, the Delta T/T method, can be used for predicting if interference between geostationary-satellite networks will occur. This method is defined in detail in Appendix 29 of the Radio Regulations and is based on CCIR Report 454-3 and its modifications.

The Delta T/T method is based on the concept that the noise temperature of the system subject to interference undergoes an apparent increase due to the effect of the interference. The interfering signals are treated as thermal noise whose power spectral density is equal to the maximum power spectral density of the signals. The method is applicable whenever two geostationary-satellites share a portion of the frequency band on at least one of their paths and can be used for any modulation characteristics of the satellite networks concerned independent of the precise frequencies employed.

The separation angle between the satellites is used to determine the sidelobe levels or the antenna gain in the direction of the other satellite. As the sidelobe level increases, the interference into the victim receiver increases or the equivalent noise temperature increases by the increased sidelobe level. Then, in order to convert a change in total separation angle to an increase in equivalent noise temperature, the increase in sidelobe level can be determined.

In Annex III of Appendix 29, the sidelobe level of the reference antennas is given as:

$$\text{Level (dB)} = -25 * \text{Log}\ \lambda \qquad (3\text{-}1)$$

where λ is the total separation angle between the satellites. The change in total separation angle, $\Delta\lambda$, can be used in equation (3-1) to compute the change in sidelobe level as:

$$\text{Change in Level} = 25*\text{Log}(\theta_S + \Delta\lambda) - 25*\text{Log}\theta_S \qquad (3\text{-}2)$$

Equation (3-2) is used in the following paragraphs to examine the increase in equivalent noise temperature due to inclination angles of the satellites.

3.2 Increase in noise temperature due to worst case separation angle

Equation (3-2) was used with the worst case change in total separation angle given by:

$$\Delta\lambda = I_1 I_2/2 \qquad (3\text{-}3)$$

to compute the increase in equivalent noise temperature. Figures 3-1(a) through (e) show contours of constant increases in the equivalent noise temperature as a function of the inclination angles of both satellites for the worst nodal phase angle. Figure 3-1(a) is for an equatorial plane separation angle of 2 degrees. The equatorial plane separation angle increases in 1 degree steps to 6 degrees in Figure 3-1(e). As an example, for inclination angles of 10 degrees for both satellites, the increase in equivalent noise temperature is:

Separation Angle	Increase in Noise temperature
2	3.9 dB
3	2.8
4	2.1
5	1.7
6	1.5

3.3 Increase in noise temperature as a function of nodal phase angle

Equation (3-2) was used with the total separation angle given by:

$$\Delta\lambda = I_1 I_2/2 \ \text{Sin}\gamma_0 \qquad (3\text{-}4)$$

to compute the increase in equivalent noise temperature as a function of the nodal phase angle, i.e., the angular difference between ascending nodes. Figures 3-2(a) through 3-2(c) show the increase in equivalent noise temperature as a function of nodal phase angle for various inclination angles. The inclination angle of satellite 2 is 5 degrees in Figure 3-2(a) increasing to 15 degrees in Figure 3-2(c). The inclination angle of satellite 1 varies from 0 to 15 degrees representing the 16 lines on the plot. The equatorial plane separation angle is 2 degrees for these figures. Notice that for nodal phase angles less than 180

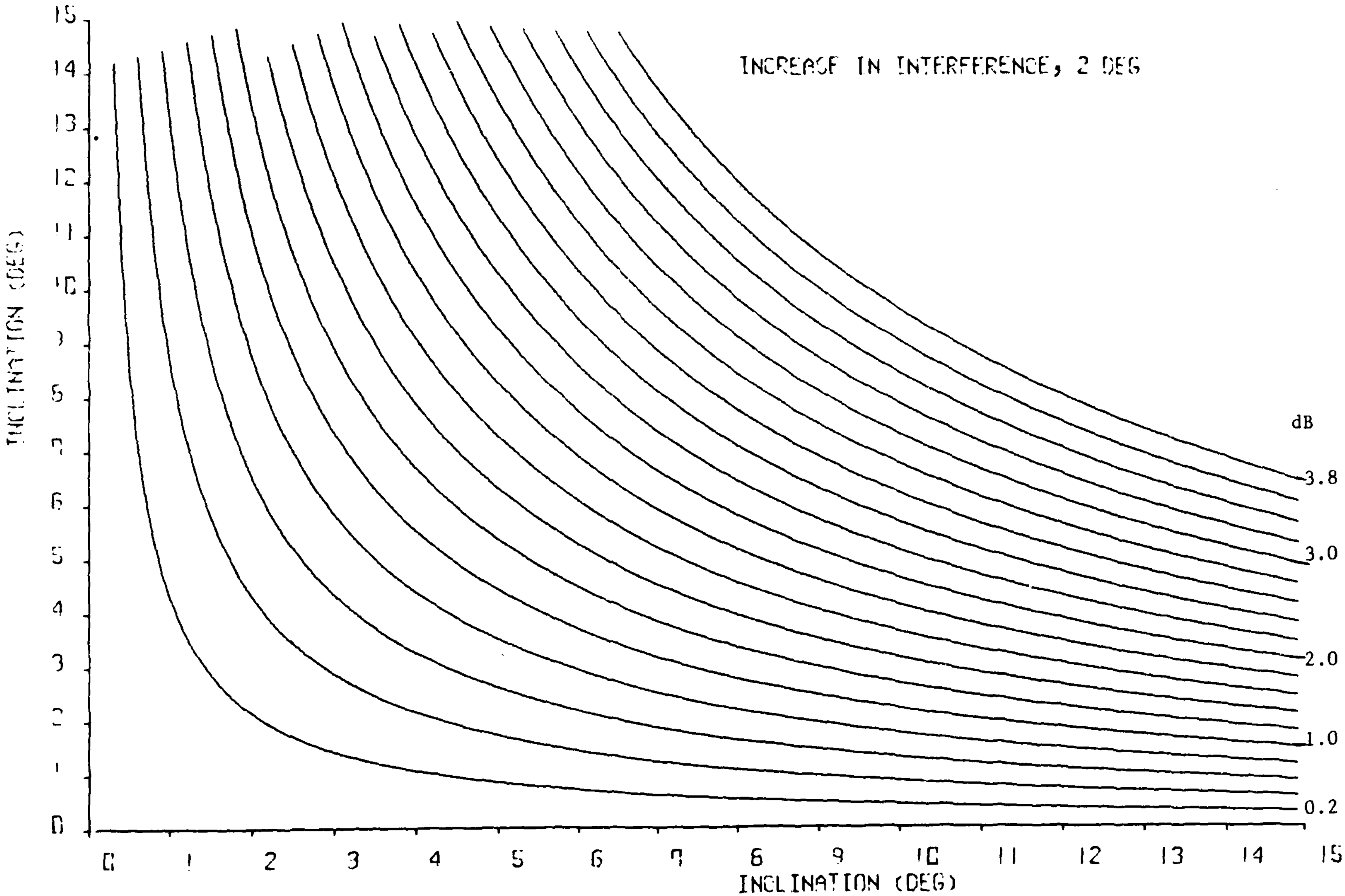


Figure 3-1(a): Increase in equivalent noise temperature for the worst case total separation angle as a function of inclination angles. Equatorial plane separation is 2 degrees.

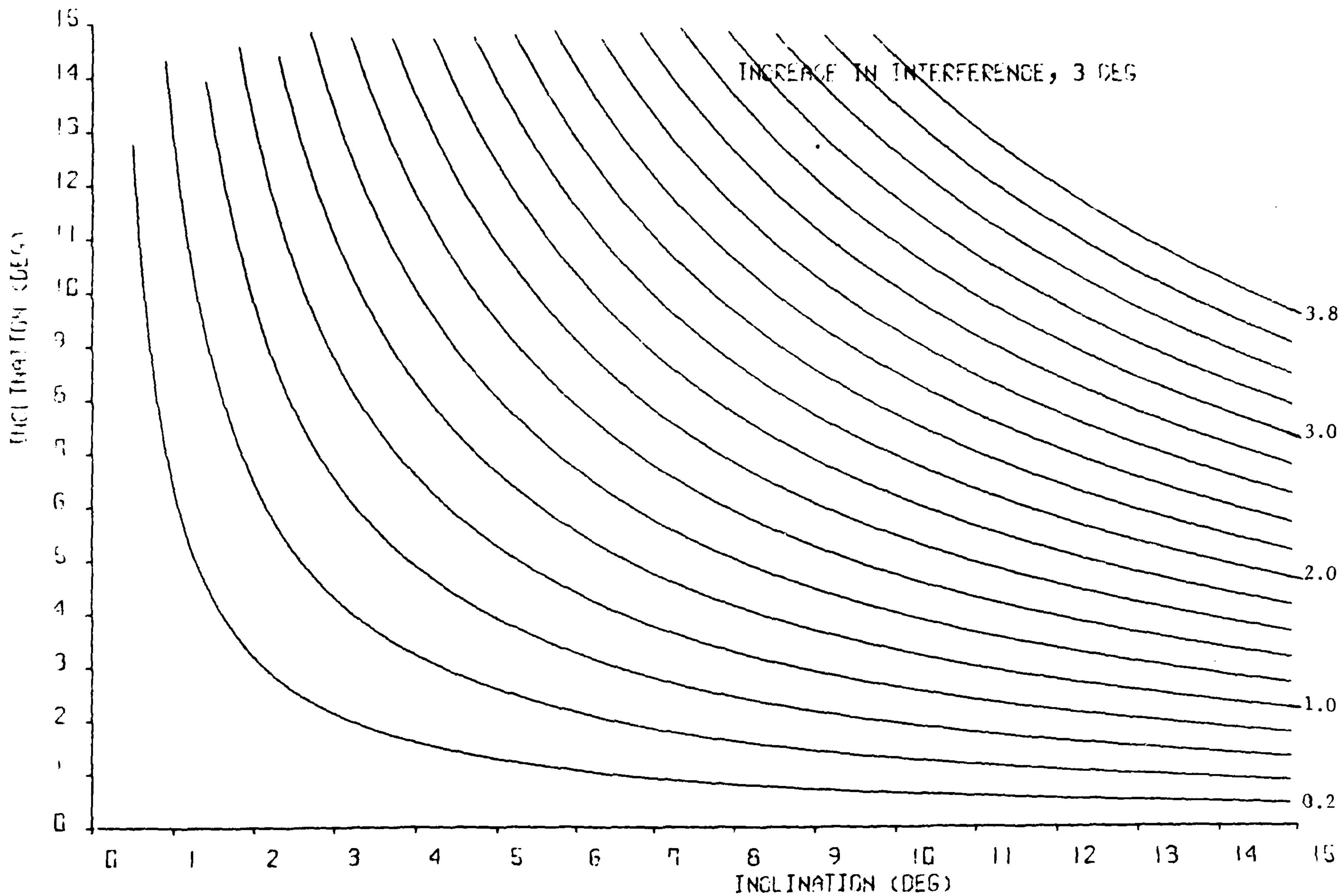


Figure 3-1(b): Increase in equivalent noise temperature for the worst case total separation angle as a function of inclination angles. Equatorial plane separation is 3 degrees.

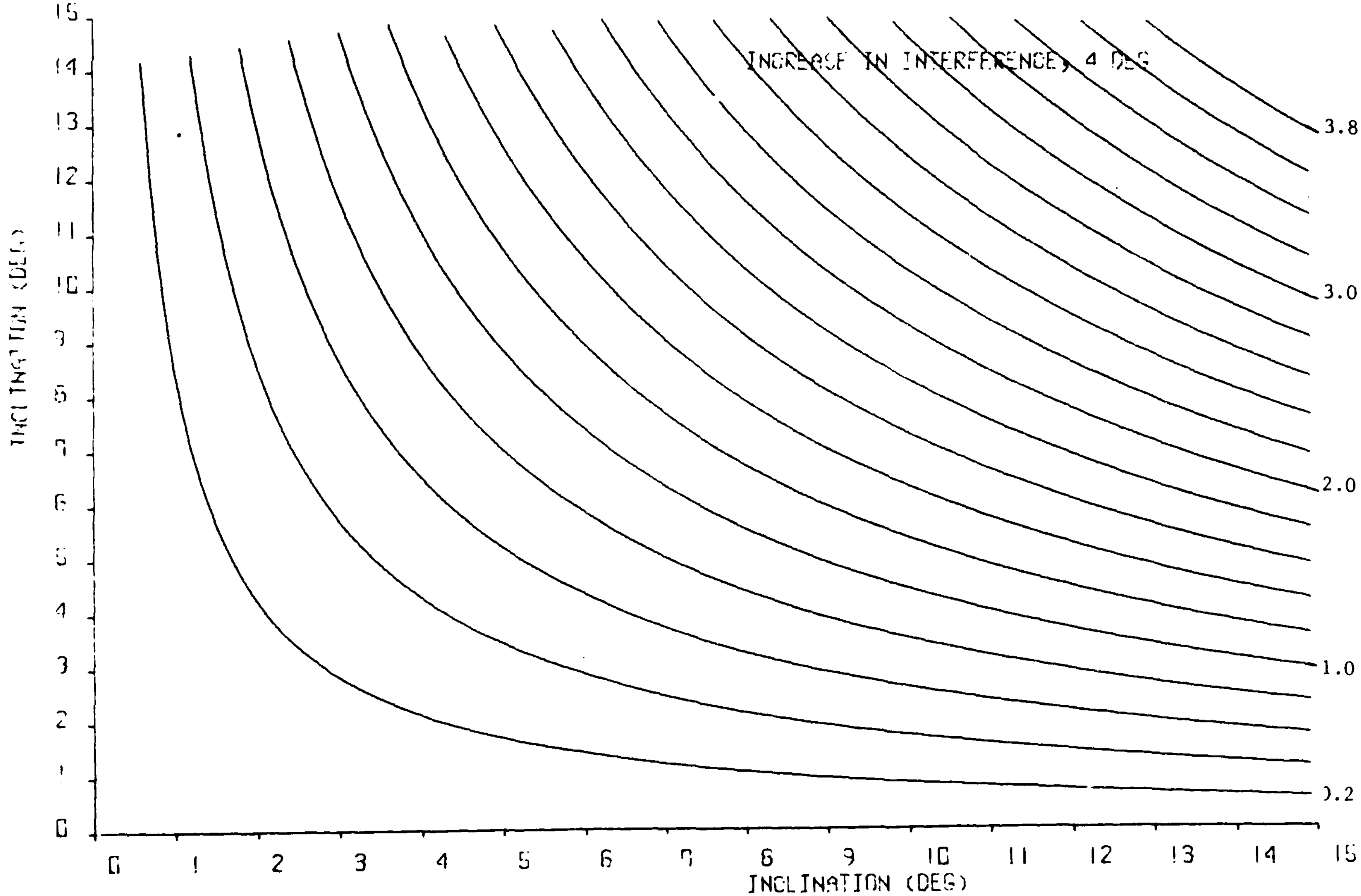


Figure 3-1(c): Increase in equivalent noise temperature for the worst case total separation angle as a function of inclination angles. Equatorial plane separation is 4 degrees.

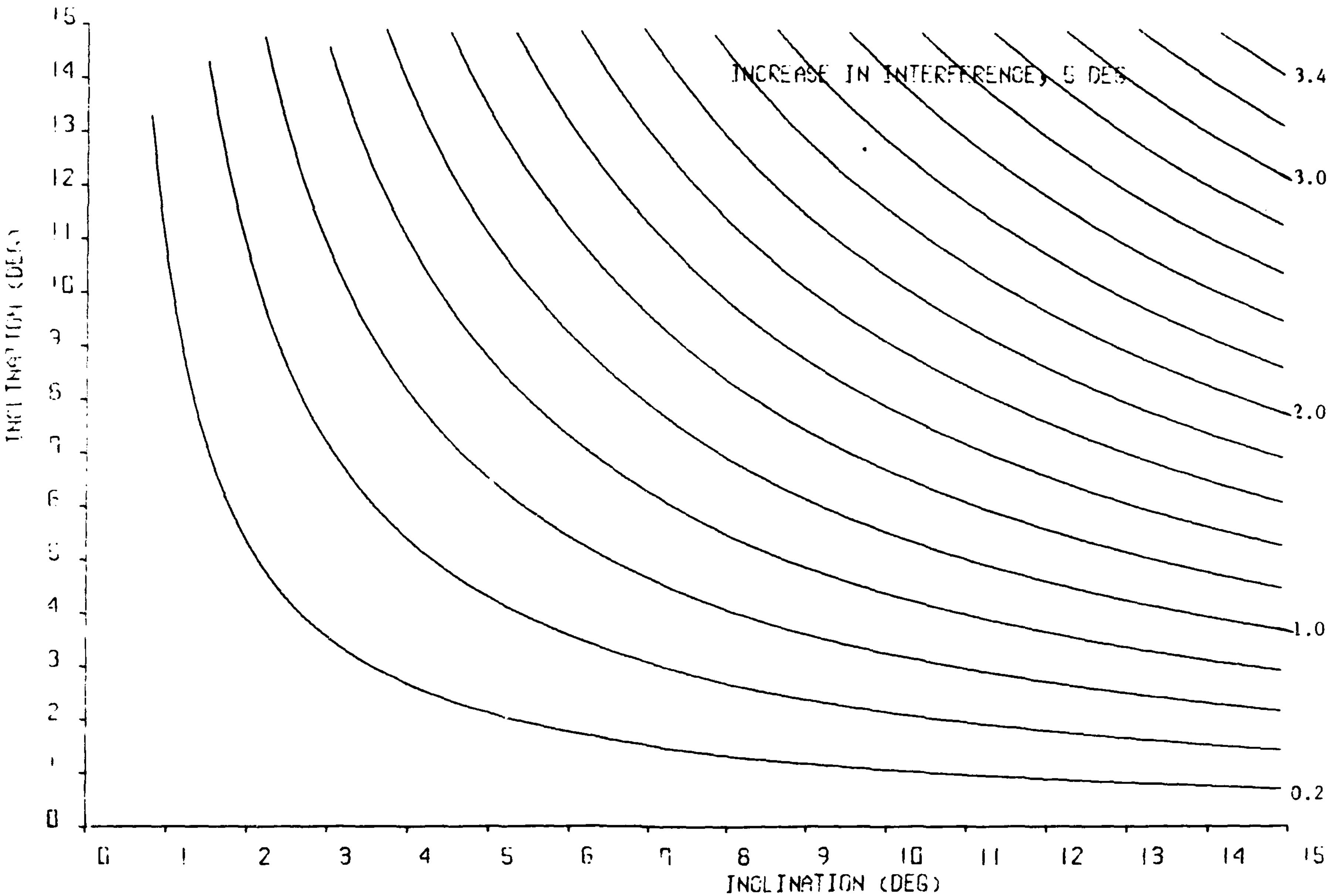


Figure 3-1(d): Increase in equivalent noise temperature for the worst case total separation angle as a function of inclination angles. Equatorial plane separation is 5 degrees.

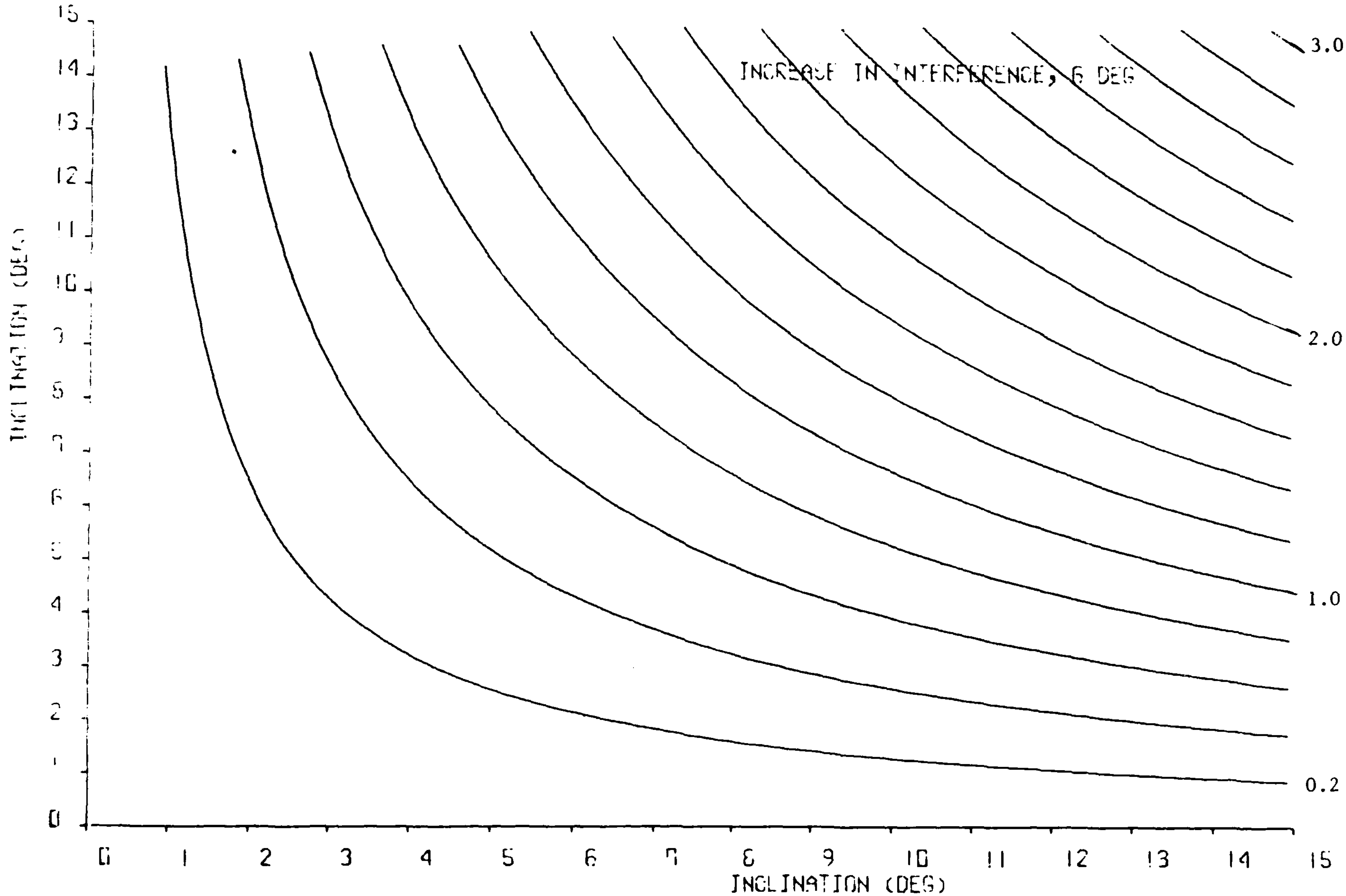


Figure 3-1(e): Increase in equivalent noise temperature for the worst case total separation angle as a function of inclination angles. Equatorial plane separation is 6 degrees.

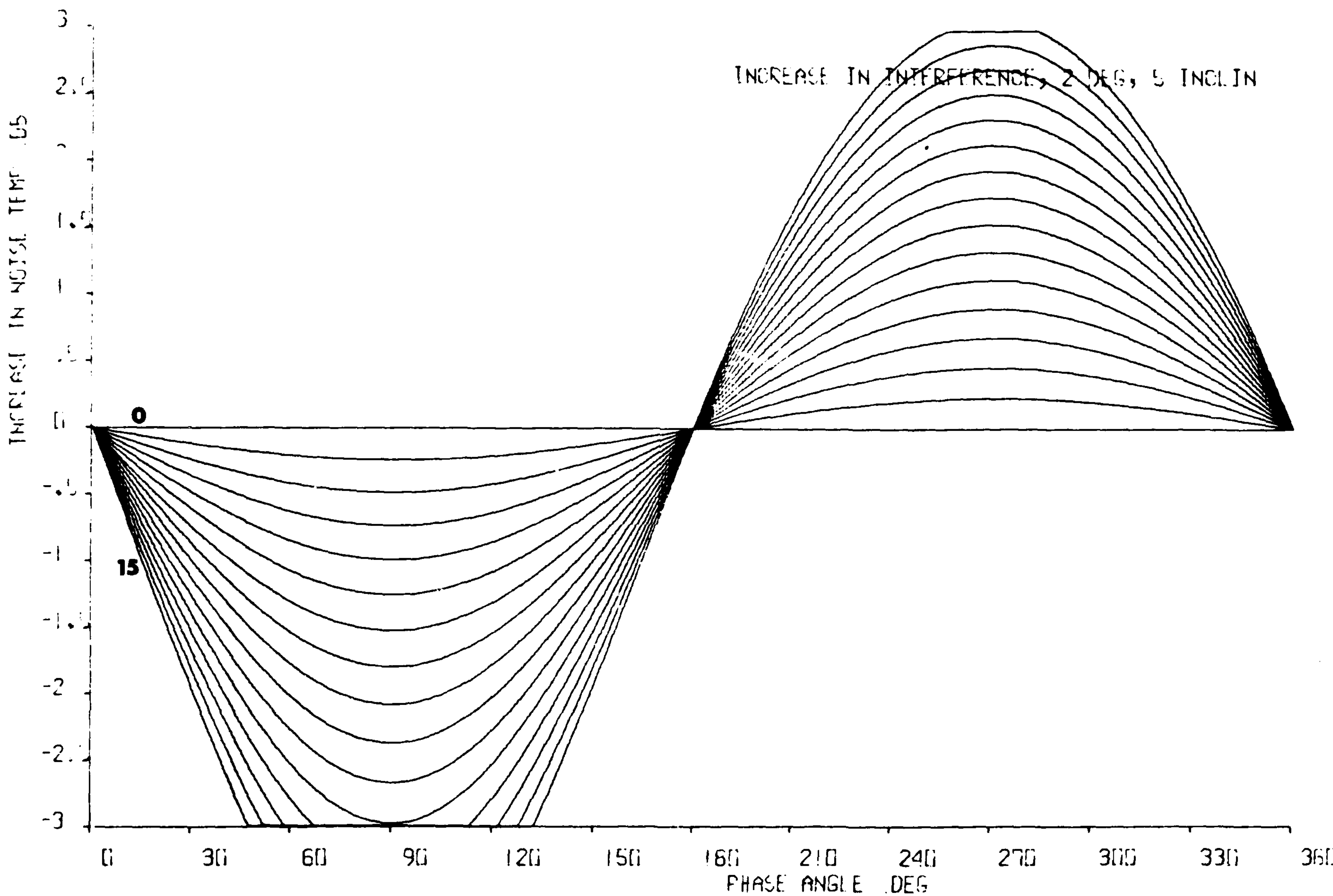


Figure 3-2(a): Increase in equivalent noise temperature as a function of nodal phase angle. The equatorial plane separation angle is 2 degrees and the inclination angle of satellite 2 is 5 degrees.

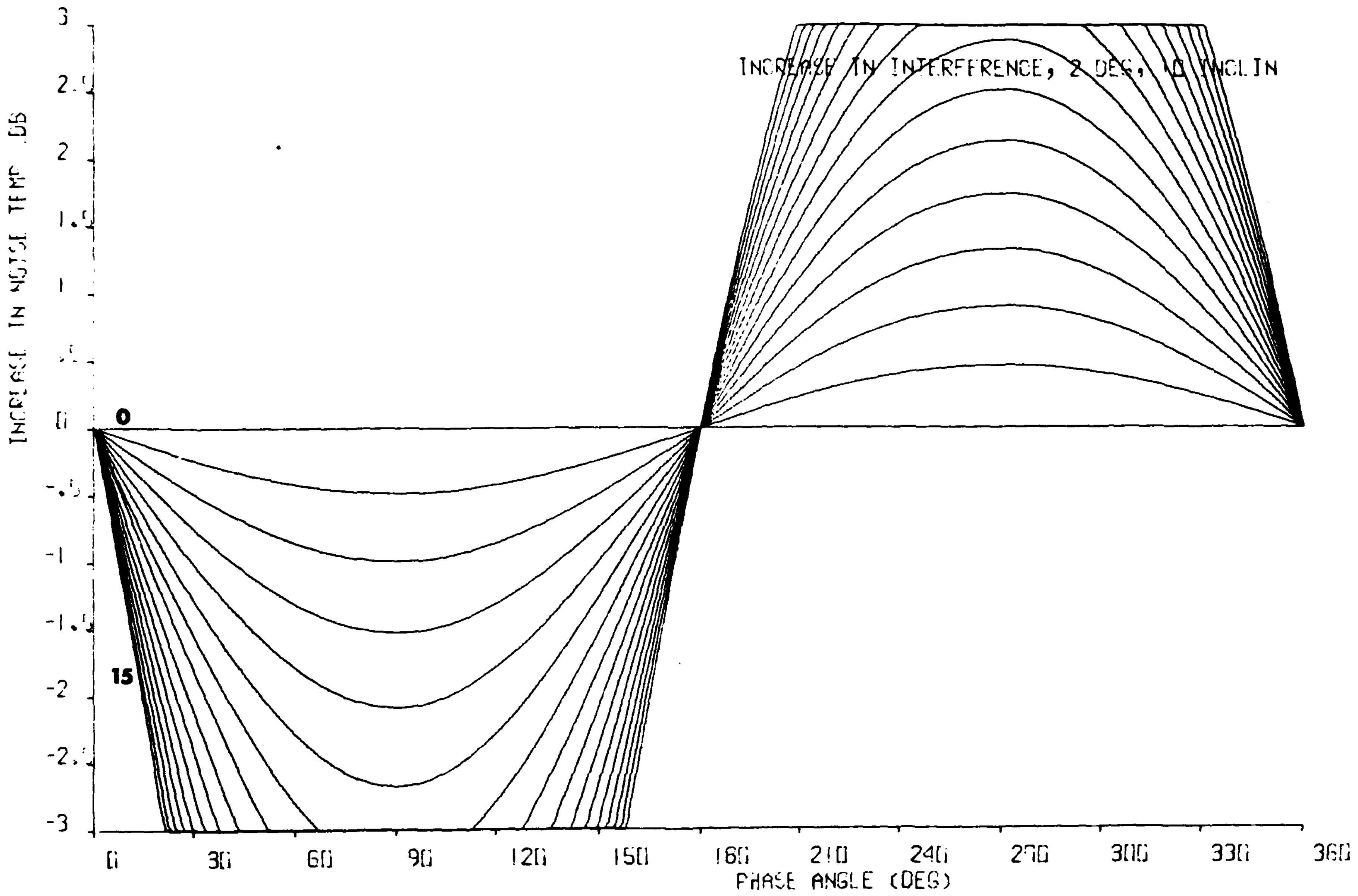


Figure 3-2(b): Increase in equivalent noise temperature as a function of nodal phase angle. The equatorial plane separation angle is 2 degrees and the inclination angle of satellite 2 is 10 degrees.

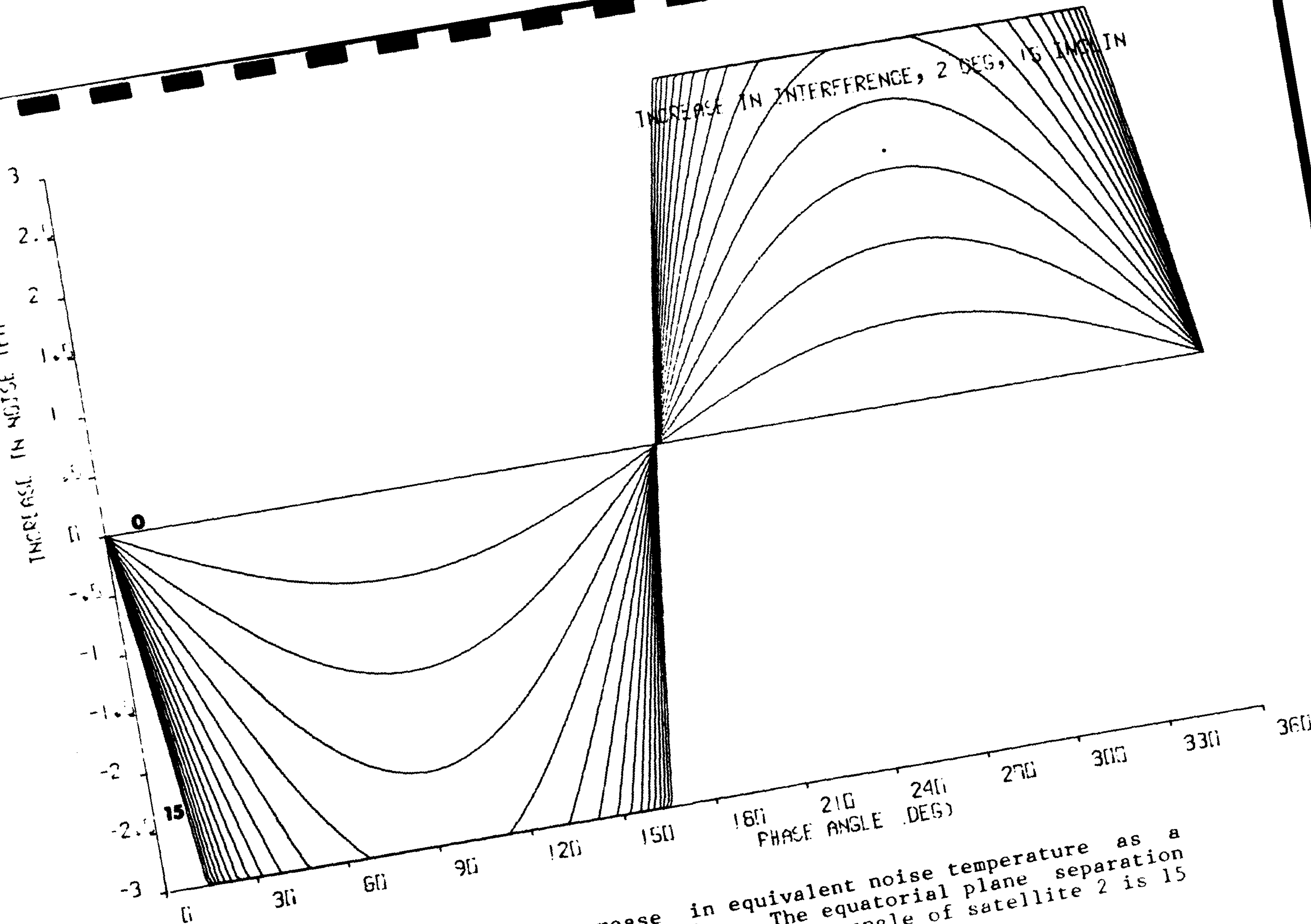


Figure 3-2(c): Increase in equivalent noise temperature as a function of nodal phase angle. The equatorial plane separation angle is 2 degrees and the inclination angle of satellite 2 is 15 degrees.

degrees, the equivalent noise temperature is decreased for any inclination angle.

Figures 3-3(a) through 3-3(d) show the increase in equivalent noise temperature as a function of nodal phase angle and the variation with equatorial plane separation angle. The inclination angle of satellite 2 is 10 degrees for each figure. The equatorial plane separation angle is 3 degrees in Figure 3-3(a) increasing to 6 degrees in Figure 3-3(d). The inclination angle of satellite 1 varies from 0 to 15 degrees in each plot.

3.4 Increase in noise temperature due to inclination angles

Equation (3-2) was used in conjunction with the total separation angle given by equation (3-3) and the percent of time the total separation angle is less than the equatorial plane separation angle given by:

$$\%Time = 200/\pi \sqrt{I_1 I_2 \theta_S / (I_1^2 + I_2^2)} \qquad (3\text{-}5)$$

to compute the increase in equivalent noise temperature as a function of the inclination angles of both satellites. The variables in this set of equations are:

- o inclination angle of satellite 1,
- o inclination angle of satellite 2,
- o equatorial plane separation angle between satellite 1 and 2
- o percent of time the total separation angle is less than the equatorial plane separation angle and
- o the increase in equivalent noise temperature.

Figures 3-4(a) through 3-4(e) show the increase in equivalent noise temperature as a function of the percent of time the total separation angle is less than the equatorial plane separation angle. Figure 3-4(a) is for an equatorial plane separation angle of 2 degrees increasing to 6 degrees in Figure 3-4(e). The inclination angle of satellite 2 varies from 1 to 15 degrees in each figure as shown by the 15 lines in each plot. The inclination angle of satellite 1 varies from 0 to 15 degrees along each line in each figure, i.e., at the origin the inclination angle of satellite 1 is 0 degrees and at the end of each line the inclination angle is 15 degrees. Notice that the increase in equivalent noise temperature is based upon the worst case change in total separation angle.

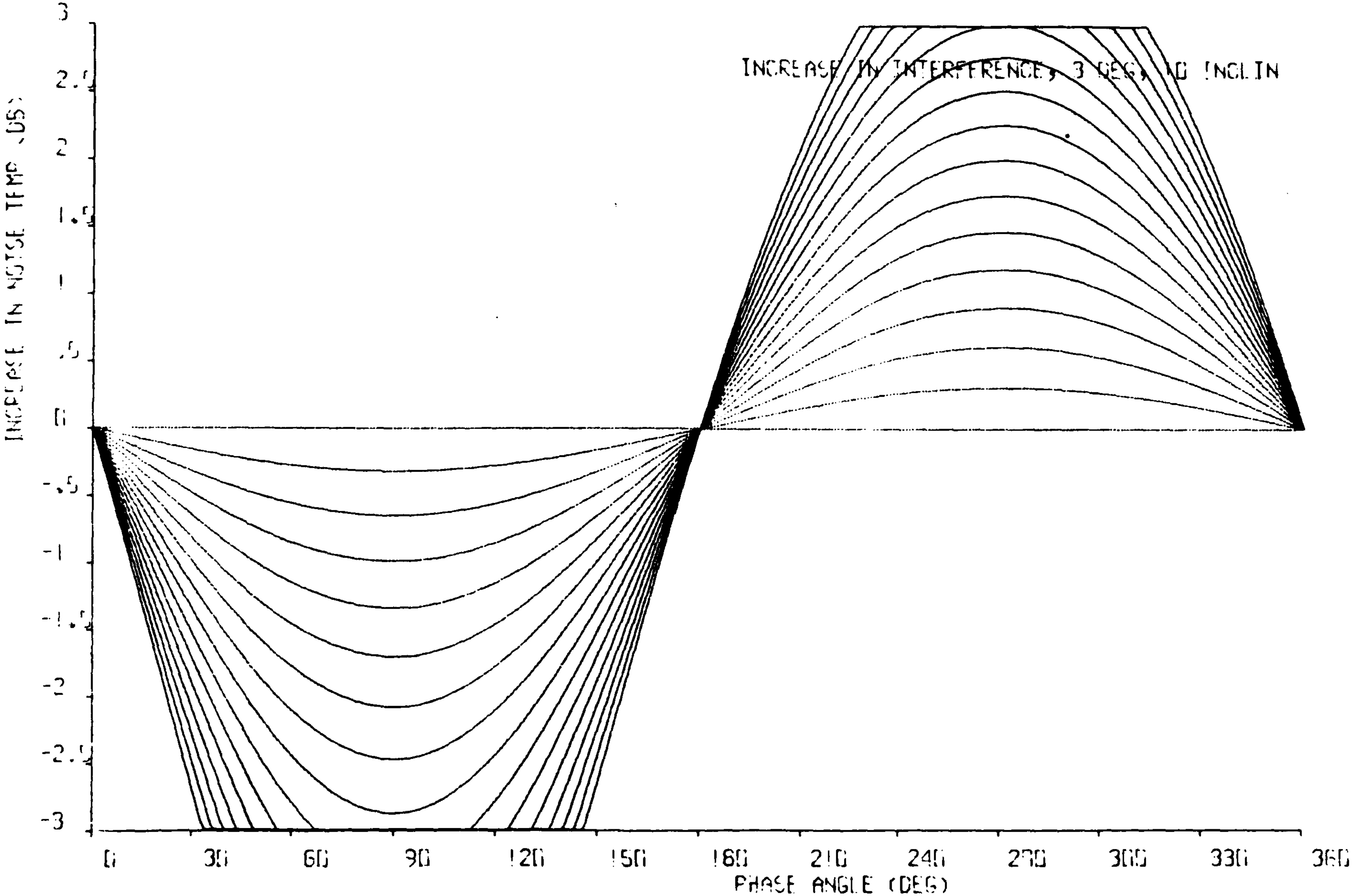


Figure 3-3(a): Increase in equivalent noise temperature as a function of nodal phase angle. The equatorial plane separation angle is 3 degrees and the inclination angle of satellite 2 is 10 degrees.

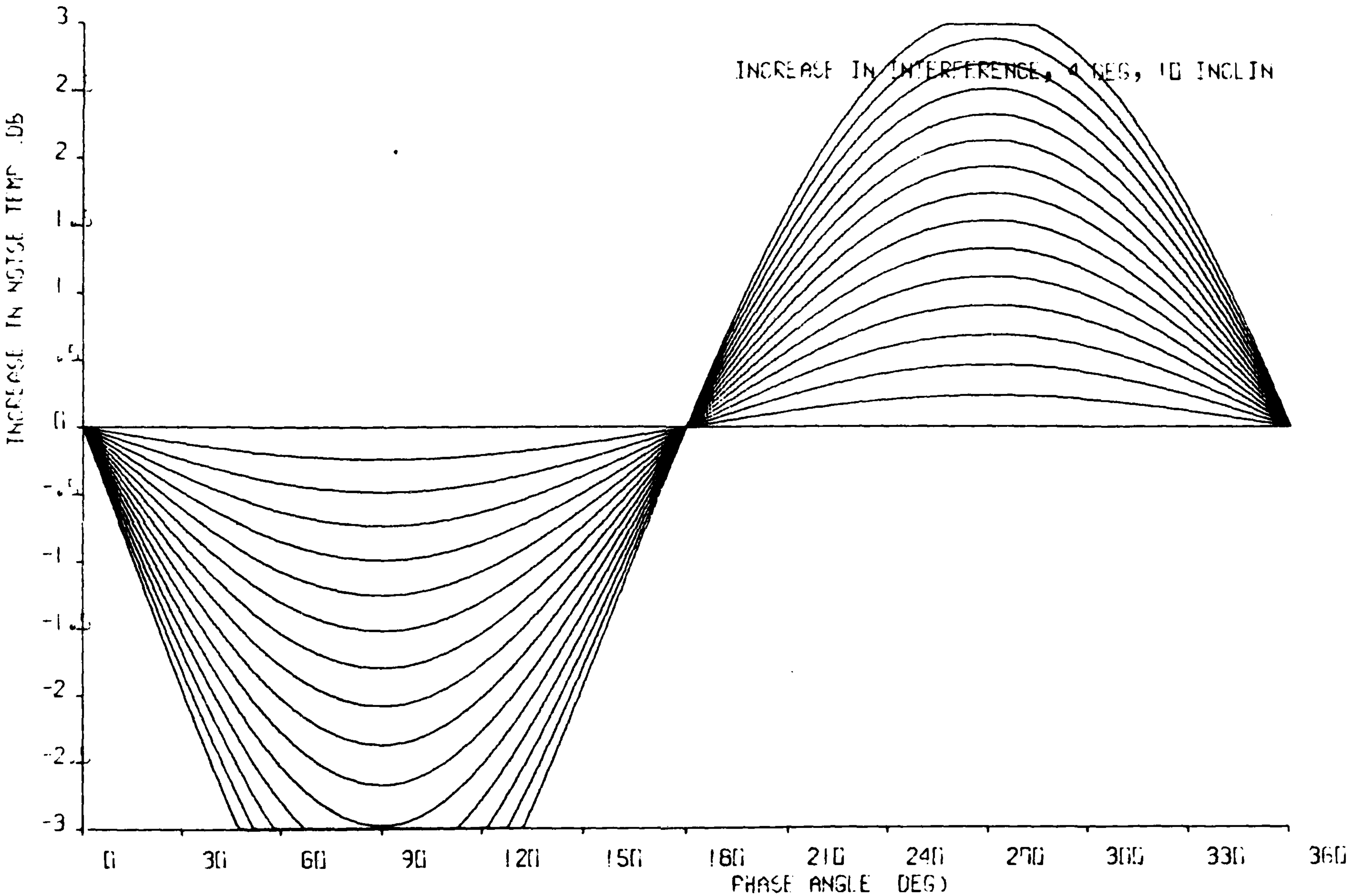


Figure 3-3(b): Increase in equivalent noise temperature as a function of nodal phase angle. The equatorial plane separation angle is 4 degrees and the inclination angle of satellite 2 is 10 degrees.

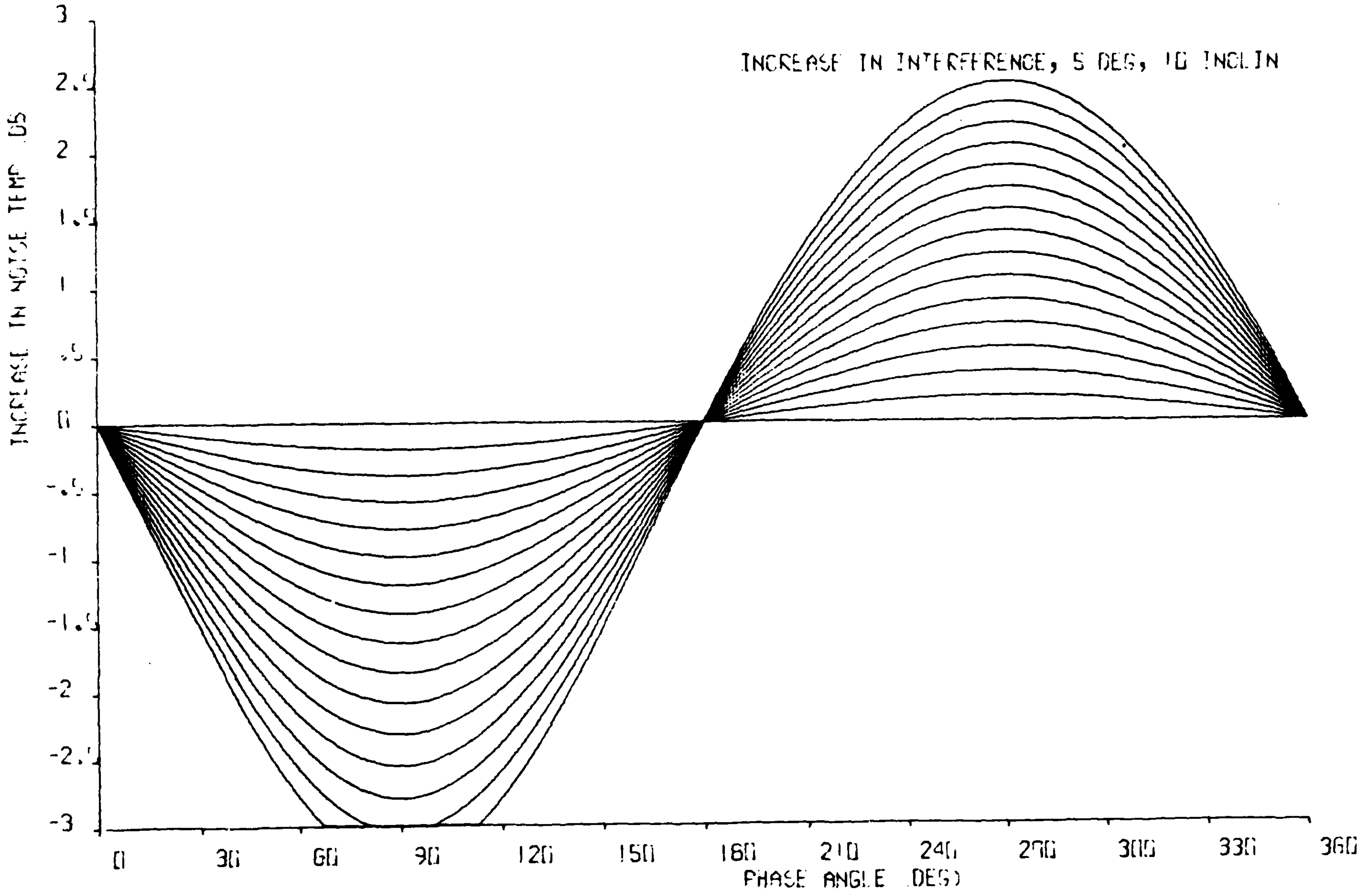


Figure 3-3(c): Increase in equivalent noise temperature as a function of nodal phase angle. The equatorial plane separation angle is 5 degrees and the inclination angle of satellite 2 is 10 degrees.

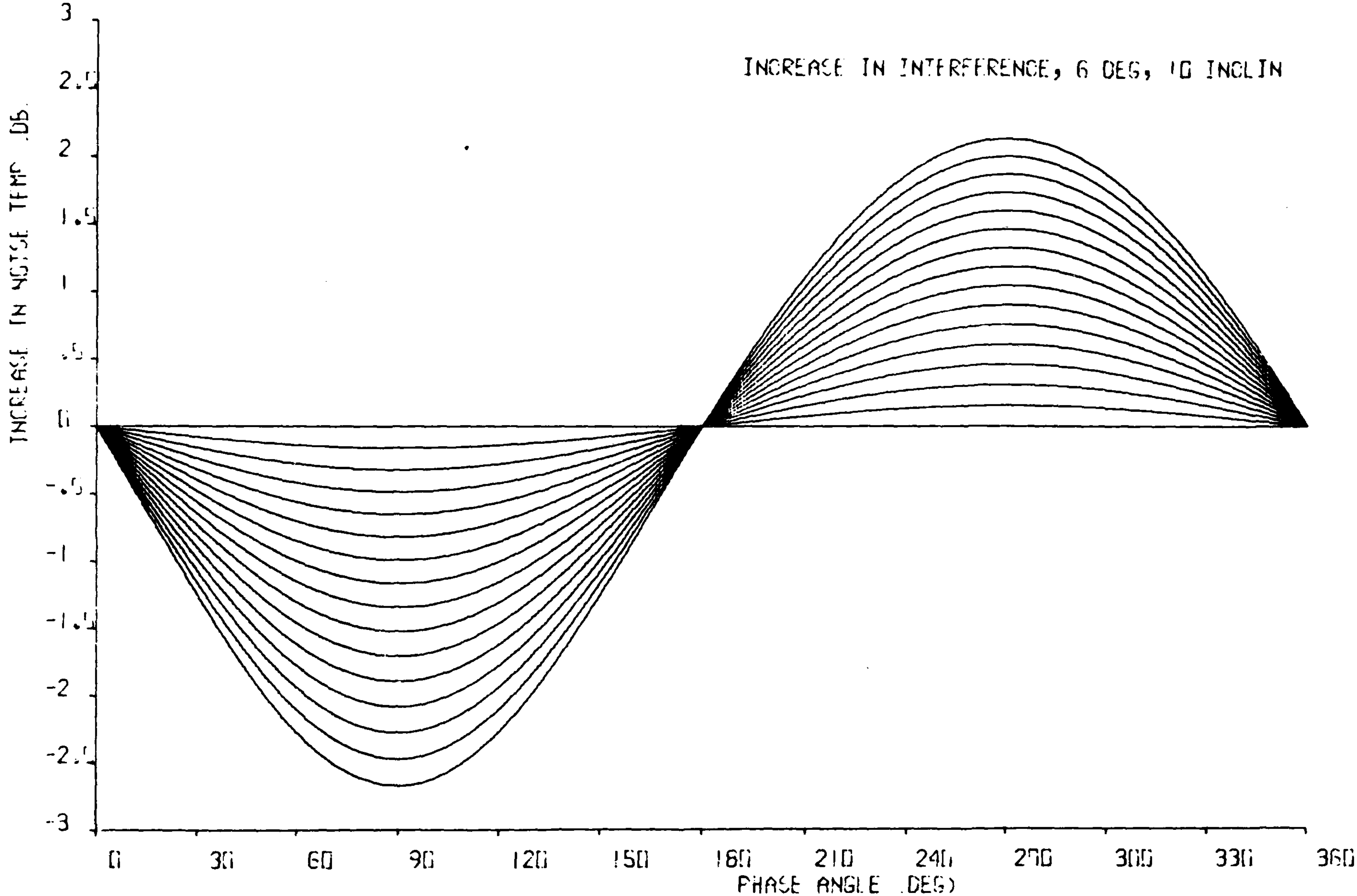


Figure 3-3(d): Increase in equivalent noise temperature as a function of nodal phase angle. The equatorial plane separation angle is 6 degrees and the inclination angle of satellite 2 is 10 degrees.

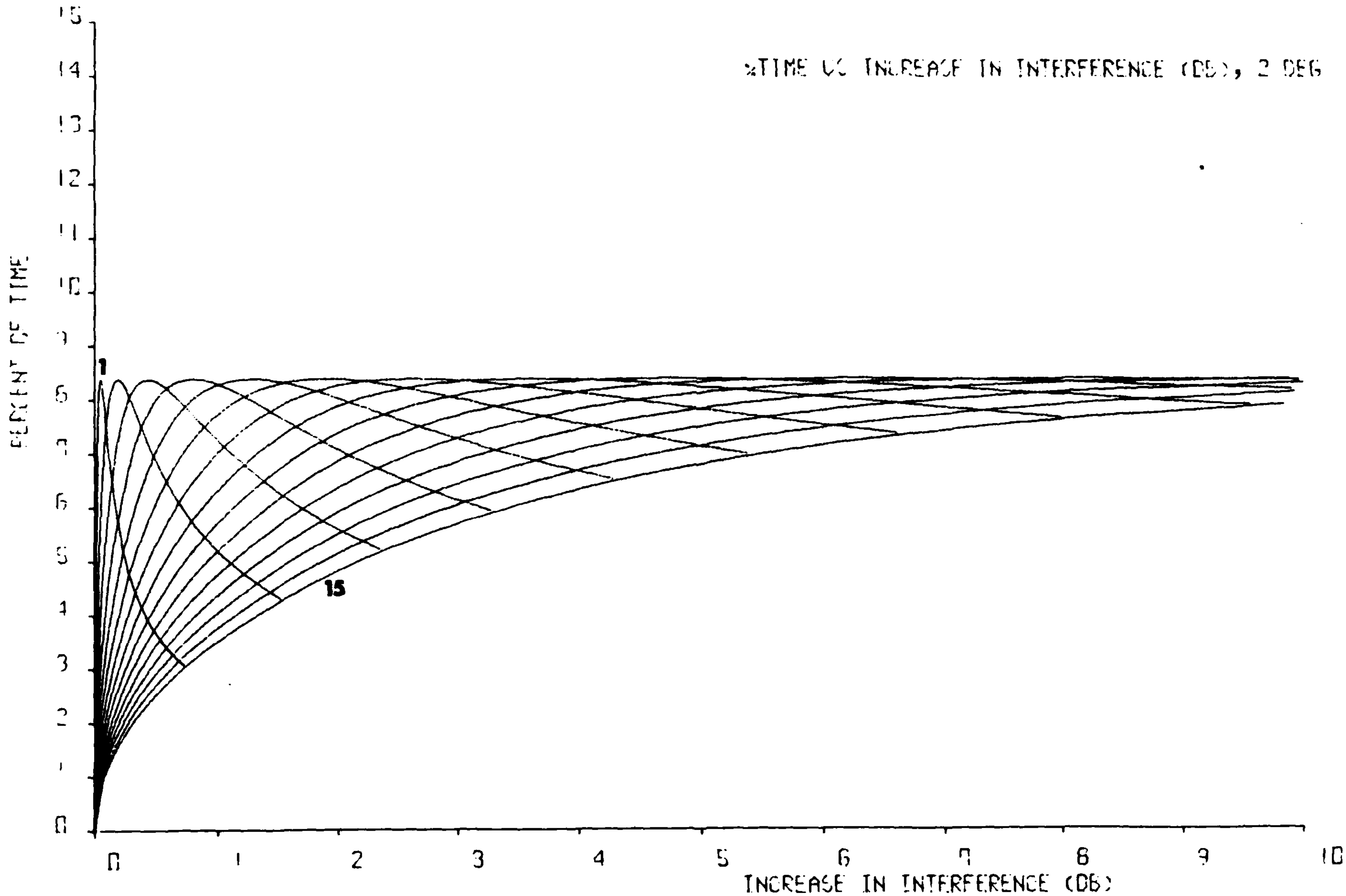


Figure 3-4(a): Percent of time that the total separation angle is less than nominal versus the worst case increase in equivalent noise temperature as a function of inclination angles. The equatorial plane separation angle is 2 degrees.

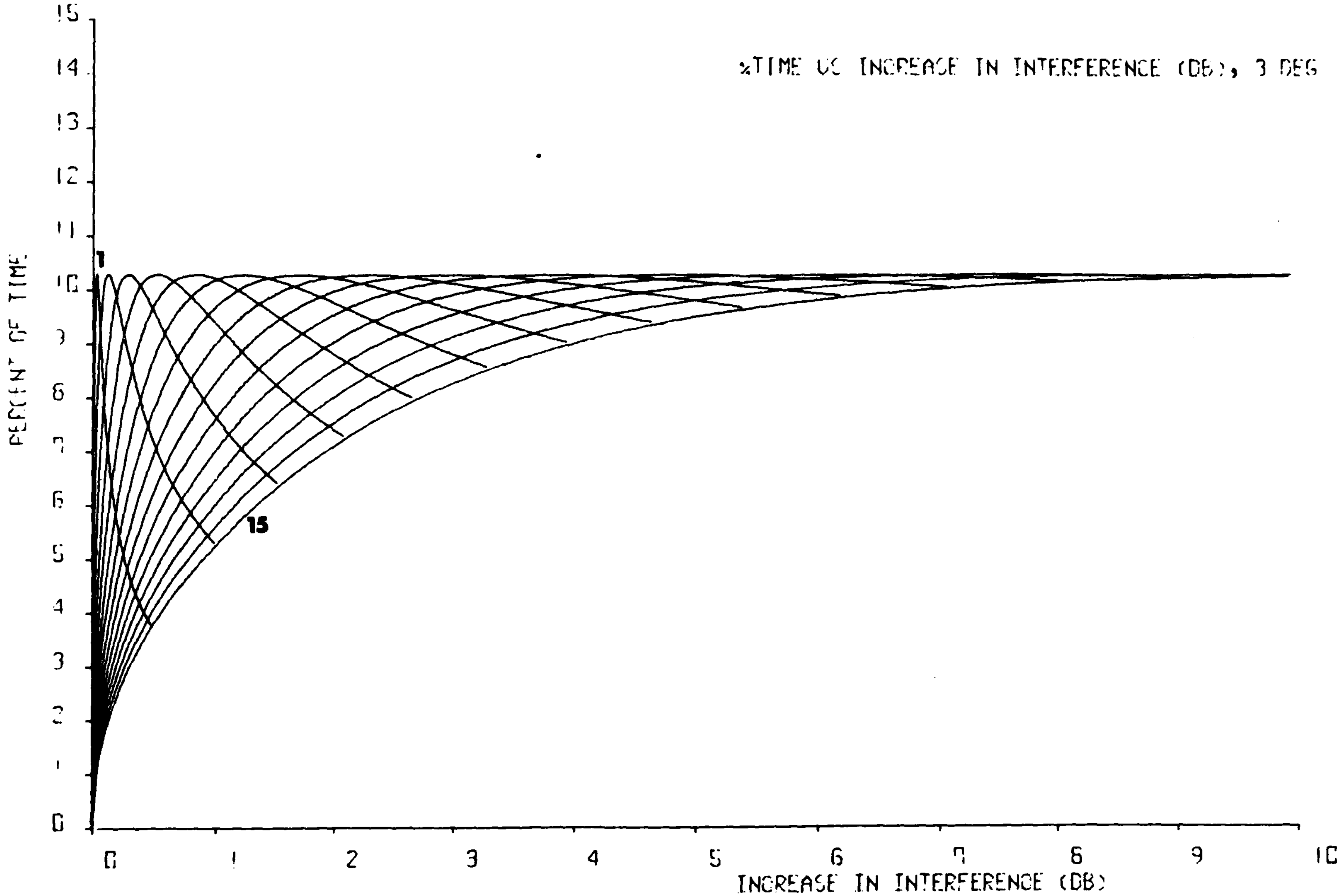


Figure 3-4(b): Percent of time that the total separation angle is less than nominal versus the worst case increase in equivalent noise temperature as a function of inclination angles. The equatorial plane separation angle is 3 degrees.

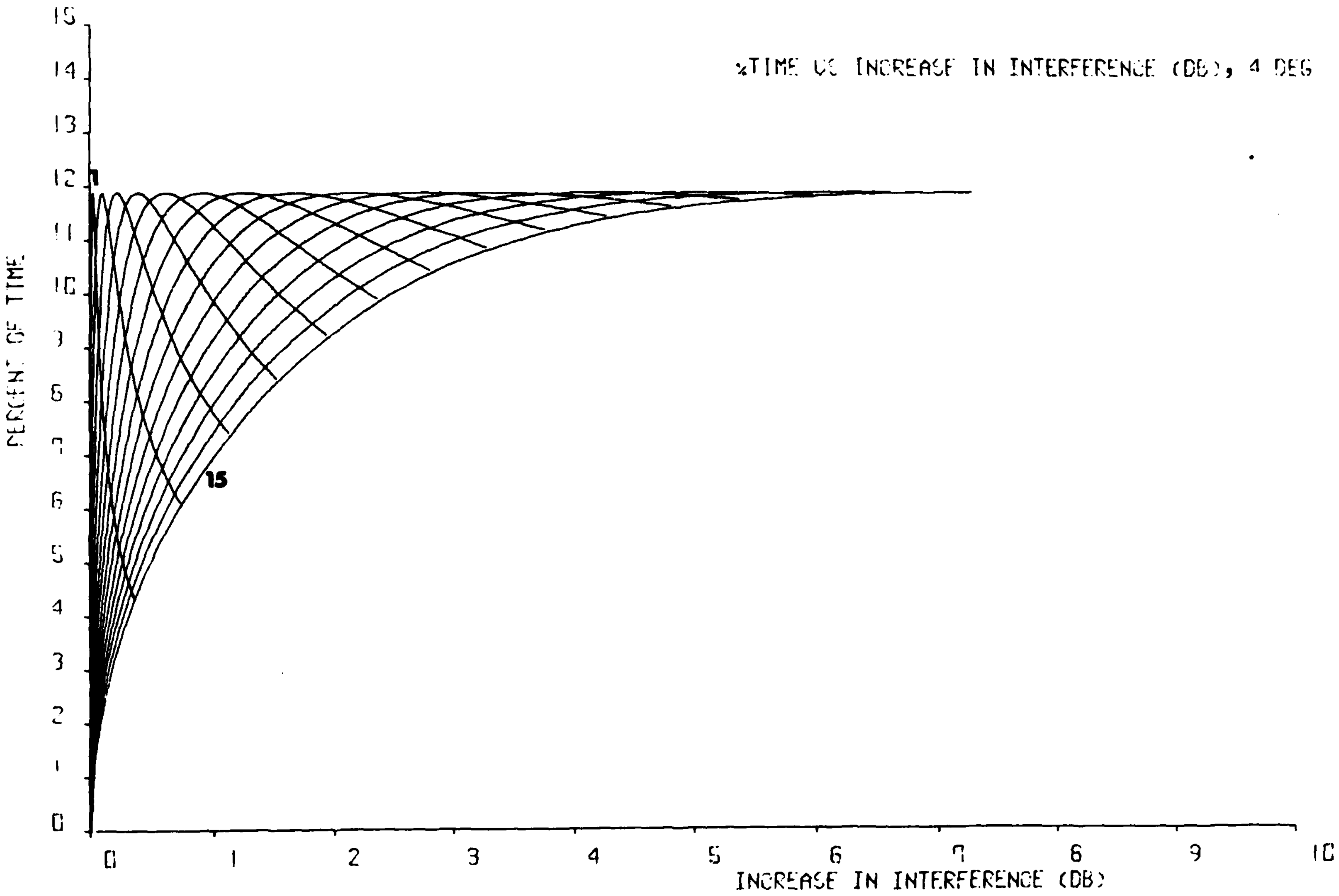


Figure 3-4(c): Percent of time that the *total separation angle* is less than nominal versus the worst case increase in equivalent noise temperature as a function of inclination angles. The equatorial plane separation angle is 4 degrees.

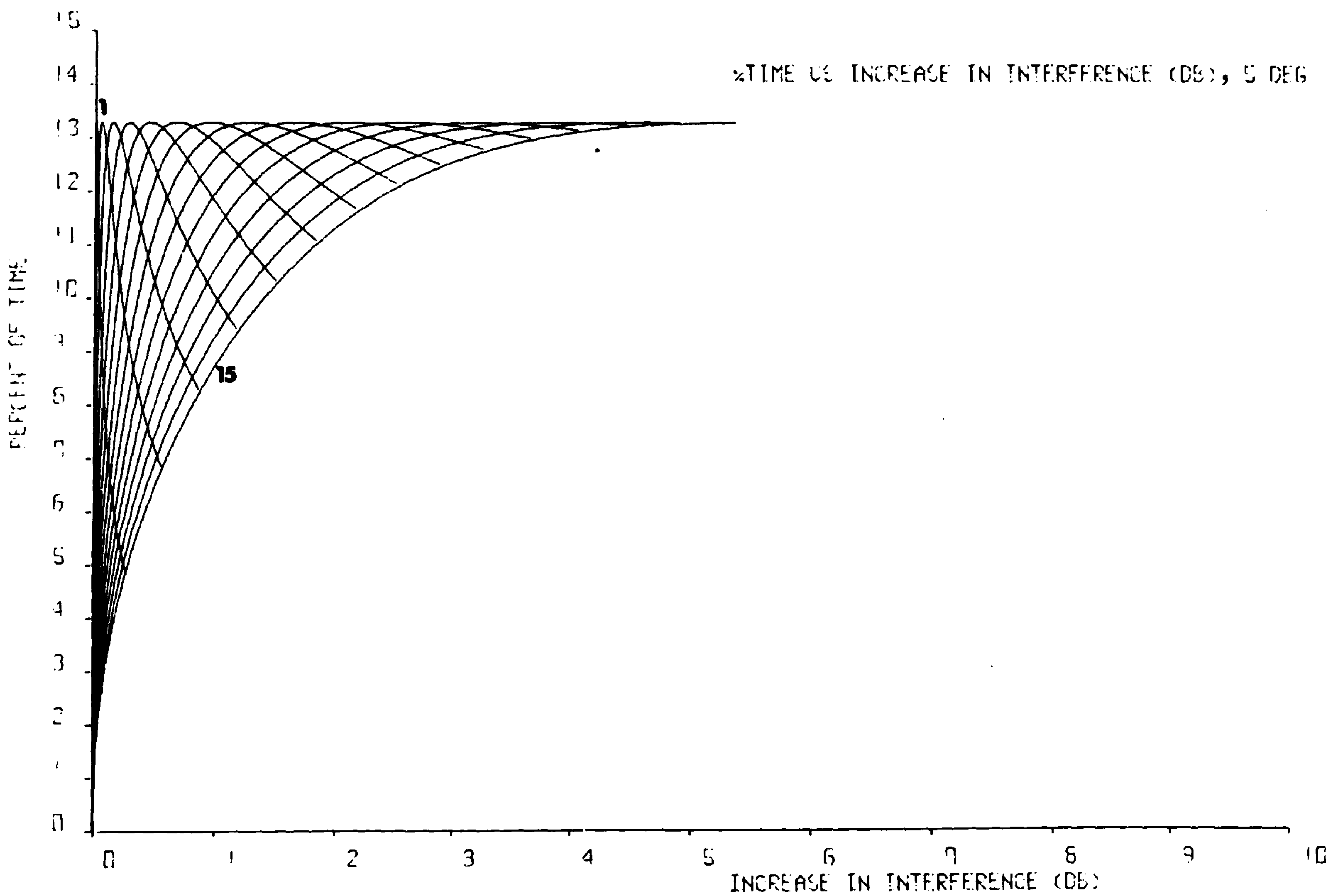


Figure 3-4(d): Percent of time that the total separation angle is less than nominal versus the worst case increase in equivalent noise temperature as a function of inclination angles. The equatorial plane separation angle is 5 degrees.

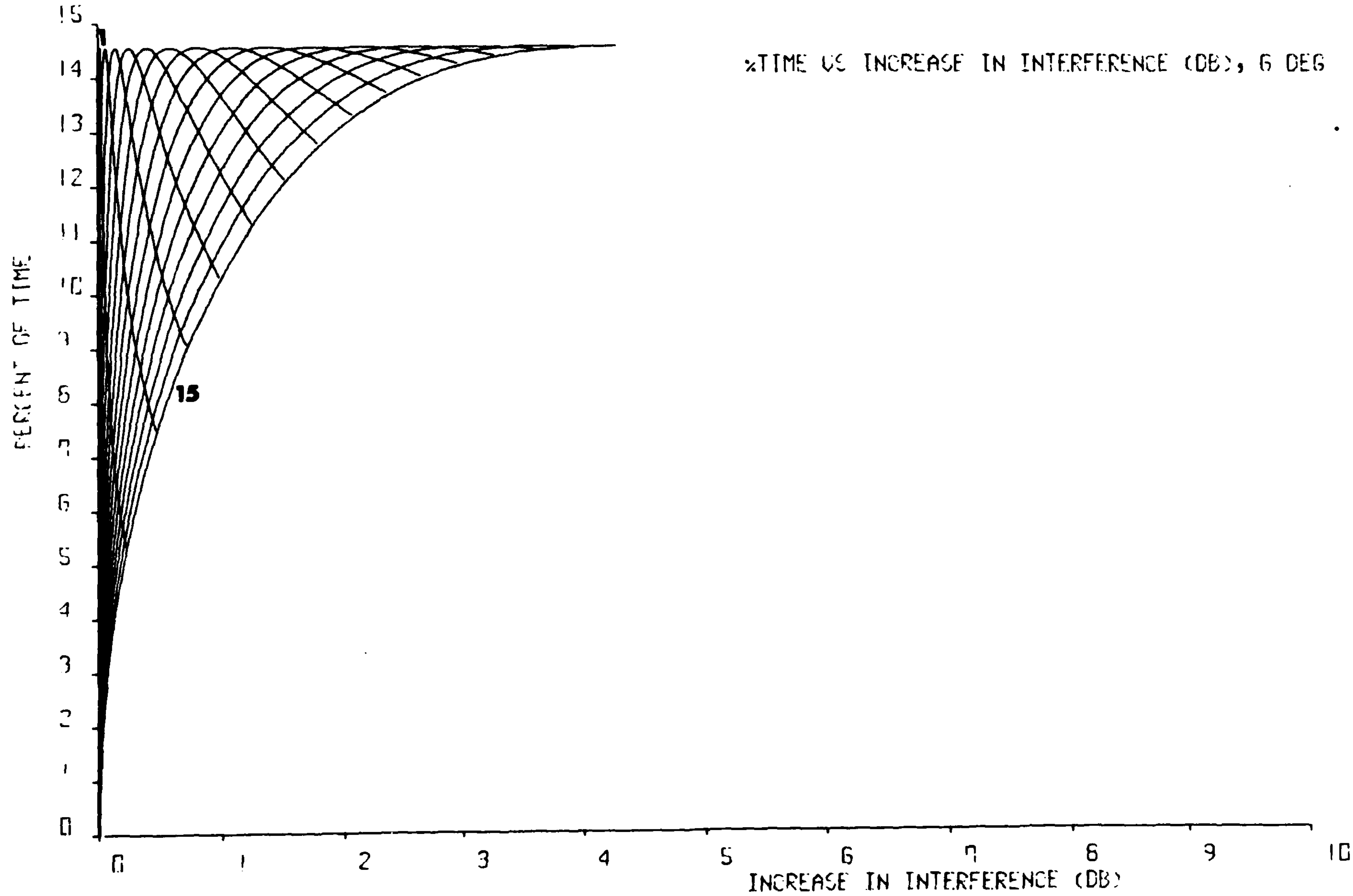


Figure 3-4(e): Percent of time that the total separation angle is less than nominal versus the worst case increase in equivalent noise temperature as a function of inclination angles. The equatorial plane separation angle is 6 degrees.

As an example, let the inclination angles of satellite 1 and 2 be 10 degrees. The following data can be extracted from the figures:

Separation Angle	%Time	Increase in Interference
2	8.4%	5.5 dB
3	10.3	3.6
4	11.9	2.6
5	13.2	2.1
6	14.5	1.8

The interpretation of these values is that with an equatorial plane separation angle of 2 degrees, 8.4% of the time an increase in equivalent noise temperature is experienced. At all other times, a decrease in equivalent noise temperature with respect to that computed with the equatorial plane separation angle is experienced. The increase in equivalent noise temperature experienced 8.4% of the time is never more than 5.5 dB for a 2 degree equatorial plane separation angle.

3.5 Statistical estimates of the increase in noise temperature due to inclination angles

Statistical estimates of the increase in equivalent noise temperature are provided as follows:

- o Nodal phase angle is uniformly distributed between -180 and 180 degrees and inclination angles are not random variables.
- o Inclination angles are random variables having either a uniform distribution or a guassian distribution.

The statistical estimates of the change in total separation angle based upon these assumptions are converted into an increase in equivalent noise temperature using equation (3-2).

Based upon statistical independence of inclination angles and nodal phase angle and the assumption that the nodal phase angle is uniformly distributed (-180, 180), the deviation of the change in total separation angle was shown to be:

$$\sigma_{\Delta\lambda} = I_1 I_2 / \sqrt{8} \qquad (3\text{-}6)$$

For inclination angles of 10 degrees for each satellite, the increase in equivalent noise temperature is:

Separation Angle	Increase in Interference
2	2.9 dB
3	2.0
4	1.6
5	1.3
6	1.1

Since for nodal phase angles between 0 and 180 degrees, the interference decreases, the data above can be interpreted as: For 84% of the nodal phase angles, the interference is less than or equal to the value shown in the table.

If the inclination angles are assumed to be random variables, the deviation of the change in total separation angle is given by:

$$\sigma_{\Delta\lambda} = \sigma_{I_1}\sigma_{I_2} / \sqrt{8} \tag{3-7}$$

If further, the inclination angles are assumed uniformly distributed, the deviation becomes:

$$\sigma_{\Delta\lambda} = I_m^2/3 \sqrt{8} \tag{3-8}$$

For maximum inclination angles of 10 degrees for each satellite, the increase in equivalent noise temperature is:

Separation Angle	Increase in Interference
2	1.1 dB
3	0.7
4	0.5
5	0.4
6	0.3

As above, the interpretation is that for 84% of the combinations of inclination angles and nodal phase angles, the interference is less than the value shown.

Section 4

CONCLUSIONS AND RECOMMENDATIONS

The study of the impact of inclination angles on the need for coordination provided answers to the following questions:

- o How large can the inclination angles become before Delta T/T computations for determining the need for coordination would be affected?
- o What are the risks of not including inclination angles in determining need for coordination?
- o What procedure can be used to include the effects of inclination angles on Delta T/T computations, when necessary?
- o Is there a need to modify the definition of geostationary orbit to include allowable inclination angles?
- o How can inclination angle be used to reduce the potential for interference?

The answers to these questions and the basis for the answers are presented in the following paragraphs.

4.1 Maximum inclination angles not requiring consideration in determining the need for coordination

During the course of this study, interest was expressed in defining the maximum inclination angles not requiring consideration in determining the need for coordination. To this end, 5 degree and 10 degree inclination angles were considered as potential candidates for the maximum inclination angles. The case for each of these angles is presented in the following paragraphs followed by consideration of the case against any maximum inclination angle.

4.1.1 Case for a 5 degree maximum

The case for 5 degree maximum inclination angles can be summarized as follows:

- o The worst case change in total separation angle for 5 degree inclination angles is 0.2 degrees. This converts to an increase in equivalent noise temperature of 1.1 dB for a 2 degree equatorial plane separation angle. This worst case value occurs at 2 points in the orbit and for equal inclination angles

of 5 degrees and for a nodal phase angle of 270 degrees. Nodal phase angles of other than 270 degrees have a smaller change in total separation angle.

- o The percent of time the total separati angle is less than the equatorial plane separation angle of 2 degrees is 8.4% for equal inclination angles, i.e., about 2 hours per day. For non-equal inclination angles, this percentage decreases. During this two hour period, the worst change in total separation angle is 0.2 degrees (1.1 dB increase in interference). In other words, 91.6% of the time, the total separation angle is greater than 2 degrees resulting in a decrease in interference.

- o For nodal phase angles between 0 and 180 degrees, the total separation angle is greater than 2 degrees. Only for a nodal phase angle of 270 degrees is the worst case change in total separation angle experienced.

The above argues for a statistical view of the increase in equivalent noise temperature due to inclination angles. The statistical case for a 5 degree maximum inclination angle can be summarized as:

- o Using the nodal phase angle as a uniformly distributed random variable, the deviation of the change in total separation angle is 0.15 degrees for equal inclination angles of 5 degrees. This can be interpreted as: For 84% of the nodal phase angles, the decrease in separation angle is less than 0.15 degrees.

- o If additionally, the inclination angles are uniformly distributed between -5 and 5 degrees, the deviation of the change in total separation angle is 0.05 degrees

- o Finally, the increase in separation angle estimated above occurs less than 8.4% of the time over an orbit.

As a result of the above arguments, a maximum inclination angle of 5 degrees may be reasonable without including the effects in the Delta T/T computations performed to determine the need for coordination.

4.1.2 Case for a 10 degree maximum

The case for 10 degree maximum inclination angles can be summarized as follows:

- o The worst case change in total separation angle for 10 degree inclination angles is 0.9 degrees. This converts to an increase in equivalent noise temperature of 3.9 dB for a 2 degree equatorial plane separation angle. This worst case value occurs at 2 points in the orbit and for equal inclination angles of 10 degrees and for a nodal phase angle of 270 degrees. Nodal phase angles of other than 270 degrees have a smaller change in total separation angle.

- o The percent of time the total separation angle is less than the equatorial plane separation angle of 2 degrees is 8.4% for equal inclination angles, i.e., about 2 hours per day. For non-equal inclination angles, this percentage decreases. During this two hour period, the worst change in total separation angle is 0.9 degrees (3.9 dB increase in interference). In other words, 91.6% of the time, the total separation angle is greater than 2 degrees resulting in a decrease in interference.

- o For nodal phase angles between 0 and 180 degrees, the total separation angle is greater than 2 degrees. Only for a nodal phase angle of 270 degrees is the worst case change in total separation angle experienced.

The above argues for a statistical view of the increase in equivalent noise temperature due to inclination angles. The statistical case for a 10 degree maximum inclination angle can be summarized as:

- o Using the nodal phase angle as a uniformly distributed random variable, the deviation of the change in total separation angle is 0.6 degrees (2.9 dB increase in interference) for equal inclination angles of 10 degrees. This can be interpreted as: For 84% of the nodal phase angles, the decrease in separation angle is less than 0.6 degrees.

- o If additionally, the inclination angles are uniformly distributed between -10 and 10 degrees, the deviation of the change in total separation angle is 0.2 degrees (1.1 dB increase in interference).

- o Finally, the 1.1 dB increase in interference estimated above occurs less than 8.4% of the time over an orbit.

As a result of the above arguments, a maximum inclination angle of 10 degrees may be reasonable without including the effects in the Delta T/T computations performed to determine the need for coordination.

4.1.3 Case against any maximum

The case against any maximum inclination angles to be ignored in determining the need for coordination is based upon the relatively static geometric relationship between the adjacent satellites and the lack of a naturally defined maximum number. The case against any maximum inclination angles can be summarized as follows:

- The satellites in orbit have a fixed nodal phase angle determined by the orbit transfer characteristics and subsequent station keeping maneuvers with each satellite tracing its figure eight once per orbit. The nodal phase angle will be slowly changing except during station keeping maneuvers. The inclination angle of each satellite will increase at about 0.86 degrees per year near the equatorial plane slowing as the inclination angle increases. The relationship between the satellites will be relatively static until station keeping maneuvers are conducted.

- The statistical arguments used for the 5 and 10 degree maximum inclination angles can be faulty in a particular application (i.e., Murphy's Law). Since the worst case change in total separation angle is significant for even a 5 degree inclination angle, the impact of inclination angle on the need for coordination should always be considered.

4.2 Impact of inclination angles on need for coordination

The analysis presented in Sections 2 and 3 identified the following characteristics of an inclined geostationary orbit which may impact the potential interference between satellite networks:

- Inclination angles change the total separation angle between adjacent satellites.

- The nodal phase angle between adjacent satellites affects the magnitude and direction of the change in total separation angle.

  - Nodal phase angles between 0 and 180 degrees result in an increase in total separation angle for any inclination angles

  - Nodal phase angles between 180 and 360 degrees result in a decrease in the total separation angle for a short period of time (2 one hour periods in 24 hours for a 2 degree equatorial plane separation angle)

- For any nodal phase angle, the period of time over which the separation angle is reduced is small (2 hours out of 24 for a 2 degree separation). At all other times, the total separation angle is significantly greater than the equatorial plane separation angle.

- The worst case change in total separation angle is significant for most inclination angles. It is approximated for inclination angles less than 15 degrees by:

$$\Delta\lambda^0 = \pi/360\ I_1^0 I_2^0$$

  where everything is in degrees. Thus, for 5 degree inclination angles, the change in separation angle is 0.22 degrees which converts to over a one dB increase in interference.

- The worst case change in separation angle occurs for a nodal phase angle of 270 degrees and at two points in the orbit (24 hour period). At other nodal phase angles the change in separation angle decreases while at times near the two worst case points the change in total separation angle decreases.

- The percent of time the total separation angle is less than the equatorial plane value is small (8.4% maximum for a 2 degree separation angle) and has a maximum for equal inclination angles. For non-equal inclination angles, the percent of time decreases.

- Inclined GSO have variable coverage moving North and South by the inclination angle each orbit.

In summary, use of inclined geostationary orbits will increase the need for coordination since the total separation angle between the satellite networks will decrease (either worst case or statistically) resulting in potentially increased interference. Thus, satellite networks in GSO not requiring coordination based upon Delta T/T computations may require coordination when inclination angles are considered.

4.3 Procedure for including inclination angles in Delta T/T

The procedure for including the effects of the worst case change in total separation angle in the Delta T/T computations is as follows:

- Compute the change in the total separation angle due to inclination angles as:

$$\Delta\lambda = I_1 I_2/2 \quad \text{(radians)}$$

- o In the Appendix 29 computations, the equatorial plane separation angle is computed either as the difference in the satellite positions in the equatorial plane or as the angle from a ground station to the victim satellite or vice versa.
- o The equatorial plane separation angle is reduced by the worst case change in total separation computed above.
- o The resulting angle is then used in the Appendix 29 computations as the separation angle to determine the various antenna gains.

Sample computations are provided in Section 3 of this report.

The worst case change in total separation angle is recommended for use in determining the need for coordination. Use of other estimates are not as conservative and may not be appropriate for all situations.

4.4 Broadened definition of geostationary orbit

The definition of geostationary orbit provided by the Radio Regulations is as follows:

- o Geosynchronous Satellite: An earth satellite whose period of revolution is equal to the period of rotation of the Earth about its axis.
- o Geostationary Satellite: A geosynchronous satellite whose circular and direct orbit lies in the plane of the Earth's equator and which thus remains fixed relative to the Earth; ...
- o Geostationary Orbit: The orbit in which a satellite must be placed to be a Geostationary Satellite.
- o Inclination of an Orbit: The angle determined by the plane containing the orbit and the plane of the Earth's equator.

The broadened definition of geostationary orbit involves the following change to the above definitions:

- o Geostationary Satellite: A geosychronous satellite whose circular and direct orbit lies within 15 degrees of the plane of the Earth's equator.

In concert with the broadened definition, Appendix 29 of the Radio Regulations should be modified to account for the effects of inclination angle. Additionally, the maximum inclination angles to be expected for the satellite network should be notified. Otherwise, the procedures for determining the need for coordination remain the same.

4.5 Use of inclined GSO to reduce potential for interference

The results presented in Section 2 indicate that if adjacent satellites are given inclination angles, then for selected nodal phase angles, the total separation angle between the adjacent satellites is always larger than the equatorial plane separation angle. This increased separation angle results in decreases in the equivalent noise temperature of the victim satellite as shown in Section 3. Thus, closer equatorial plane separation angles can be tolerated if inclination angles for both adjacent satellites are acceptable and nodal phase angle control is feasible for one or both satellites.

As an example, Figure 2-7(c) shows that the minimum total changė in separation angle is 1.96 degrees for a 90 degree nodal phase angle with both satellites having 15 degree inclination angles. Thus, if the equatorial plane separation angle is 2 degrees, the total separation angle is never less than 3.96 degrees. If the equatorial plane separation angle is 1 degree, the total separation angle is never less than 2.96 degrees.

The impact of implementing this nodal phase angle control to gain increased separation angles is:

- o Satellite network coverage moves an amount equal to the inclination angles both North and South during an orbit.

- o Earth stations experience higher dynamics including accelerations, doppler, etc. as well as larger changes in pointing angles.

- o Nodal phase angle contol requires continuous coordination between the satellite networks.

- o Satellites adjacent to the two satellites using nodal phase control would experience a decrease in total separation angle and thus potentially increased interference.

The various tradeoffs implied by the above would need to be investigated in detail for each application.

Appendix A-1

MATHEMATICAL DERIVATIONS

A-1: Maximum longitude excursions

A-2: Total separation angle

A-3: Minimum separation angle for a nodal phase angle of 270 degrees

A-4: Minimum separation angle for any nodal phase angle

A-5: Angle where total separation angle is less than nominal

Appendix A-2

MAXIMUM LONGITUDE EXCURSIONS

The longitude excursion due to inclination angle was given as:

$$\theta = \mathrm{Tan}^{-1}(\mathrm{Cos}\, I\, \mathrm{Tan}\,\gamma) - \gamma \qquad \text{(A-1-1)}$$

where I is the inclination angle and γ is the angle from the ascending node. Several trigonometry identities and small angle approximations will be used to simplify equation (A-1-1). Remember that:

$$\mathrm{Tan}^{-1}\, a/b = \mathrm{Sin}^{-1} a/\sqrt{a^2 + b^2} \qquad \text{(A-1-2)}$$

and that

$$\mathrm{Sin}^{-1}\, a \pm \mathrm{Sin}^{-1}\, b = \mathrm{Sin}^{-1}[a(1-b^2)^{1/2} \pm b(1-a^2)^{1/2}] \qquad \text{(A-1-3)}$$

Equation (A-1-1) can be rewritten as:

$$\theta = \mathrm{Sin}^{-1}\, [0.5\, \mathrm{Sin}2\gamma(\mathrm{Cos}\, I-1)/\sqrt{1-\mathrm{Sin}^2 I \mathrm{Sin}^2\gamma}\,]. \qquad \text{(A-1-4)}$$

Figure 2-1 shows that the maximum longitude excursions are small. Using this assumption, the sine is approximately the angle. Then:

$$\theta = 0.5\, \mathrm{Sin}2\gamma(\mathrm{Cos}I-1)/\sqrt{1-\mathrm{Sin}^2 I \mathrm{Sin}^2\gamma} \qquad \text{(A-1-5)}$$

If the inclination is assumed to be small (<15 degrees), the trig functions can be approximated as:

$$\mathrm{Cos}\, I \simeq 1 - I^2/2 \qquad \text{(A-1-6)}$$

$$\mathrm{Sin}\, I \simeq I.$$

Then:

$$\theta \simeq -I^2/4\, \mathrm{Sin}2\gamma/\sqrt{1-I^2\mathrm{Sin}^2\gamma}\;. \qquad \text{(A-1-7)}$$

Remember that:

$$1/(1-a)^{1/2} \simeq 1 + a/2$$

so that equation (A-1-7) becomes

$$\theta \simeq -I^2/4\, \mathrm{Sin}\, 2\gamma(1+I^2/2\, \mathrm{Sin}^2\gamma). \qquad \text{(A-1-8)}$$

Neglecting terms of the order of I^4 gives the longitude excursions as:

$$\theta \simeq -0.25\ I^2\ \mathrm{Sin}2\gamma\ . \qquad \text{(A-1-9)}$$

The maximum value of equation (A-1-9) can be found by recognizing that the minimum and maximums occur at

$$\gamma = n\pi/2.$$

The maximum longitude excursions are then

$$\theta_{max} = I^2/4 \qquad \text{(radians)}. \qquad \text{(A-1-10)}$$

Appendix A-3

TOTAL SEPARATION ANGLE

The total separation angle can be written as:

$$\text{Cos}\,\lambda = \text{Cos}\phi_1 \text{Cos}\phi_2 \text{Cos}(\theta_1 - \theta_2) + \text{Sin}\,\phi_1 \text{Sin}\,\phi_2 \qquad \text{(A-2-1)}$$

where:
$$\phi_1 = \text{Sin}^{-1}(\text{Sin}\ I_1\ \text{Sin}\,\gamma_1)$$
$$\theta_1 = \theta_0 + \Delta\theta_1$$
$$\Delta\theta_1 = \text{Tan}^{-1}(\text{Cos}I_1\ \text{Tan}\ \gamma_1) - \gamma_1$$
$$\theta_2 = \theta_0 + \theta_S + \Delta\theta_2 .$$

The latitude excursion is on the order of the inclination angle and can be approximated as:

$$\phi_1 \simeq I_1\ \text{Sin}\,\gamma_1 . \qquad \text{(A-2-2)}$$

and the longitude excursion was shown in Appendix A-1 to be approximately:

$$\Delta\theta_1 \simeq -0.25 I_1^2\ \text{Sin}2\,\gamma_1 . \qquad \text{(A-2-3)}$$

The term $\text{Cos}(\theta_1 - \theta_2)$ is given by:

$$\text{Cos}(\theta_1 - \theta_2) = \text{Cos}(\Delta\theta_1 - \theta_S - \Delta\theta_2)$$

where $\Delta\theta_1$, θ_S and $\Delta\theta_2$ are small. This term can then be approximated as:

$$\text{Cos}(\theta_1 - \theta_2) \simeq 1 - (\Delta\theta_1 - \theta_S - \Delta\theta_2)^2/2 \qquad \text{(A-2-4)}$$

Substituting in equation (A-2-1) and rearranging gives:

$$\text{Cos}\,\lambda \simeq \text{Cos}\phi_1 \text{Cos}\phi_2 + \text{Sin}\,\phi_1 \text{Sin}\,\phi_2$$
$$- \text{Cos}\,\phi_1 \text{Cos}\,\phi_2\ (\Delta\theta_1 - \theta_S - \Delta\theta_2)^2/2 \qquad \text{(A-2-5)}$$

Grouping the first two terms results in:

$$\text{Cos}\,\lambda = \text{Cos}(\phi_1 - \phi_2) - \text{Cos}\,\phi_1 \text{Cos}\phi_2\ (\Delta\theta_1 - \theta_S - \Delta\theta_2)^2/2 \qquad \text{(A-2-6)}$$

Since the latitude excursion is small:

$$\text{Cos}(\phi_1 - \phi_2) \simeq 1 - (\phi_1 - \phi_2)^2/2 \qquad \text{(A-2-7)}$$

and

$$\text{Cos}\,\phi_1 \simeq 1 - \phi_1^2/2.$$

Substituting into equation (A-2-6) and discarding terms of the order of a small angle to the fourth power gives:

$$\mathrm{Cos}\,\lambda = 1-(\phi_1 - \phi_2)^2/2 - (\Delta\theta_1 - \theta_S - \Delta\theta_2)^2/2. \qquad \text{(A-2-8)}$$

The final assumption is that the total separation angle is small so that:

$$\mathrm{Cos}\,\lambda \simeq 1 - \lambda^2/2. \qquad \text{(A-2-9)}$$

Finally, the total separation angle is approximately:

$$\lambda^2 \simeq (\phi_1 - \phi_2)^2 + (\Delta\theta_1 - \theta_S - \Delta\theta_2)^2. \qquad \text{(A-2-10)}$$

Substituting equations (A-2-2) and (A-2-3) into equation (A-2-10) gives the total separation angle as:

$$\lambda^2 = [I_1 \mathrm{Sin}\gamma_1 - I_2 \mathrm{Sin}(\gamma_1 + \gamma_0)]^2$$
$$+ [-I_1^2/4\ \mathrm{Sin}2\gamma_1 + I_2^2/4\ \mathrm{Sin}2(\gamma_1 + \gamma_0) - \theta_S]^2. \qquad \text{(A-2-11)}$$

Appendix A-4

MINIMUM SEPARATION ANGLE FOR A NODAL PHASE ANGLE OF 270 DEGREES

The total separation angle for a 270 degree nodal phase angle was given as:

$$\lambda^2 = [I_1 \mathrm{Sin}\gamma_1 + I_2 \mathrm{Cos}\gamma_1]^2$$
$$+ [-\mathrm{Sin}2\gamma_1 (I_1^2/4 + I_2^2/4) - \theta_s]^2 \qquad \text{(A-3-1)}$$

This appendix will show that the minimum total separation angle is found when:

$$\mathrm{Tan}\,\gamma_1 = -I_2/I_1 \qquad \text{(A-3-2)}$$

That equation (A-3-2) yields the minimum of equation (A-3-1) will be demonstrated by showing that equation (A-3-2) is the approximate solution of:

$$d\lambda^2/d\gamma_1 = 0. \qquad \text{(A-3-3)}$$

Performing the indicated derivative gives the following:

$$2[I_1 \mathrm{Sin}\,\gamma_1 + I_2 \mathrm{Cos}\,\gamma_2][I_1 \mathrm{Cos}\gamma_1 - I_2 \mathrm{Sin}\,\gamma_1]$$
$$+ 2[-\mathrm{Sin}2\gamma_1 (I_1/4 + I_2/4) - \theta_s]$$
$$\times [-2\mathrm{Cos}2\gamma_1 (I_1/4 + I_2/4)] = 0 \qquad \text{(A-3-4)}$$

Neglecting terms of the order of a small angle to the third and fourth power results in:

$$2[I_1 \mathrm{Sin}\,\gamma_1 + I_2 \mathrm{Cos}\,\gamma_1][I_1 \mathrm{Cos}\gamma_1 - I_2 \mathrm{Sin}\gamma_1] = 0 \qquad \text{(A-3-5)}$$

Substituting equation (A-3-2) into equation (A-3-5) shows that equation (A-3-3) is satisfied. That the solution is the minimum total separation angle can be seen from the plots.

Appendix A-5

MINIMUM SEPARATION ANGLE FOR ANY NODAL PHASE ANGLE

The total separation angle for any nodal phase angle was given as:

$$\lambda^2 = [I_1 \mathrm{Sin}\gamma_1 - I_2 \mathrm{Sin}(\gamma_1 + \gamma_0)]^2$$
$$+ [-I_1/4\ \mathrm{Sin}2\gamma_1 + I_2/4\ \mathrm{Sin}2(\gamma_1 + \gamma_0) - \theta_s]^2 \qquad \text{(A-4-1)}$$

This appendix will show that the minimum total separation angle is found when:

$$\mathrm{Tan}\gamma_1 = I_2 \mathrm{Sin}\gamma_0/(I_1 - I_2 \mathrm{Cos}\gamma_0) \qquad \text{(A-4-2)}$$

That equation (A-4-2) yields the minimum of equation (A-4-1) will be demonstrated by showing that equation (A-4-2) is the approximate solution of:

$$d\lambda^2/d\gamma_1 = 0. \qquad \text{(A-4-3)}$$

Performing the indicated derivative gives the following:

$$2[I_1 \mathrm{Sin}\gamma_1 - I_2 \mathrm{Sin}(\gamma_1 + \gamma_0)][I_1 \mathrm{Cos}\gamma_1 - I_2 \mathrm{Cos}(\gamma_1 + \gamma_0)]$$
$$+ 2[-I_1^2/4\ \mathrm{Sin}2\gamma_1 + I_2^2/4\ \mathrm{Sin}2(\gamma_1 + \gamma_0) - \theta_s]$$
$$\times [-I_1^2/2\ \mathrm{Cos}2\gamma_1 + I_2^2/2\ \mathrm{Cos}2(\gamma_1 + \gamma_0)] = 0 \qquad \text{(A-4-4)}$$

Neglecting terms of the order of a small angle to the third and fourth power results in:

$$2[I_1 \mathrm{Sin}\gamma_1 - I_2 \mathrm{Sin}(\gamma_1 + \gamma_0)][I_1 \mathrm{Cos}\gamma_1 - I_2 \mathrm{Cos}(\gamma_1 + \gamma_0)] = 0$$
$$\text{(A-4-5)}$$

Substituting equation (A-4-2) into equation (A-4-5) shows that equation (A-4-3) is satisfied. That the solution is the minimum total separation angle can be seen from the plots.

Appendix A-6

ANGLE WHERE TOTAL SEPARATION ANGLE IS LESS THAN NOMINAL

The total separation angle for a 270 degree nodal phase angle was given as:

$$\lambda^2 = [I_1 \mathrm{Sin}\gamma_1 + I_2 \mathrm{Cos}\gamma_2]^2$$
$$+[-\mathrm{Sin}2\gamma_1 (I_1^2/4 + I_2^2/4) - \theta_S\ ]^2 \qquad \text{(A-5-1)}$$

As discussed in Section 2.5, the angle near when the separation angle is minimum can be approximated as:

$$\gamma_1 = \gamma_{min} + \Delta\gamma \qquad \text{(A-5-2)}$$

where γmin is the angle which has the minimum separation angle and $\Delta\gamma$ is a small variation around the minimum. The angle where the minimum separation angle occurs is given by:

$$\mathrm{Tan}\ \gamma_{min} = -I_2/I_1 \qquad \text{(A-5-3)}$$

Substituting equation (A-5-2) into equation (A-5-1) and using the following approximations:

$$\mathrm{Sin}(\gamma_{min} + \Delta\gamma) \simeq \mathrm{Sin}\gamma_{min} + \Delta\gamma \mathrm{Cos}\gamma_{min}$$

$$\mathrm{Cos}(\gamma_{min} + \Delta\gamma) \simeq \mathrm{Cos}\gamma_{min} - \Delta\gamma \mathrm{Sin}\gamma_{min}$$

yields:

$$\lambda^2 = \Delta\gamma^2 [I_1^2 + I_2^2 + (I_1^2 - I_2^2)^2/4]$$
$$+ \Delta\gamma[\theta_S (I_1^2 - I_2^2) - I_1 I_2/2\ (I_1^2 - I_2^2)]$$
$$+ \theta_S^2 + I_1^2 I_2^2/4 - \theta_S I_1 I_2 \qquad \text{(A-5-4)}$$

Neglecting terms of small angles to higher powers gives:

$$\lambda^2 = \Delta\gamma^2 (I_1^2 + I_2^2) + \Delta\gamma\theta_S (I_1^2 - I_2^2)$$
$$+ \theta_S^2 - \theta_S I_1 I_2 \qquad \text{(A-5-5)}$$

Solving equation (A-5-5) for $\Delta\gamma$ when the total separation angle is equal to the equatorial plane separation angle gives:

$$\Delta\gamma_{1\ 2} = [-\theta_S (I_1^2 - I_2^2) \pm 2\sqrt{\theta_S I_1 I_2}\ ]/2(I_1^2 + I_2^2). \qquad \text{(A-5-6)}$$

Appendix B
Multiple Entry to Single Entry Interference Ratios

CONTENTS/STATUS

- o General
  - Single entry interference
  - Multiple entry interference
- o Homogeneous Case – Beyond First Sidelobe
  - Definitions
  - Single entry interference
  - Multiple entry interference
  - ME/SE ratio
  - Maximum ME/SE ratio
  - Plots of ME/SE vs. satellite antenna discrim.
  - Orbit utilization
  - Plots of orbit utilization vs. sat. ant. discrim
- o Homogeneous Case – Appendix 29 Antenna Patterns
  - Single entry interference
  - Multiple entry interference
  - ME/SE ratio

CONTENTS/STATUS (CON'T)

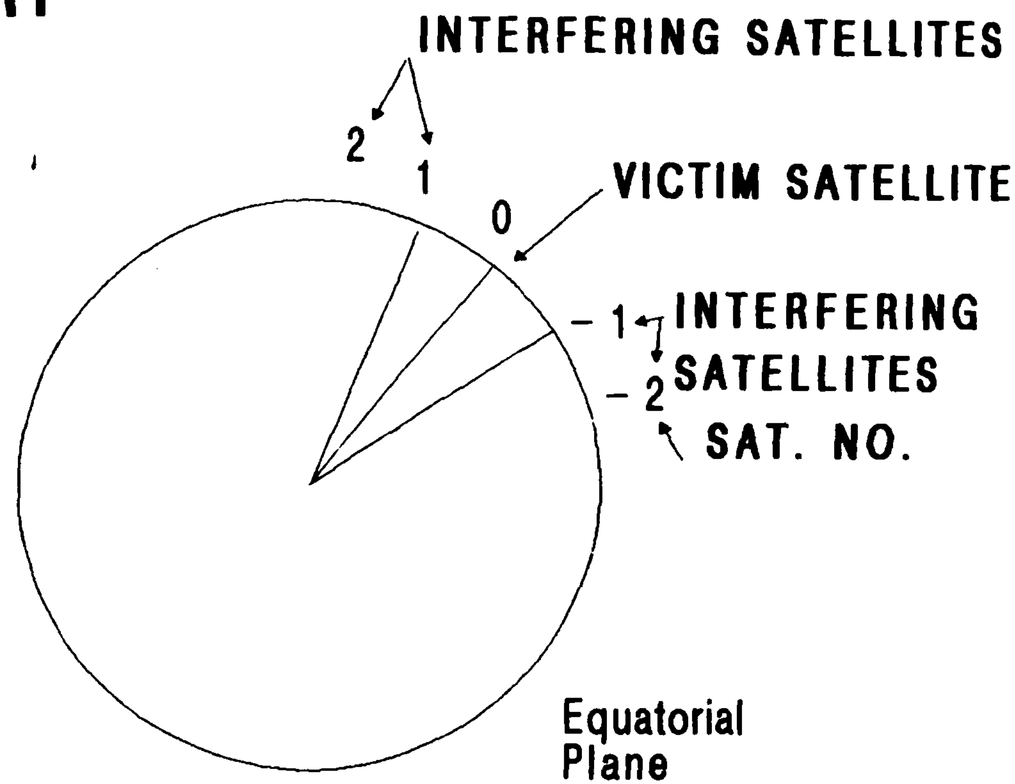
- o Homogeneous Case – Appendix 29 (Con't)
  - –Plots of ME/SE ratio vs satellite ant. discrim.
  - –Orbit utilization
  - –*Plots of orbit utilization vs sat. ant. discrim.*

- o East – West Stationkeeping Error
  - –Definitions
  - –Formulation of the problem
  - –Worst case ME/SE ratio with plots
    - – –*Homogeneous Case – beyond sidelobe*
    - – –Appendix 29 antenna patterns
  - –*Expected value of ME/SE ratio given worst case SE*
  - –*Orbit utilization with plots*

CONTENTS/STATUS (CON'T)

- o Earth Station Tracking Error
  - *Definitions*
  - *Formulation of the problem*
  - *Worst case ME/SE ratio with plots*
    - *Homogeneous Case – beyond sidelobe*
    - *Appendix 29 antenna patterns*
  - *Expected value of ME/SE ratio given worst case SE*
  - *Orbit utilization with plots*
- o Combined Stationkeeping and Tracking Errors
  - *Formulation of the problem*
  - *Worst case ME/SE ratio with plots*
    - *Homogeneous Case – beyond sidelobe*
    - *Appendix 29 antenna patterns*
  - *Expected value of ME/SE ratio given worst case SE*
  - *Orbit utilization with plots*

GEOMETRY


INTERFERING SATELLITES
2
1
0
VICTIM SATELLITE
- 1
INTERFERING
SATELLITES
- 2
SAT. NO.
Equatorial
Plane

SINGLE ENTRY INTERFERENCE

o Single Entry Interference is a Function of

- Earth Station Parameters
  - Transmitted power density
  - Transmitting antenna gain
  - Receiving antenna gain

- Satellite Parameters
  - Satellite antenna gain (receiving)
  - Satellite antenna gain (transmitting)
  - Transmitted power density
  - Transmission gain
  - Polarization isolation

- Geometry
  - Free space loss
  - Angular separation

SINGLE ENTRY INTERFERENCE (CON'T)

- Single Entry Interference has the form

$$SE = K\theta^{-2.5}$$

where K includes the effect of the parameters and θ is the angular separation of the satellites (assumed beyond the first sidelobe).

- Showing the satellite antenna discrimination gives

$$SE = \alpha K\theta^{-2.5}$$

where α is the satellite antenna discrimination.

- Other parameters can be shown such as earth station transmitting antenna gain.

GENERAL CASE

- o Multiple Entry (ME) Interference

$$ME = \sum_{\substack{n=-N \\ n \neq 0}}^{n=N} \text{Interference from Satellite } n$$

- o Interference from Satellite n when separation is beyond the first sidelobe

$$\alpha_n |n\theta|^{-2.5}$$

- o Then

$$ME = \theta^{-2.5} \sum_{n=-N}^{N} \alpha_n |n|^{-2.5}$$

$$ME = 2\theta^{-2.5} \sum_{n=1}^{N} \alpha_n \; n^{-2.5}$$

HOMOGENEOUS CASE

- Antenna Discrimination value is α_n
  - equals either 1 for no discrimination and α for discrimination
- Sequences are postulated arranging satellites with and without discrimination. For the sequences below satellites with the same letter have no discrimination
  - A,B,A,B,A,B,... (Sequence 1)
  - A,B,C,A,B,C,A,B,C,... (Sequence 2)
  - A,B,C,D,A,B,C,D,... (Sequence 3)
  - A,B,C,D,E,A,B,C,D,E,... (Sequence 4)
- The satellite with the highest interference can be the adjacent satellite or the first satellite with no discrimination

HOMOGENEOUS CASE

- For each sequence, the antenna discrimination is

  - $A, \alpha, 1, \alpha, 1, \ldots$
  - $A, \alpha, \alpha, 1, \alpha, \alpha, 1, \ldots$
  - $A, \alpha, \alpha, \alpha, 1, \alpha, \alpha, \alpha, 1, \ldots$
  - $A, \alpha, \alpha, \alpha, \alpha, 1, \alpha, \alpha, \alpha, \alpha, 1, \ldots$

- Then

$$ME = 2\alpha\theta^{-2.5} \sum_{\substack{s/c \\ \text{with} \\ \text{discrim.}}} n^{-2.5} + 2\theta^{-2.5} \sum_{\substack{s/c \\ \text{without} \\ \text{discrim.}}} n^{-2.5}$$

- Single Entry (SE) Interference

  - Adjacent Satellite

$$SE = \alpha\theta^{-2.5} \quad , \alpha \geq n_c^{-2.5}$$

  - First Satellite without discrimination

$$SE = n_c^{-2.5}\,\theta^{-2.5} \quad , \alpha \leq n_c^{-2.5}$$

where $n_c = 2,3,4,5,\ldots$ depending on sequence

HOMOGENEOUS CASE

- ME/SE Ratio

  - Adjacent Satellite

$$ME/SE = 2 \sum_{\substack{s/c \\ with \\ discrim}} n^{-2.5} + 2 \sum_{\substack{s/c \\ without \\ Discrim}} n^{-2.5}/\alpha \;; \alpha \geq n_c^{-2.5}$$

  - First Satellite without discrimination

$$ME/SE = 2\alpha n_c^{2.5} \sum_{\substack{s/c \\ with \\ discr.}} n^{-2.5} + 2 n_c^{2.5} \sum_{\substack{s/c \\ without \\ discr.}} n^{-2.5} \;; \alpha \leq n_c^{-2.5}$$

- Maximum ME/SE Ratio

  - The maximum ME/SE ratio occurs when adjacent satellite ME/SE equals the first satellite ME/SE or

$$ME/SE = 2 \sum_{\substack{s/c \\ with \\ discr.}} n^{-2.5} + 2n_c^{2.5} \sum_{\substack{s/c \\ without \\ discr.}} n^{-2.5}$$

HOMOGENEOUS CASE

o The ME/SE ratio can be written in the form

$$ME/SE = W + X/\alpha \quad ; \ \alpha \geq n_c^{-2.5}$$

$$ME/SE = Y\alpha + Z \quad ; \ \alpha \leq n_c^{-2.5}$$

o For 24 satellites the parameters are

Sequence	W	X	Y	Z
1	2.19272	.460116	12.4039	2.60281
2	2.48961	.16322	38.8093	2.54435
3	2.57528	.0775579	82.4089	2.48185
4	2.61073	.0421017	145.944	2.35355
5	2.62615	.0266898	231.578	2.35355
6	2.63741	.0154271	341.918	2
7	2.64179	.0110485	478.215	2
8	2.6446	8.23045E-03	642.639	2
9	2.64651	6.32455E-03	836.901	2

o For 48 satellites the parameters are

Sequence	W	X	Y	Z
1	2.20302	.468959	12.4622	2.65283
2	2.50331	.168669	39.0228	2.62929
3	2.59065	.0813378	82.9007	2.60281
4	2.62647	.0455148	146.824	2.54435
5	2.64313	.0288536	233.075	2.54435
6	2.65284	.0191439	343.919	2.48185
7	2.65827	.0137104	481.199	2.48185
8	2.6623	9.68541E-03	646.938	2.35355
9	2.66454	7.44259E-03	842.6[illegible]	2.35355

HOMOGENEOUS CASE

- o The maximum ME/SE ratio is

Sequence ME/SE max

NUMBER OF SATELLITES IS 12

1	4.64593
2	4.80538
3	4.54031
4	4.56703
5	4.58013

NUMBER OF SATELLITES IS 24

1	4.79553
2	5.03397
3	5.05713
4	4.96429
5	4.9797
6	4.63741
7	4.64179
8	4.6446
9	4.64651

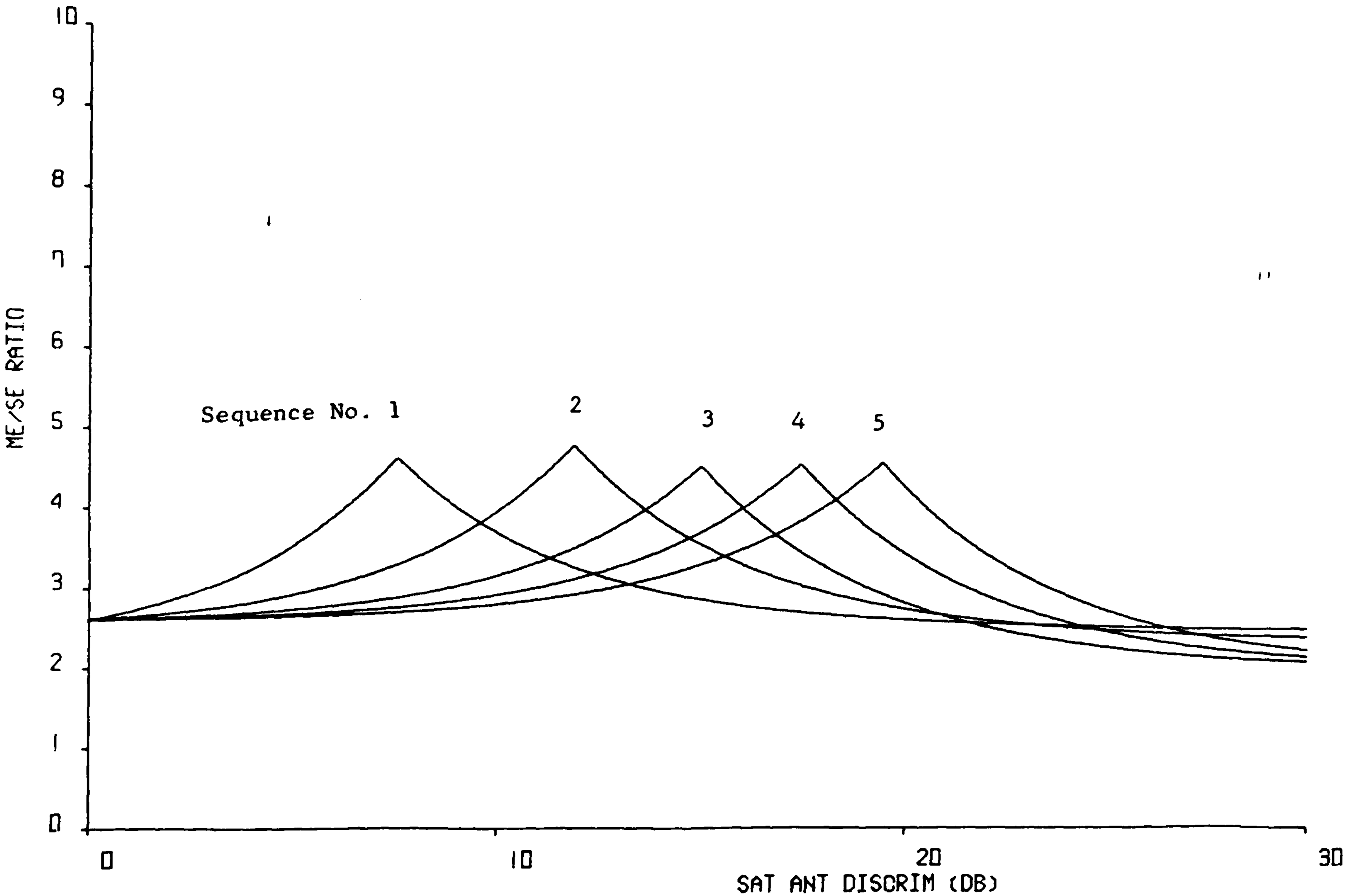
NUMBER OF SATELLITES IS 48

1	4.85586
2	5.1326
3	5.19346
4	5.17082
5	5.18748
6	5.13469
7	5.14013
8	5.01585
9	5.01809

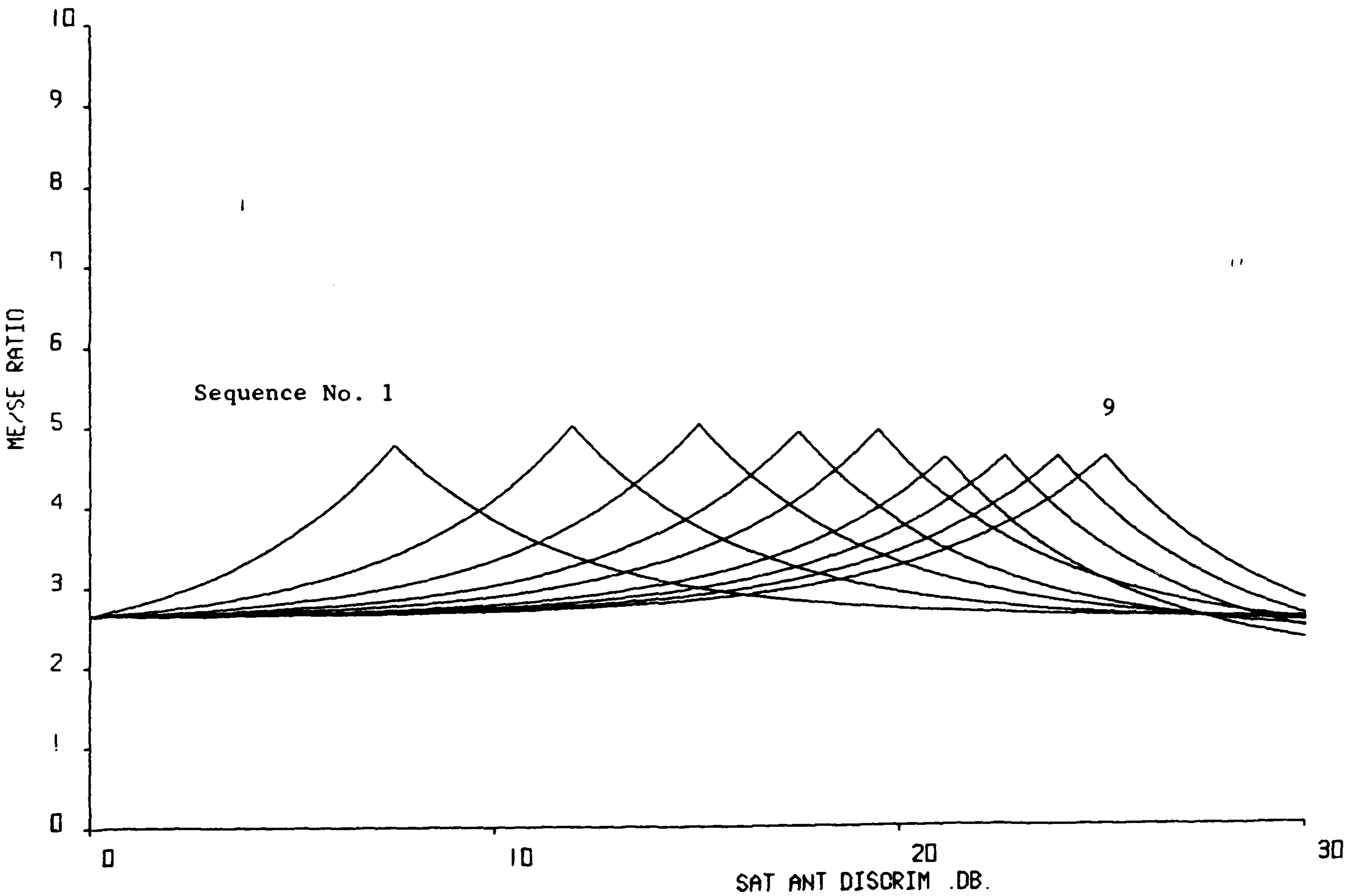
- o The maximum ME/SE ratio exhibits a maximum as a function of the sequence number

HOMOGENEOUS CASE – PLOTS

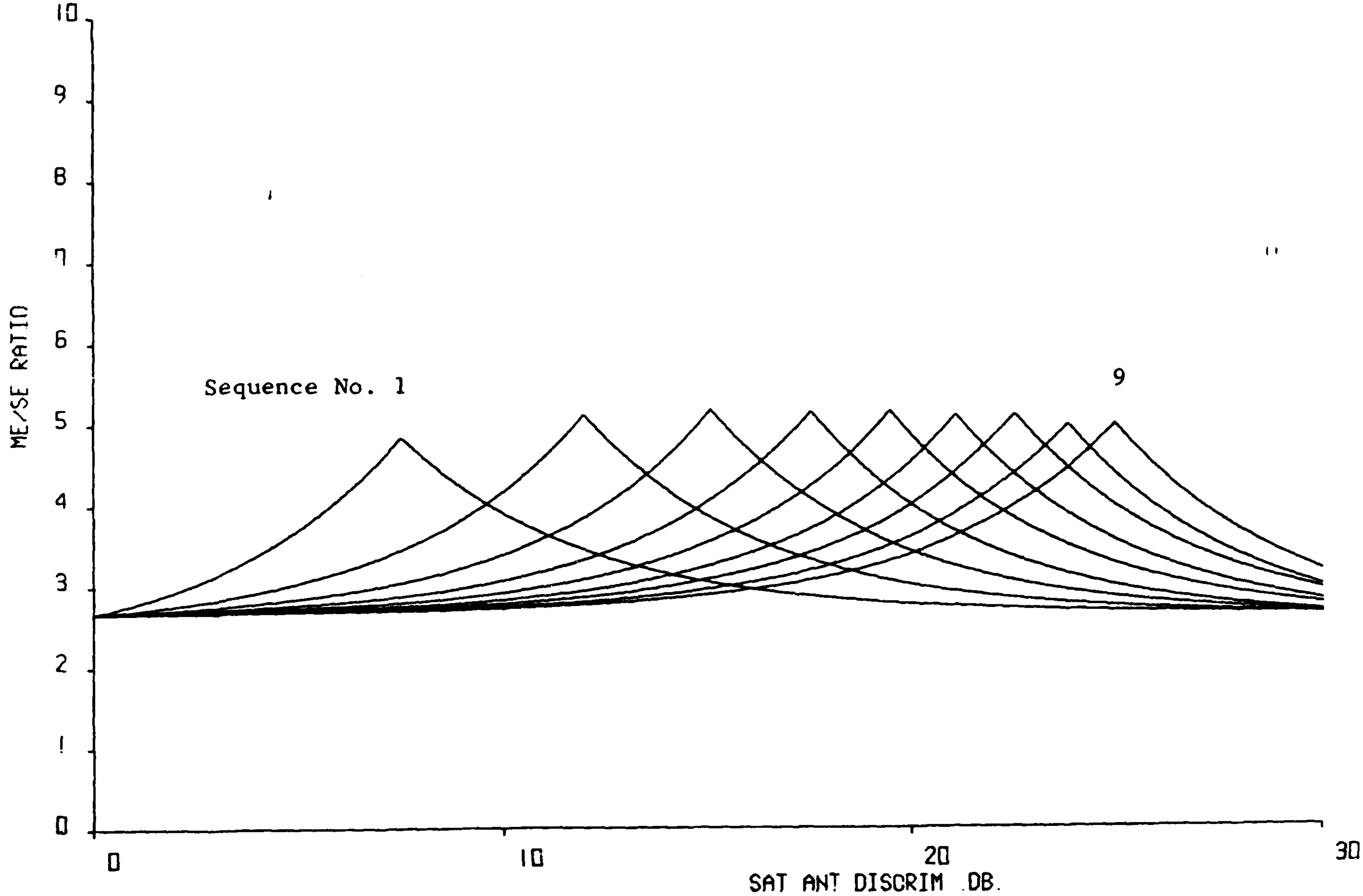
- o ME/SE Ratio versus Satellite Antenna Discrimination (dB)
  - Number of Satellites is 12, 24 and 50 in the three plots
  - Each line is for a sequence number of 1 through 9 as shown
- o Maximum ME/SE ratio versus Sequence Number
  - Each line is for a different total number of satellites



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. There are 6 satellites on either side of the victim.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. There are 12 satellites on either side of the victim.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. There are 24 satellites on either side of the victim.

HOMOGENEOUS CASE – ORBIT UTILIZATION

- The Multiple Entry interference is of the form

$$ME = (\alpha W + X)\,\theta^{-2.5}$$

where $W = 2 \sum_{\substack{\text{with} \\ \text{Discrim}}} n^{-2.5}$

$$X = 2 \sum_{\substack{\text{without} \\ \text{Discrim}}} n^{-2.5}$$

$$Y = n_c^{2.5} W = W/\alpha_c$$

$$Z = n_c^{2.5} X = X/\alpha_c$$

$$\alpha_c = n_c^{-2.5}$$

- When the discrimination, α, is one (0 dB), the angle, θ, can be adjusted to give the value of ME required for proper operation of the networks, or

$$ME_o = (W + X)\,\theta_o^{-2.5}$$

where θ_o is the minimum angle

HOMOGENEOUS CASE – ORBIT UTILIZATION

- When discrimination is used, the angle required to give the value of ME for proper operation can be found from

$$ME_o = (\alpha W + X)\,\theta_1^{-2.5}$$

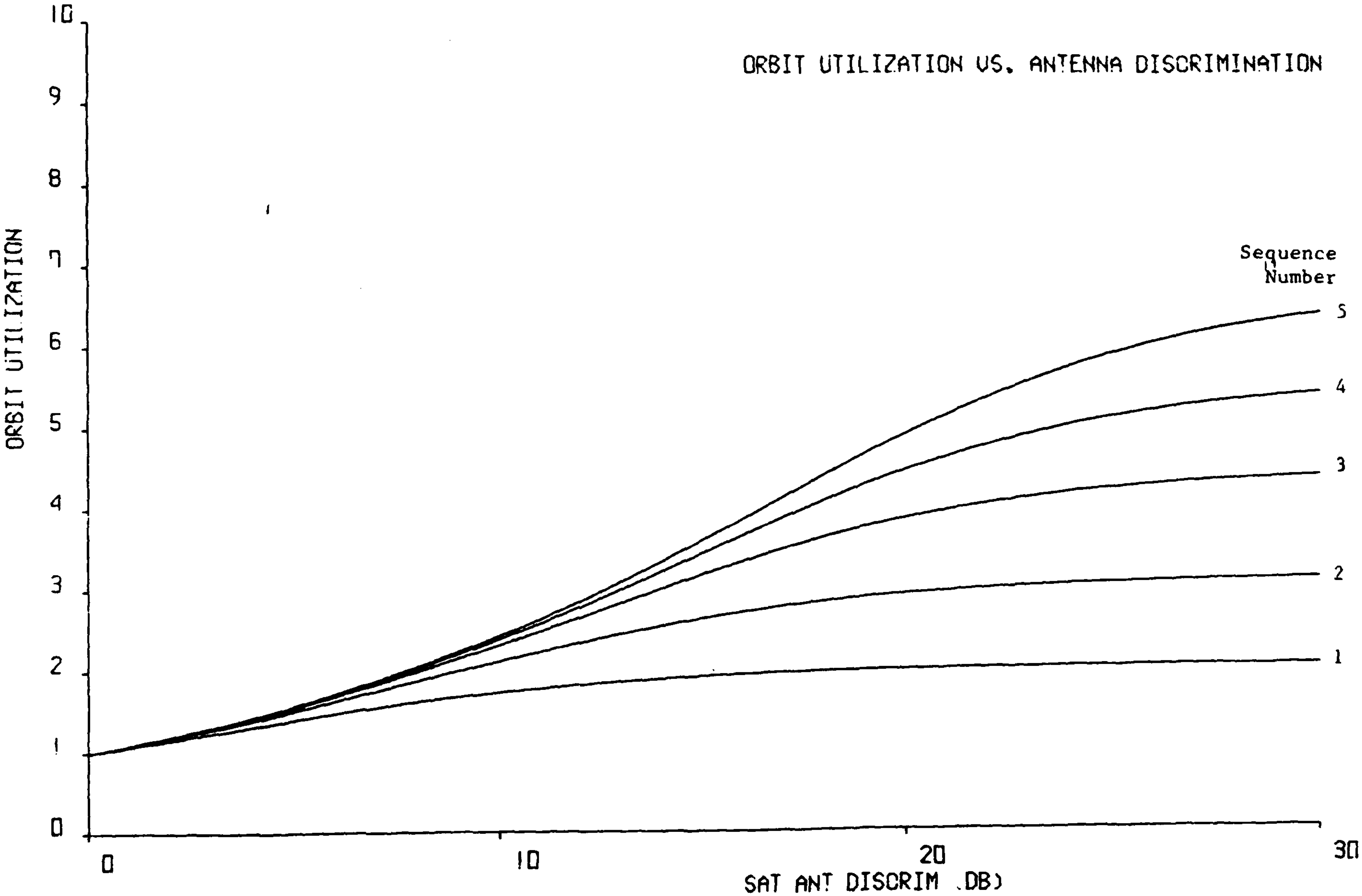
where θ_1 is this angle

- The ratio, θ_o/θ_1, is a measure of the increase in orbit utilization and is given by

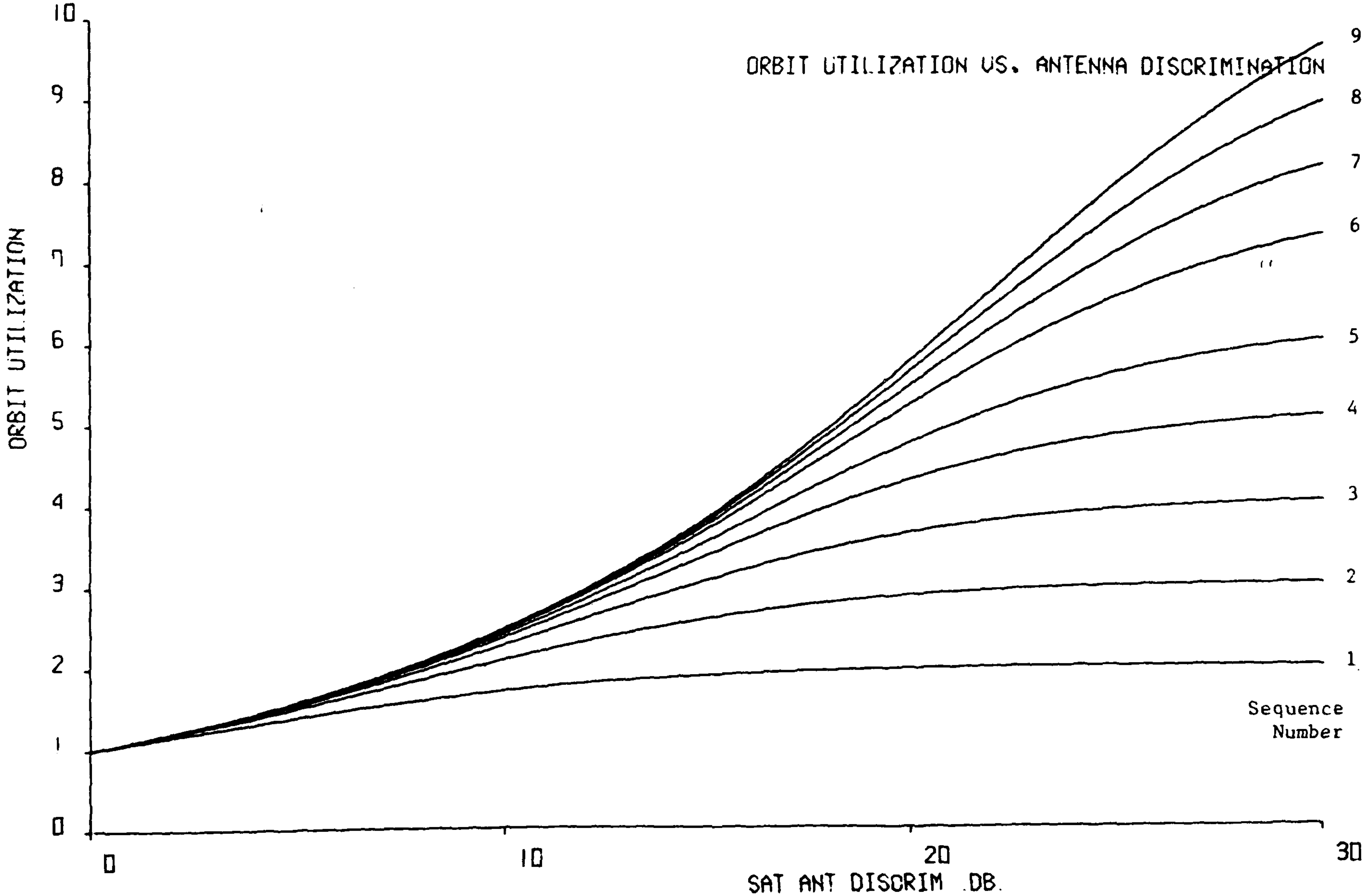
$$\frac{\theta_o}{\theta_1} = \left[\frac{W + X}{\alpha W + X}\right]^{0.4}$$

Notice that $\alpha W + X = \alpha_c(\alpha Y + Z)$

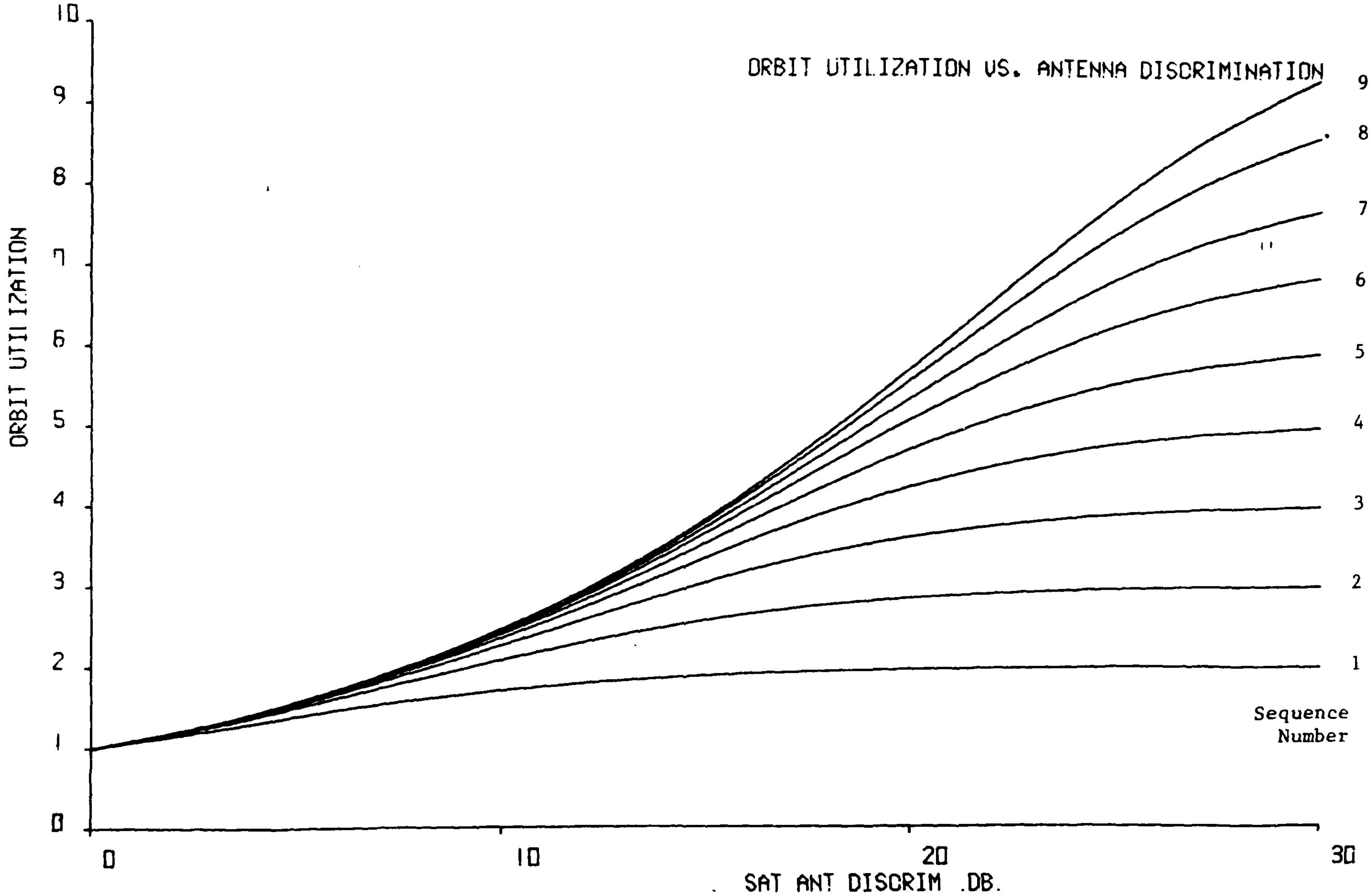
- The plot shows the increase in orbit utilization, θ_o/θ_1 as a function of the satellite antenna discrimination for the various sequences



Orbit utilization factor as a function of the satellite antenna discrimination for 12 satellites (6 on either side of the victim).



Orbit utilization factor as a function of the satellite antenna discrimination for 24 satellites (12 on either side of the victim).



Orbit utilization factor as a function of the satellite antenna discrimination for 48 satellites (24 on either side of the victim).

HOMOGENEOUS CASE – APPENDIX 29 ANTENNA PATTERNS

- Single entry interference has the form

$$SE = \alpha K G(\theta)$$

where α is the satellite antenna discrimination, $G(\theta)$ is the normalized earth station antenna discrim. from Appendix 29 and all other parameters are contained in K including the maximum earth station gain

- It will be assumed that the composite earth station and satellite antenna required discrimination is 30 dB for proper operation of the satellite networks

- The required separation angle, θ_R , is found as the solution to

$$\alpha + G(\theta) = 30 \text{ dB} \quad \text{(all in dB)}$$

HOMOGENEOUS CASE – APPENDIX 29 ANTENNA PATTERNS

- Multiple entry interference is given by

$$ME = 2\sum_{n=1}^{N} \alpha_n KG(n\theta_R)$$

where θ_R is the solution of $G(\theta_R) = 30 - \alpha_1$ for adjacent satellite having the largest interference or $\theta_R = \theta_R' / n_c$ and θ_R' is the solution of $G(\theta_R') = 30$ for the first satellite without discrimination having the largest interference. n_c is the number of satellites between the first satellite without discrimination and the victim satellite plus one and α_n is as before.

HOMOGENEOUS CASE – APPENDIX 29 ANTENNA PATTERNS

- o Single entry interference is

$SE_A = \alpha_1 KG(\theta)$	for adjacent sat. interfer.
$SE_F = KG(\theta)$	for first sat. without discrimination

- o The largest single entry interference is found as
  - The first satellite without discrimination is placed such that the earth station discrimination is 30 dB. The angle is θ_R'.
  - The adjacent satellite for this case would be at an angle of θ_R' / n_c and have a total discrimination of $\alpha_1 + G(\theta_R' / n_c)$.
  - If this discrimination is less than 30 dB, the adjacent satellite has a larger single entry interference and SE_A is used in the ME/SE ratio. Otherwise, SE_F is used.

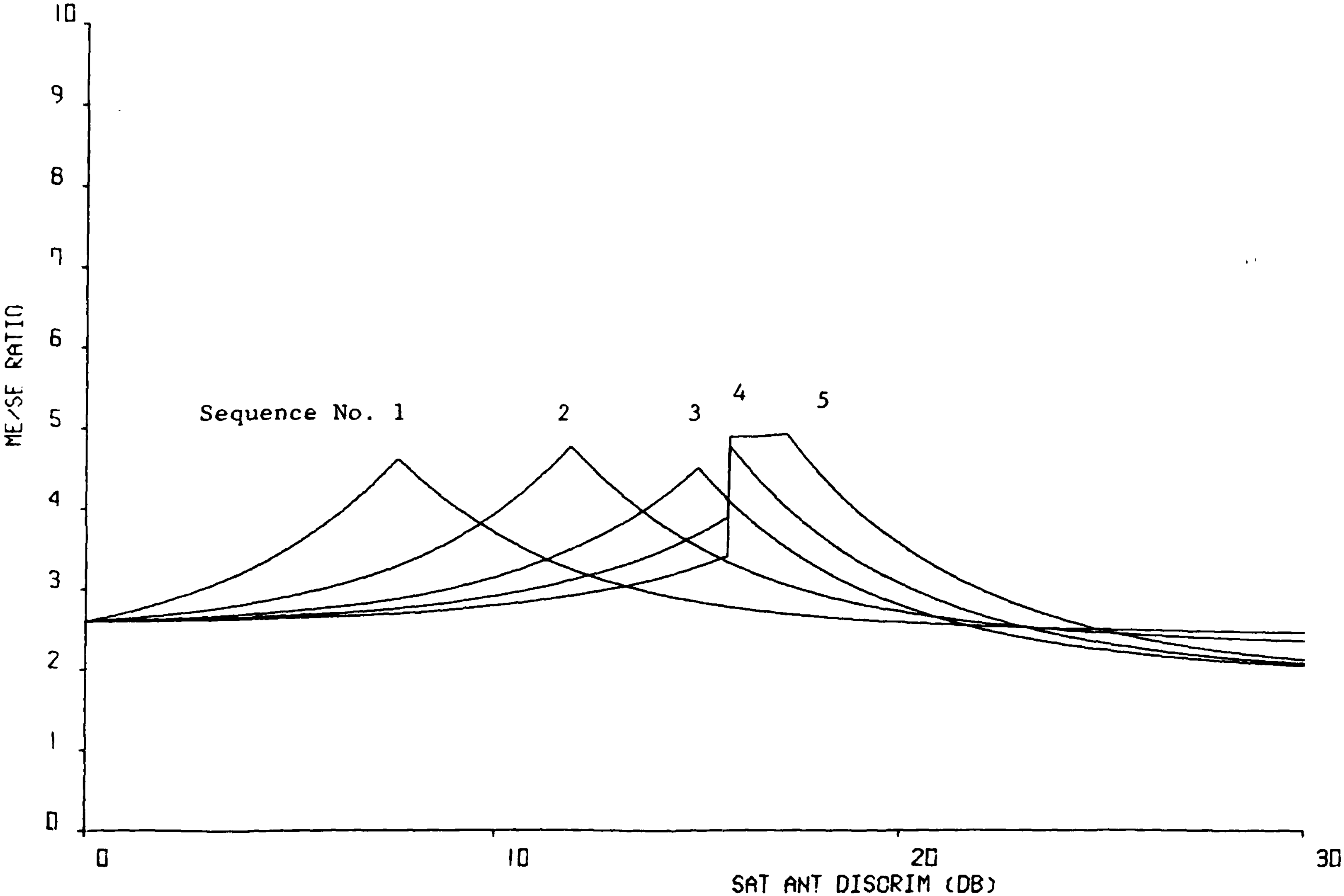
HOMOGENEOUS CASE – APPENDIX 29 ANTENNA PATTERNS

- o The ME/SE ratio is then

$ME/SE = ME/SE_A$ Adjacent Satellite

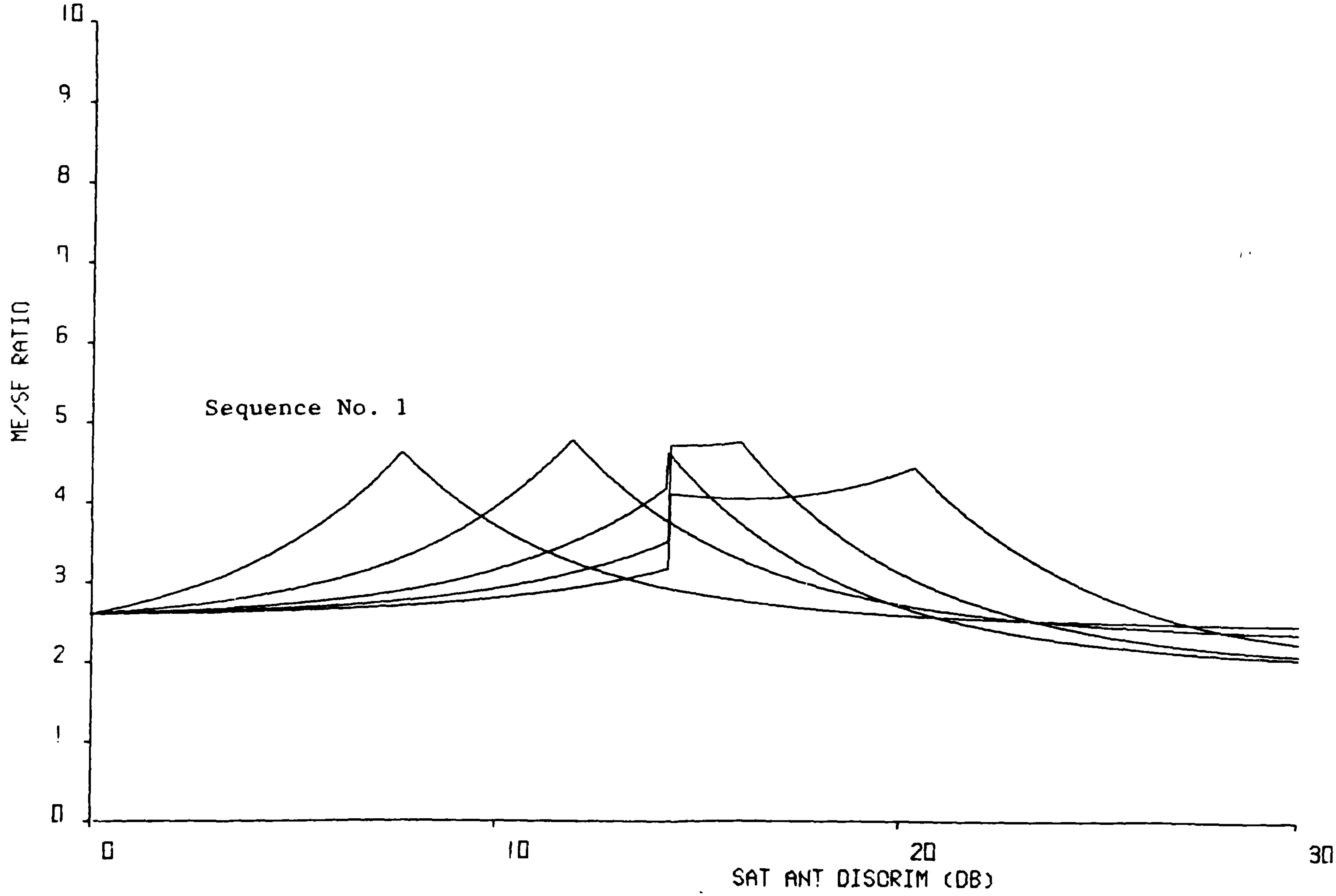
$= ME/SE_F$ First sat. w/o discrim.

- o Plots of ME/SE ratio as a function of the satellite antenna discrimination for various earth station sizes (D/λ) follow
  - – D/λ is 50, 100, 150, 200
  - – The sequence number ranges from 1 to 9 in each plot
  - – A total of 12, 24 or 50 satellites, half on each side of the victim, are used
  - – Required total discrimination is 30 dB

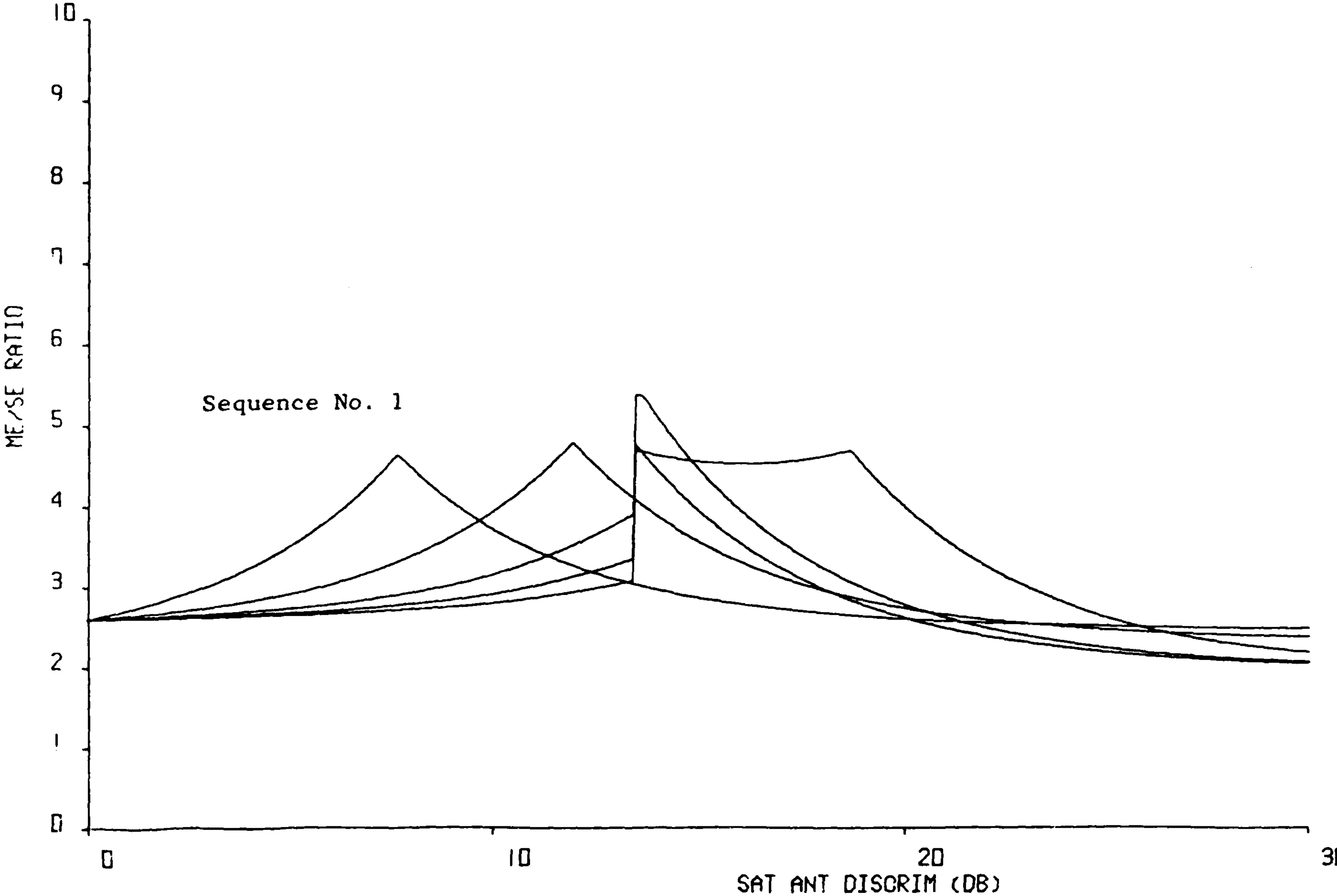


Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 6 satellites on either side of the victim. Required total discrimination is 30 db.

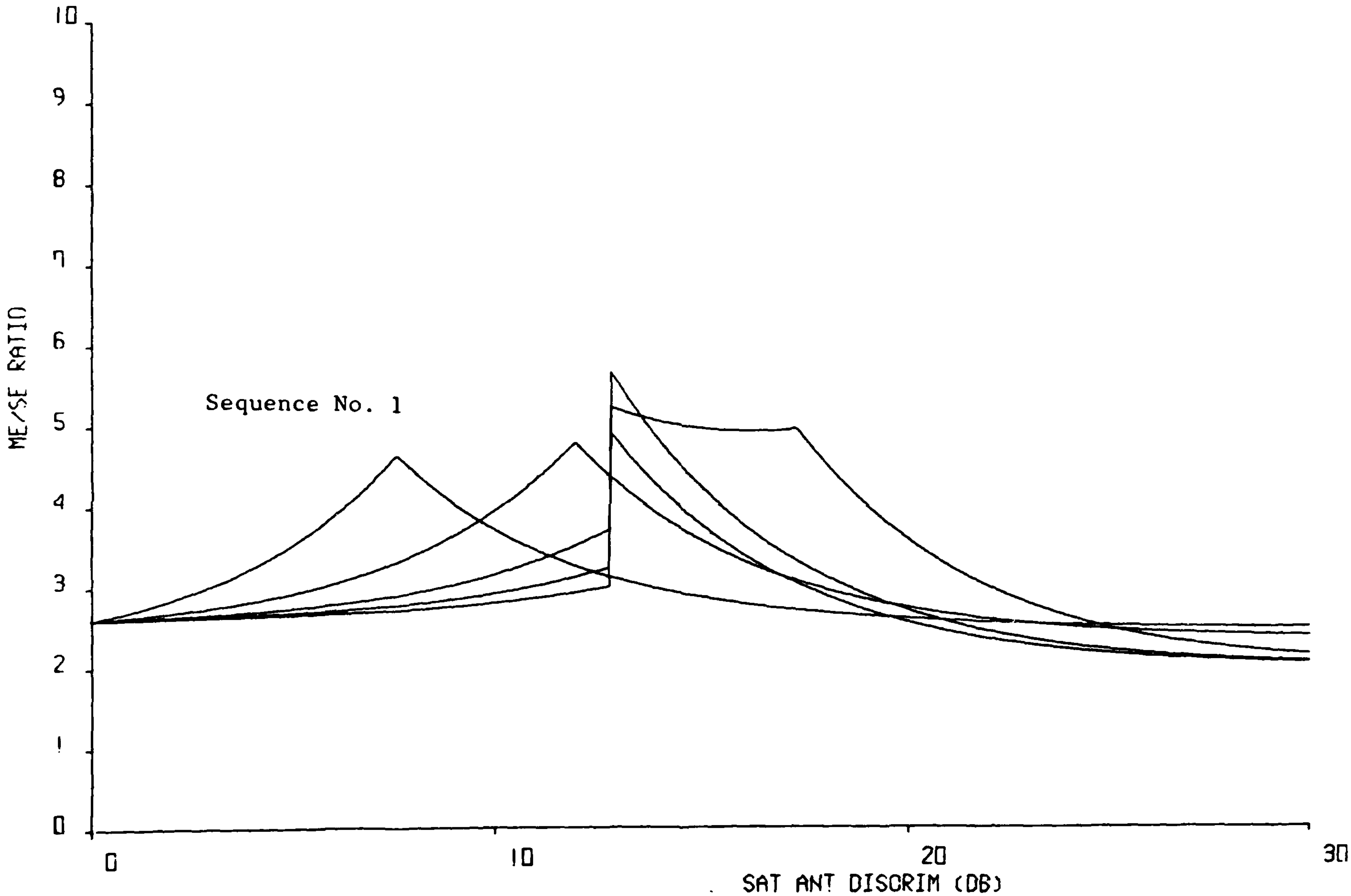
26



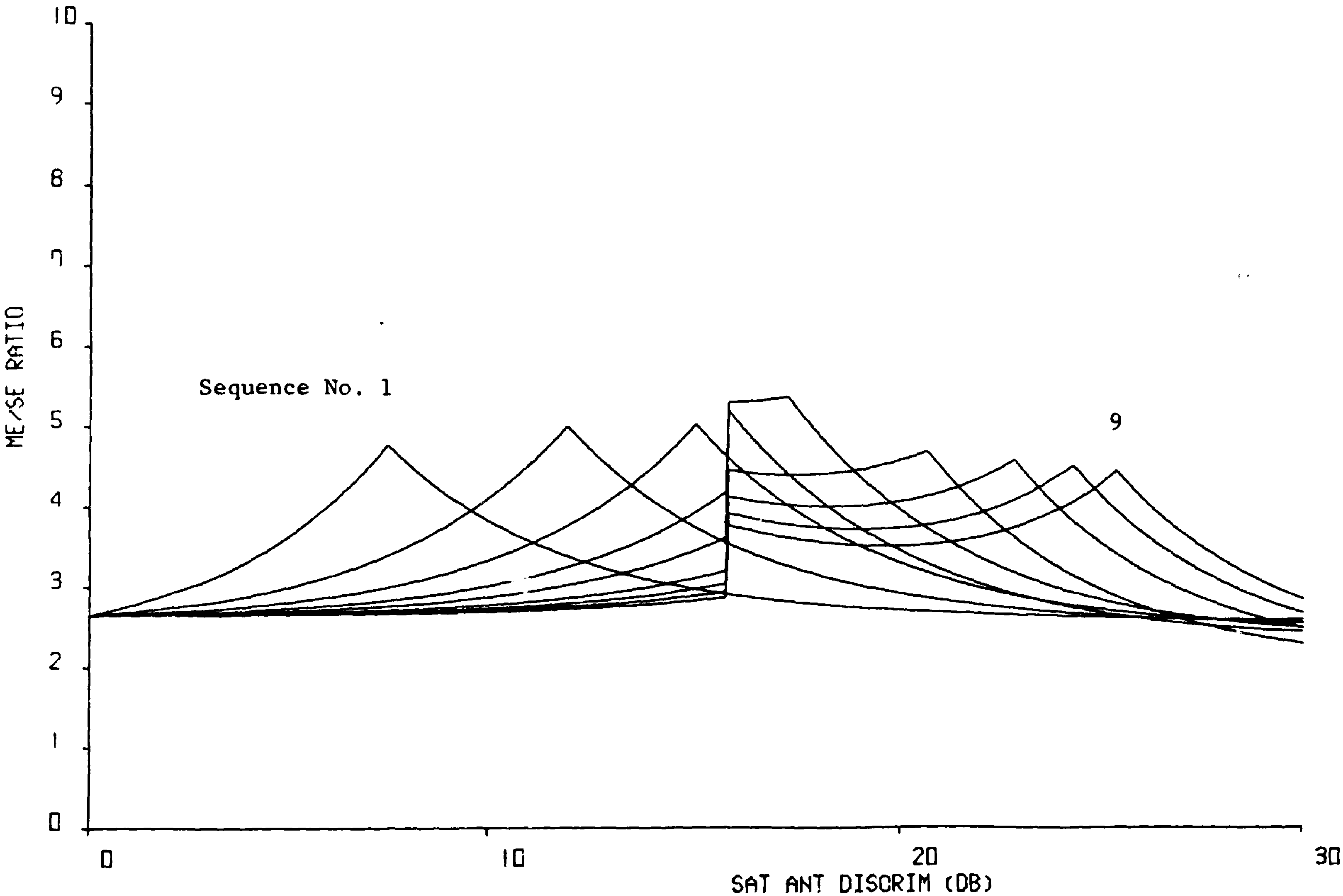
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 6 satellites on either side of the victim. Required total discrimination is 30 db.



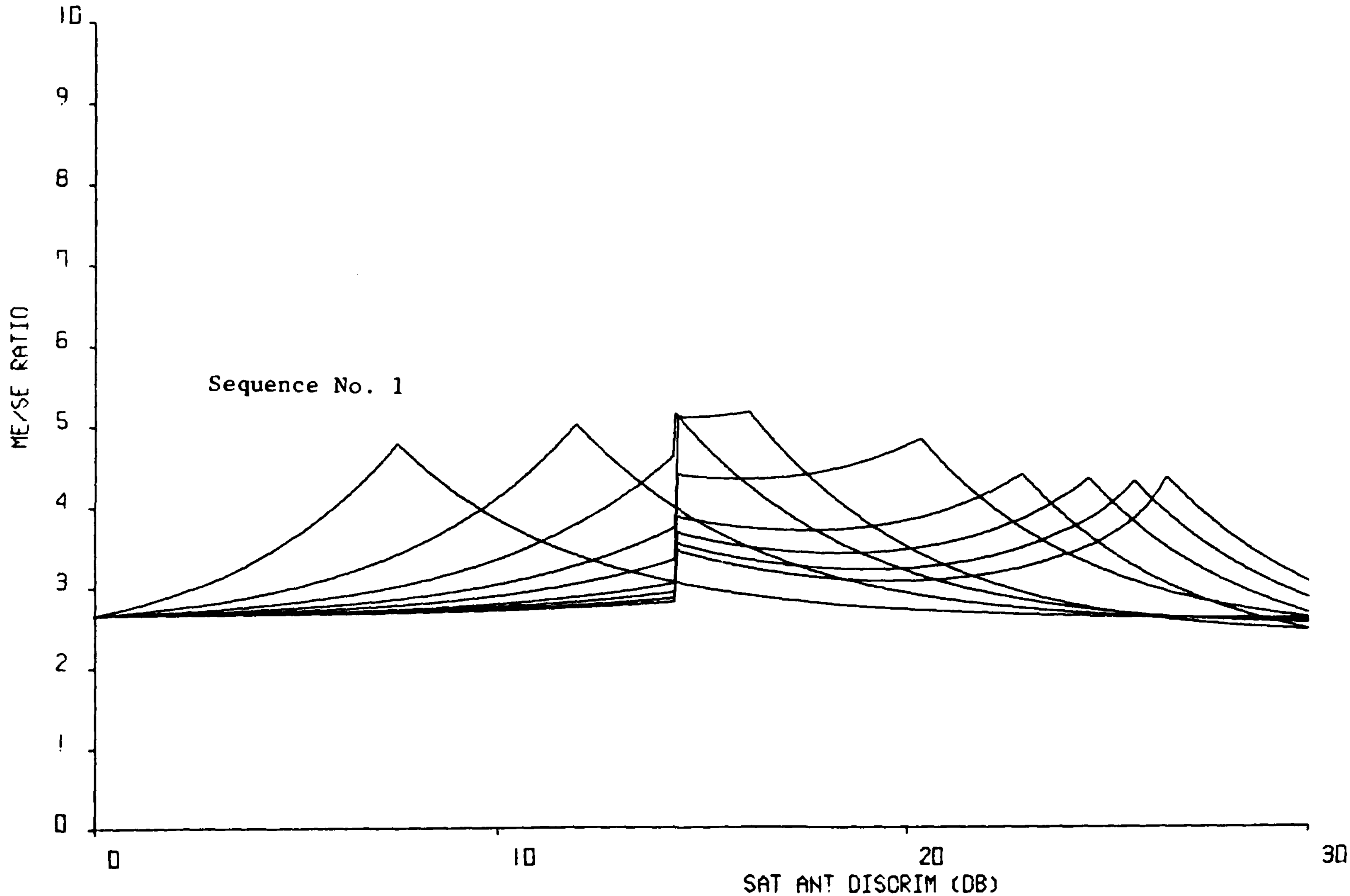
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 6 satellites on either side of the victim. Required total discrimination is 30 db.



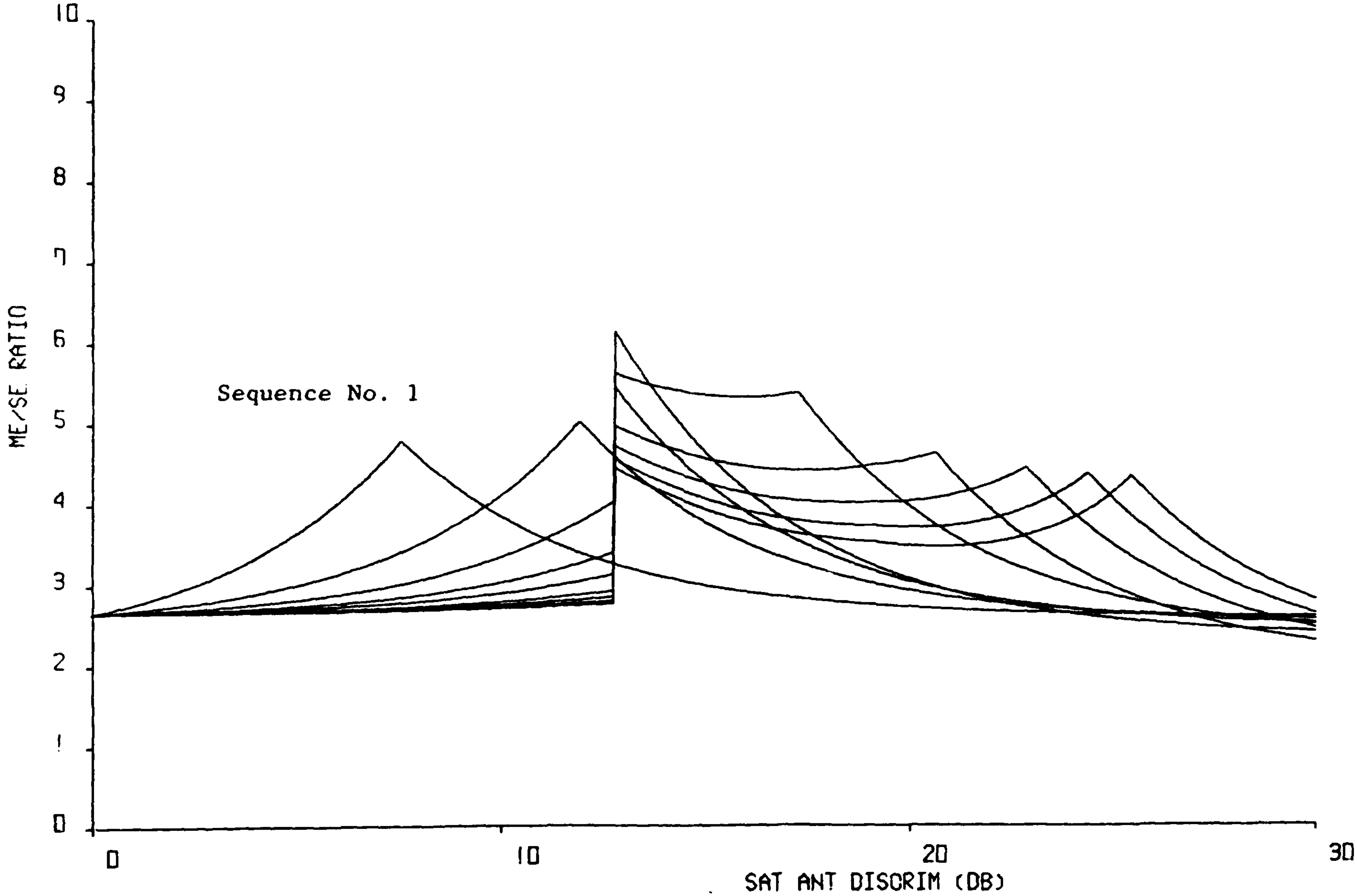
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 6 satellites on either side of the victim. Required total discrimination is 30 db.



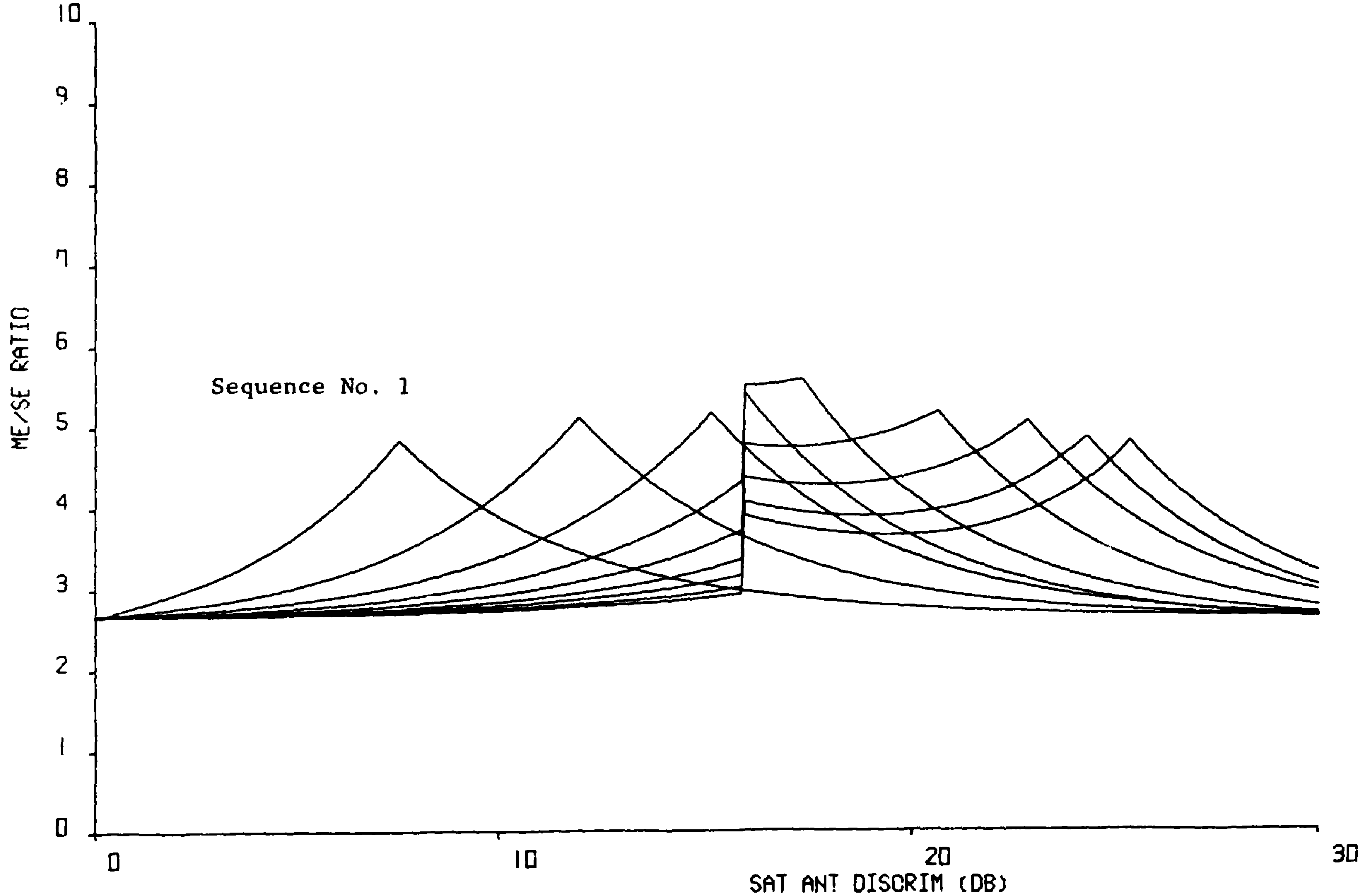
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 12 satellites on either side of the victim. Required total discrimination is 30 db.



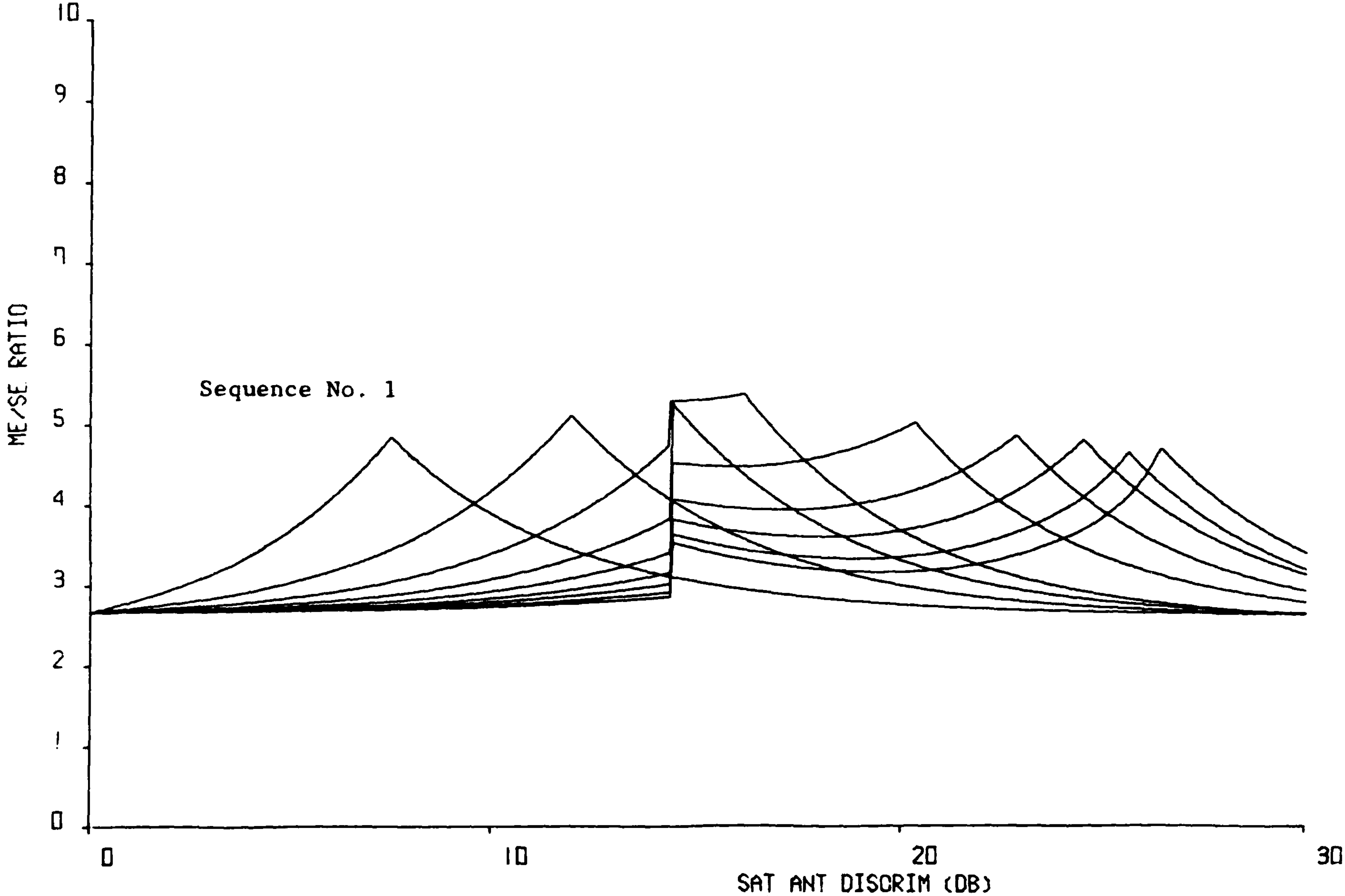
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db.



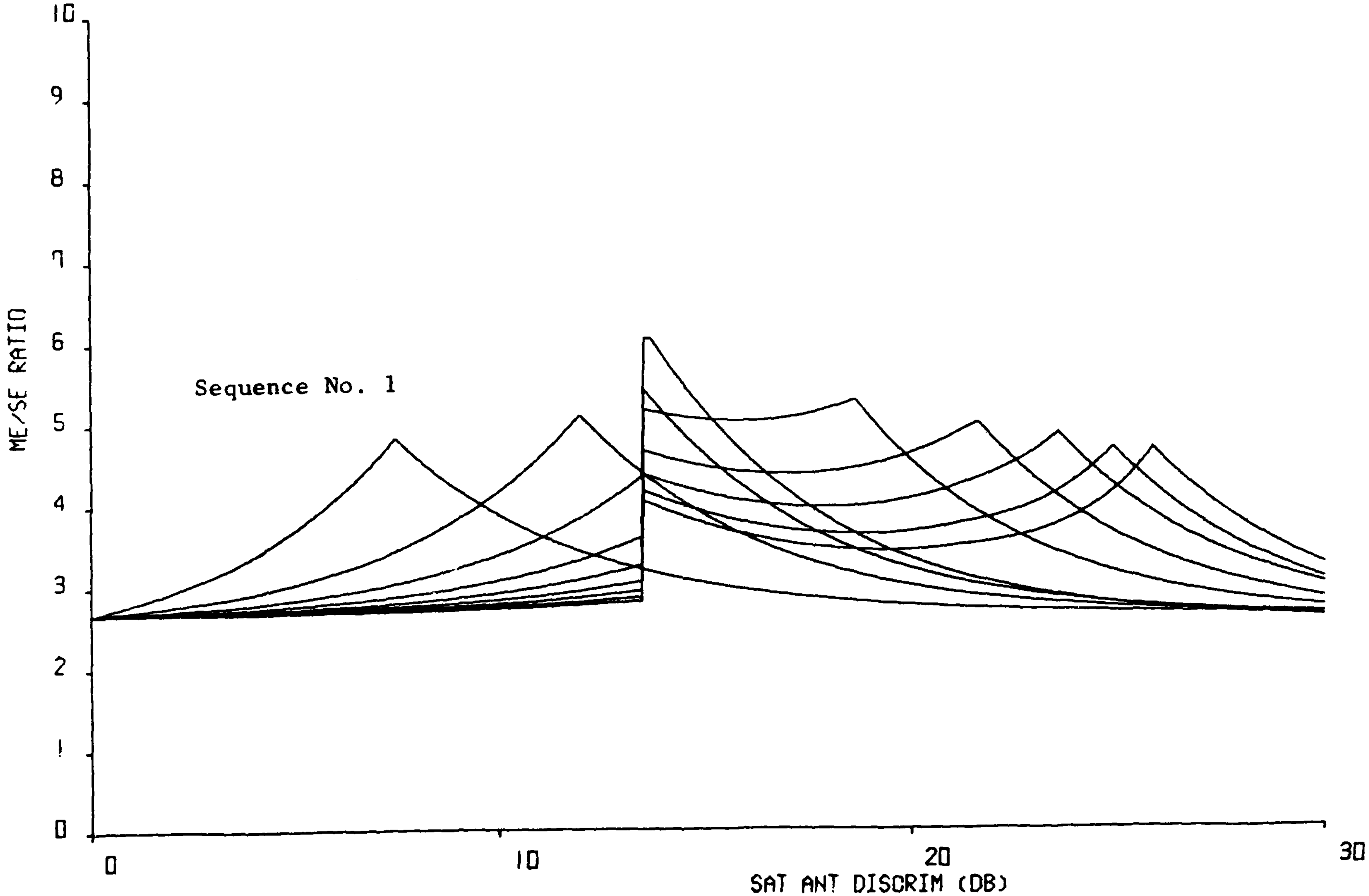
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 12 satellites on either side of the victim. Required total discrimination is 30 db.



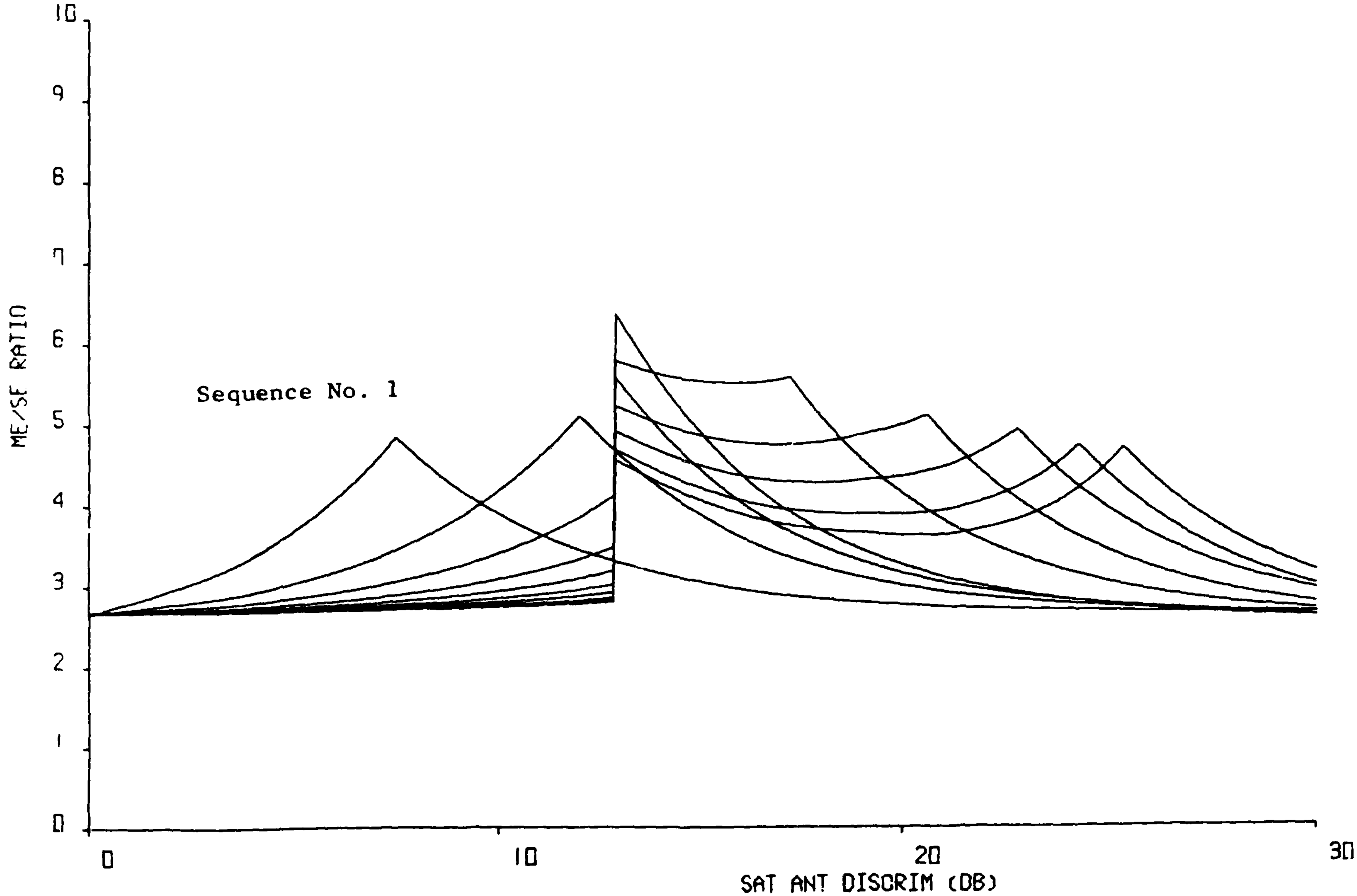
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 25 satellites on either side of the victim. Required total discrimination is 30 db.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 25 satellites on either side of the victim. Required total discrimination is 30 db.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 25 satellites on either side of the victim. Required total discrimination is 30 db.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 25 satellites on either side of the victim. Required total discrimination is 30 db.

APPENDIX 29 ANTENNA PATTERNS

- o WARC-79 Reference Pattern
- o Plot of Antenna Patterns
  - Earth station discrimination as a function of off-boresight angle
  - Various diameter to wavelength ratios of 50 to 250

WARC – 79 REFERENCE PATTERN

o $D/\lambda \geqslant 100$

$$
\begin{aligned}
G(\theta) &= G_{max} - 2.5\times10^{-3} \cdot \left(\frac{D}{\lambda}\theta\right)^2 && , 0 < \theta < \theta_m \\
&= G_1 && , \theta_m < \theta < \theta_r \\
&= 32 - 25 \operatorname{Log} \theta && , \theta_r \le \theta < 48^\circ \\
&= -10 && , 48^\circ \le \theta \le 180^\circ
\end{aligned}
$$

where:

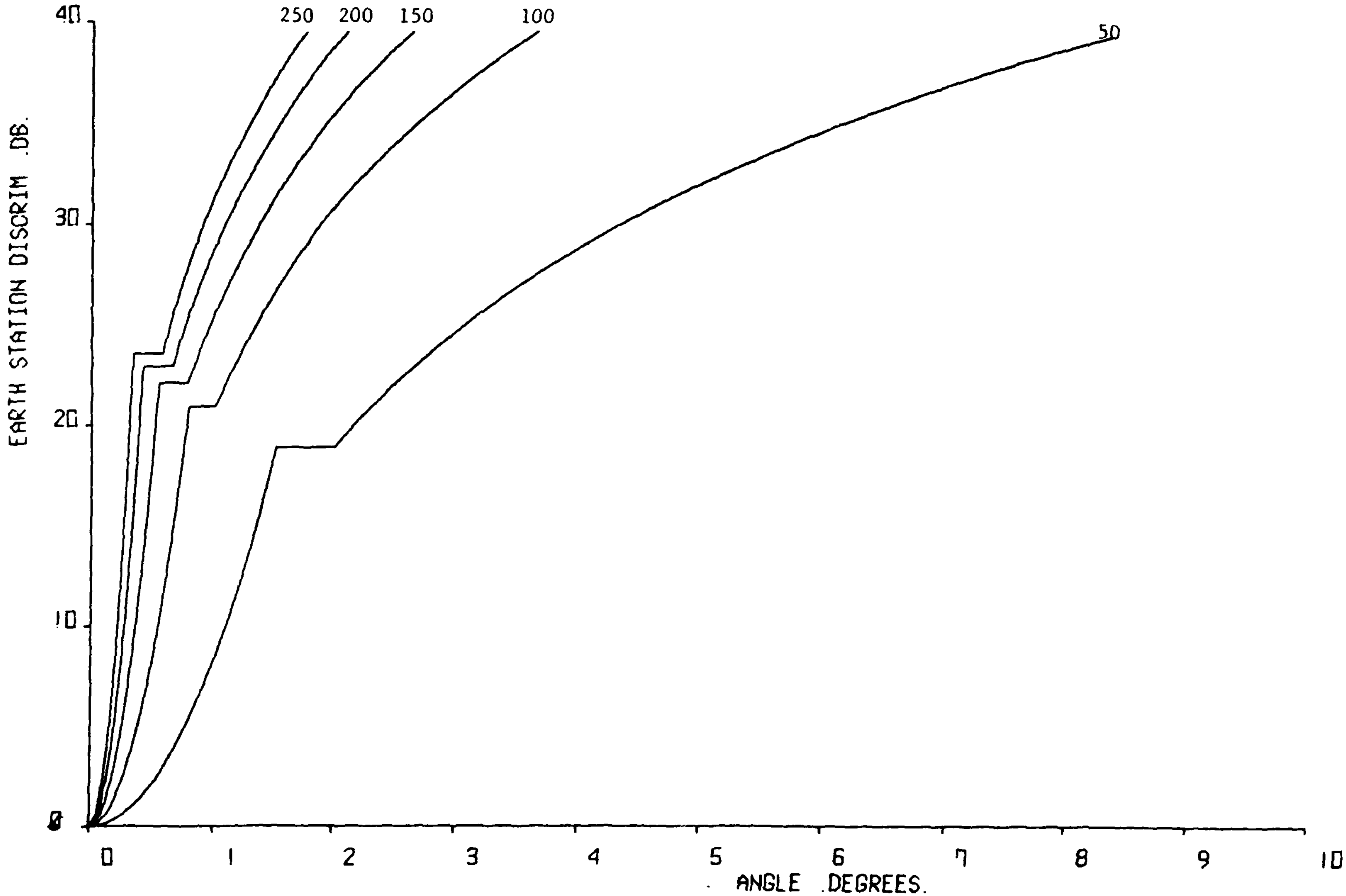
$$
\begin{aligned}
G_1 &= 2 + 15 \operatorname{Log} D/\lambda \\
\theta_m &= 20\lambda/D \sqrt{G_{max} - G_1} \quad \text{(degrees)} \\
\theta_r &= 15.85 \, (D/\lambda)^{-0.6}
\end{aligned}
$$

o $D/\lambda < 100$

$$
\begin{aligned}
G(\theta) &= G_{max} - 2.5\times10^{-3} \left(\frac{D}{\lambda}\theta\right)^2 && , 0 < \theta < \theta_m \\
&= G_1 && , \theta_m < \theta < 100\,\lambda/D \\
&= 52 - 10 \operatorname{Log} (D/\lambda) - 25 \operatorname{Log} \theta && , 100\,\lambda/D \le \theta < 48^\circ \\
&= 10 - 10 \operatorname{Log} D/\lambda && , 48^\circ \le \theta \le 180^\circ
\end{aligned}
$$

o When D/λ is not provided

$$20 \operatorname{Log} D/\lambda = G_{max} - 7.7$$



Earth station antenna discrimination as a function of off-boresight angle from Appendix 29 of the Radio Regulations for earth station antenna diameter to wavelength ratios of 50 to 250

HOMOGENEOUS CASE – ORBIT UTILIZATION APPENDIX 29 PATTERNS

- Multiple entry interference is of the form

$$ME = (\alpha W + X)$$

where $W = 2 \sum_{\substack{\text{with} \\ \text{discrim}}} G(n\theta)$

$$X = 2 \sum_{\substack{\text{without} \\ \text{discrim}}} G(n\theta)$$

- When the discrimination, α , is one (0 dB), the angle, θ, can be adjusted to give the value of ME required for proper operation of the networks, or

$$ME_o = (W_o + X_o)$$

where W_o and X_o are functions of θ_o

HOMOGENEOUS CASE – ORBIT UTILIZATION APPENDIX 29 PATTERNS

- θ_o is found by setting the total discrimination to −30 dB, setting the satellite antenna discrimination to 0 dB and solving for the angle or

$$G(\theta_o) = -30 \text{ dB}$$

- The multiple entry interference for this angle is computed and used as the reference interference as

$$ME_o = W_o + X_o$$

- When discrimination is used, the angle required to give the value of ME for proper operation can be found from

$$ME_o = (\alpha W_I + X_I)$$

where W_I and X_I are functions of θ_I, the required angle.

HOMOGENEOUS CASE – ORBIT UTILIZATION APPENDIX 29 PATTERNS

- The plots show the increase in orbit utilization θ_o / θ_1 , as a function of the satellite antenna discrimination for various sequences and various earth station antenna sizes

EAST – WEST STATIONKEEPING ERROR

o Impact of East – West stationkeeping error on the ME/SE ratio

o Performance measures

- Worst case ME/SE ratio
  - – Maximize ME, minimize SE
- Worst case SE, worst case ME
  - – Maximize ME, maximize SE
- Expected value of ME/SE ratio
  - – E{ME/SE}
- Expected value of ME/SE ratio given worst case SE

o East – West stationkeeping error

- less than 0.1 degrees (Radio Regulations)
- independent, zero mean, uniform distribution between +0.1 and –0.1 degrees

EAST–WEST STATIONKEEPING ERROR FORMULATION OF THE PROBLEM

- The angular distance from the victim satellite (number 0) to satellite n is given by

$$
\begin{aligned}
&n\theta + \Delta\theta_n + \Delta\theta_o && ;\ n>0 \\
&n\theta + \Delta\theta_n - \Delta\theta_o && ;\ n<0
\end{aligned}
$$

where $\Delta\theta_n$ is a random variable representing the East–West stationkeeping error of satellite n

- The interference from satellite n into the victim satellite 0 is

$$\alpha_n KG(n\theta + \Delta\theta_n \pm \Delta\theta_o)$$

where the sign depends on n

EAST–WEST STATIONKEEPING ERROR FORMULATION OF THE PROBLEM

- The multiple entry interference is then

$$ME = K \sum_{\substack{n=-N \\ n \neq 0}}^{N} \alpha_n G(n\theta + \Delta\theta_n + n/|n|\,\Delta\theta_o)$$

- The ME/SE ratio is given by

$$ME/SE = \frac{\sum_{\substack{n=-N \\ n \neq 0}}^{N} \alpha_n G(n\theta + \Delta\theta_n + \Delta\theta_o\, n/|n|)}{\alpha_m G(m\theta + \Delta\theta_m + \Delta\theta_o\, m/|m|)}$$

EAST–WEST STATIONKEEPING ERROR
WORST CASE ME/SE RATIO

o Assumptions

- Victim satellite is at nominal location
- All other satellites are moved 0.1 degrees toward the victim satellite
- Appendix 29 antenna patterns are used

WORST CASE ME/SE RATIO – APPENDIX 29 PATTERNS

- o Worst case single entry interference is given by

$$SE_{wc} = \alpha KG(\theta_R' - 0.1)$$

- o Worst case multiple entry interference is given by

$$ME = 2K \sum_{n=1}^{N} \alpha_n G(n\theta_R - 0.1)$$

- o The co – coverage satellite is nominally at an angle of θ_R' from the victim satellite where θ_R' is chosen to provide 30 dB of discrimination

- o For this case, the adjacent satellite would be located at an angle of θ_R' / n_c giving discrimination of $G(\theta_R' / n_c) + A$ (dB)

- o If the adjacent satellite provides less descrimination it is used in the ME/SE ratio and reset at an angle providing 30 dB of discrimination

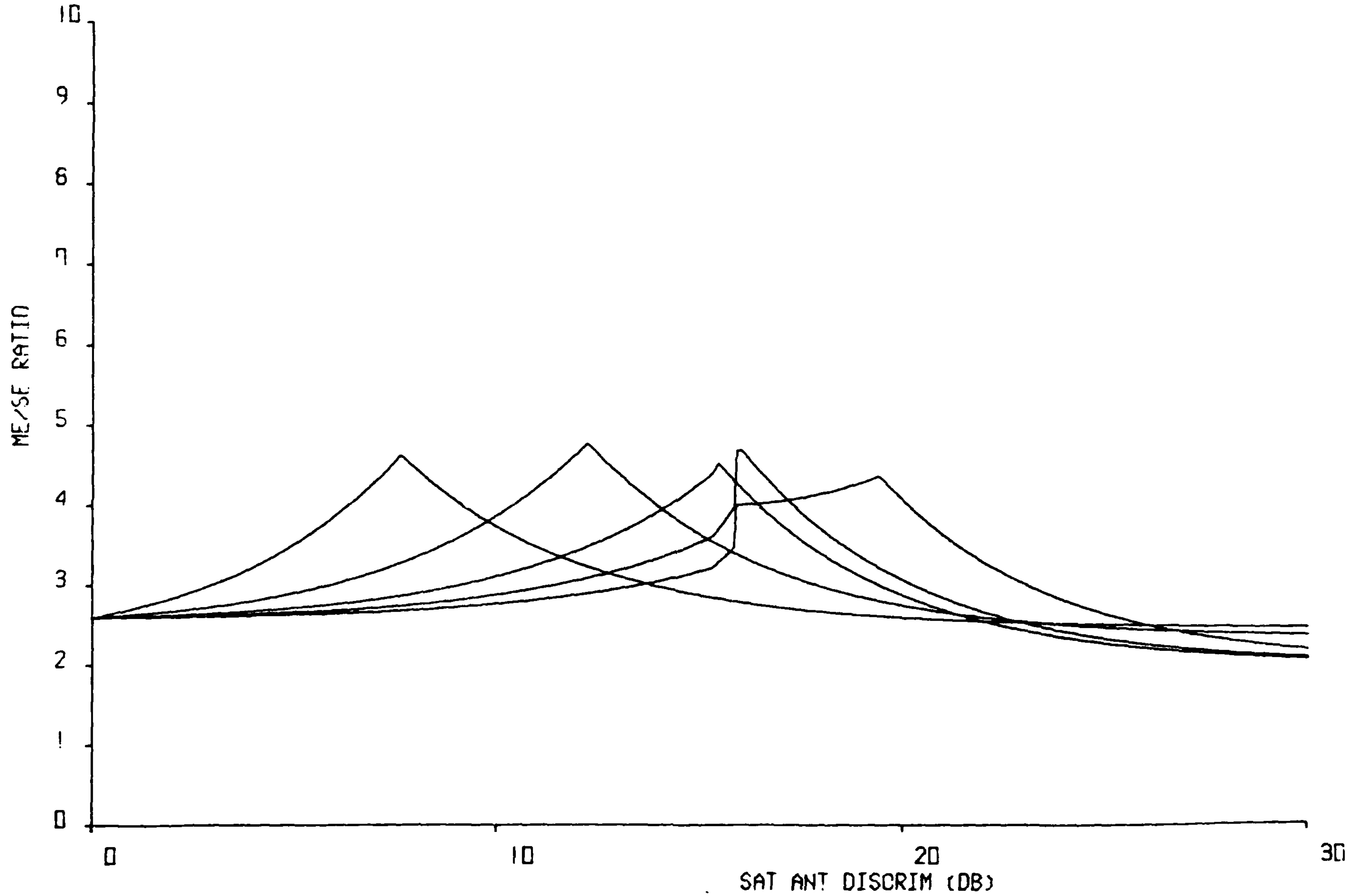
WORST CASE ME/SE RATIO – APPENDIX 29 PATTERNS

- o With the worst case Stationkeeping error, the co-coverage satellite has discrimination of $G(\theta_R' - 0.1)$

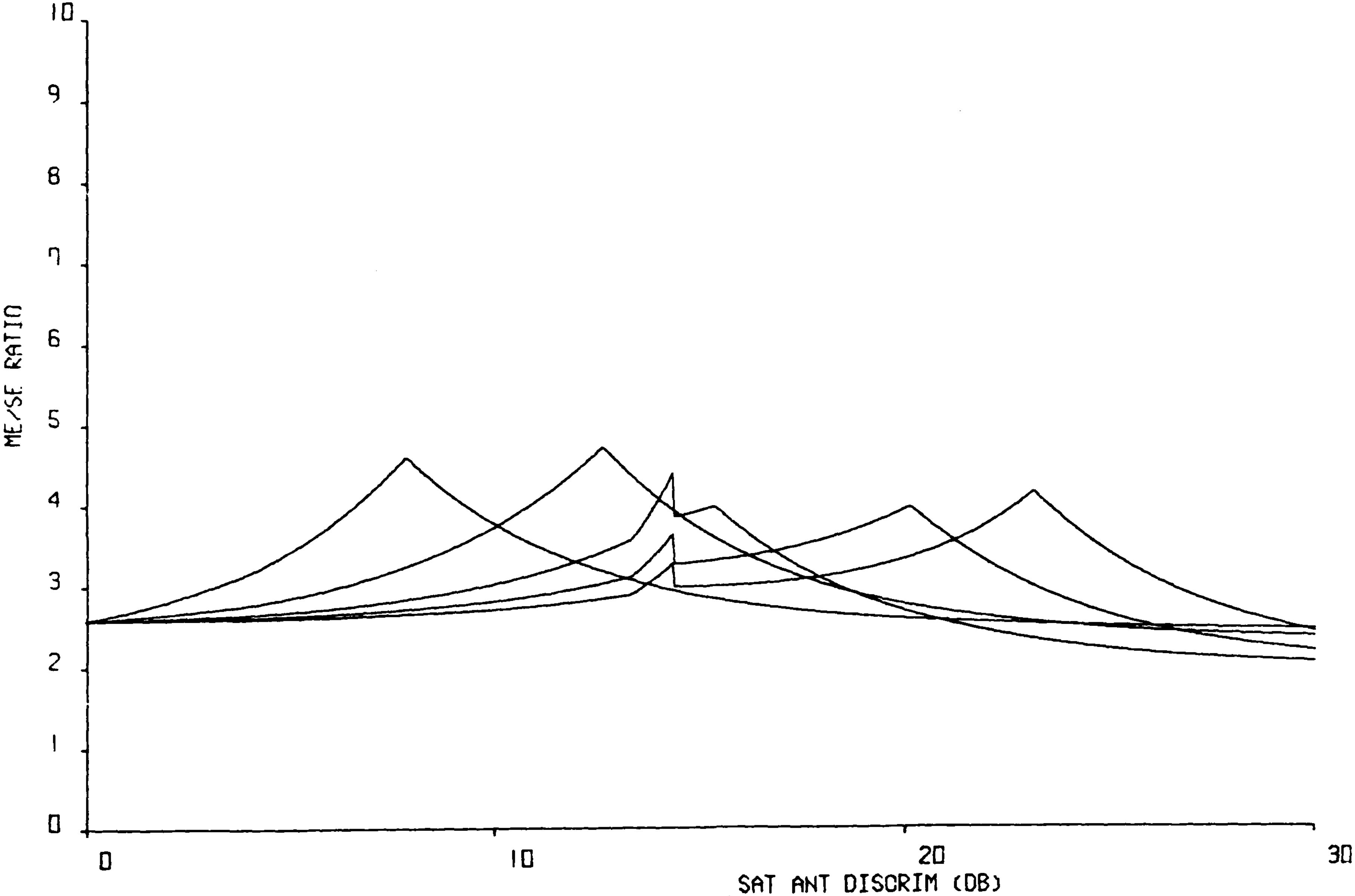
- o With the worst case stationkeeping error, the adjacent satellite has discrimination of $G(\theta_R' / n_c - 0.1) + A$ (dB)

- o If the adjacent satellite with the worst case stationkeeping error provides less discrimination than the co-coverage satellite with the worst case stationkeeping error, it is used in the ME/SE ratio and reset to a nominal angle providing 30 dB of discrimination. The SE used would have this nominal angle minus the stationkeeping error.

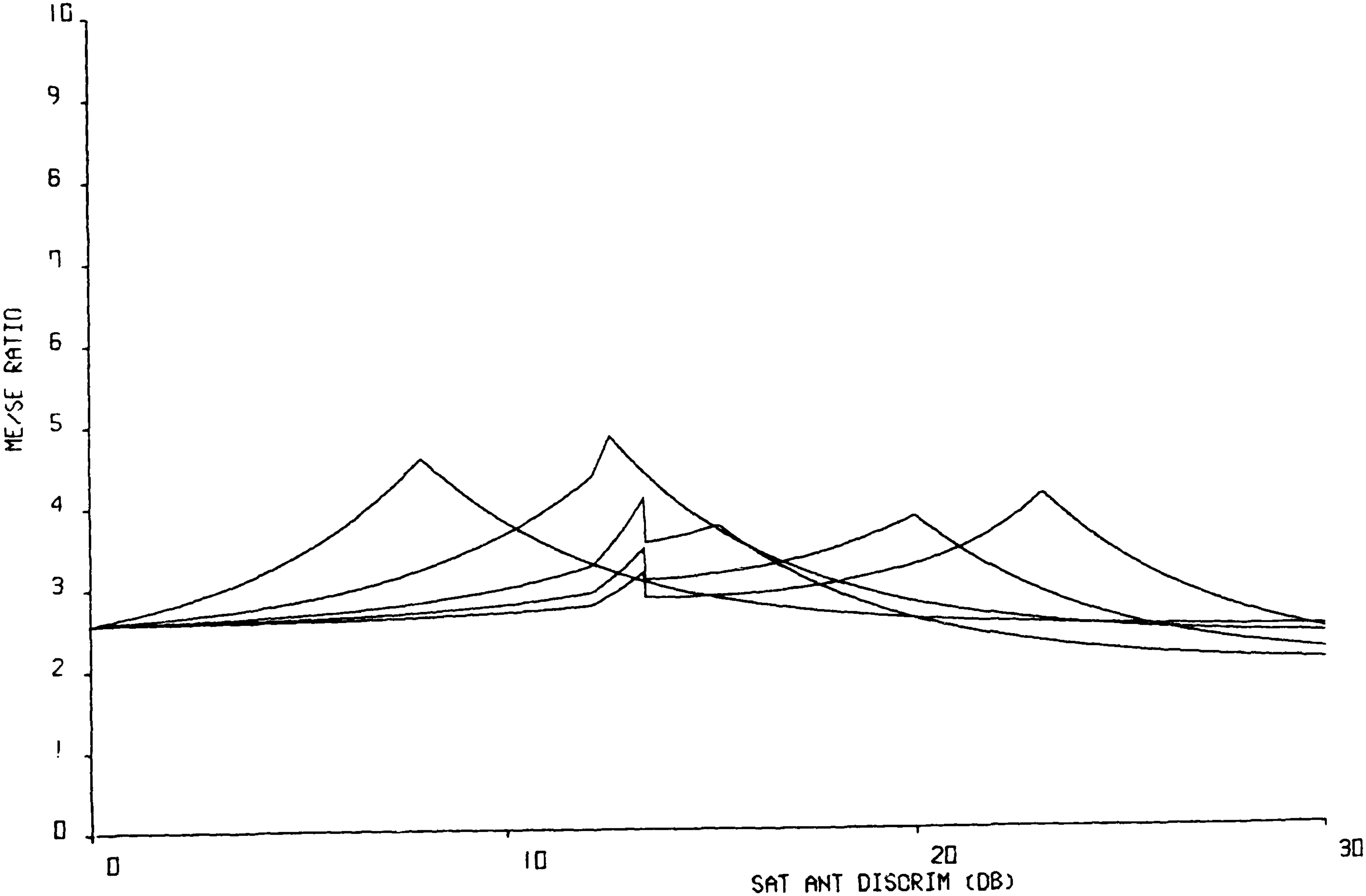
- o Plots show the worst case ME/SE ratio for various earth station antenna sizes and for various sequences



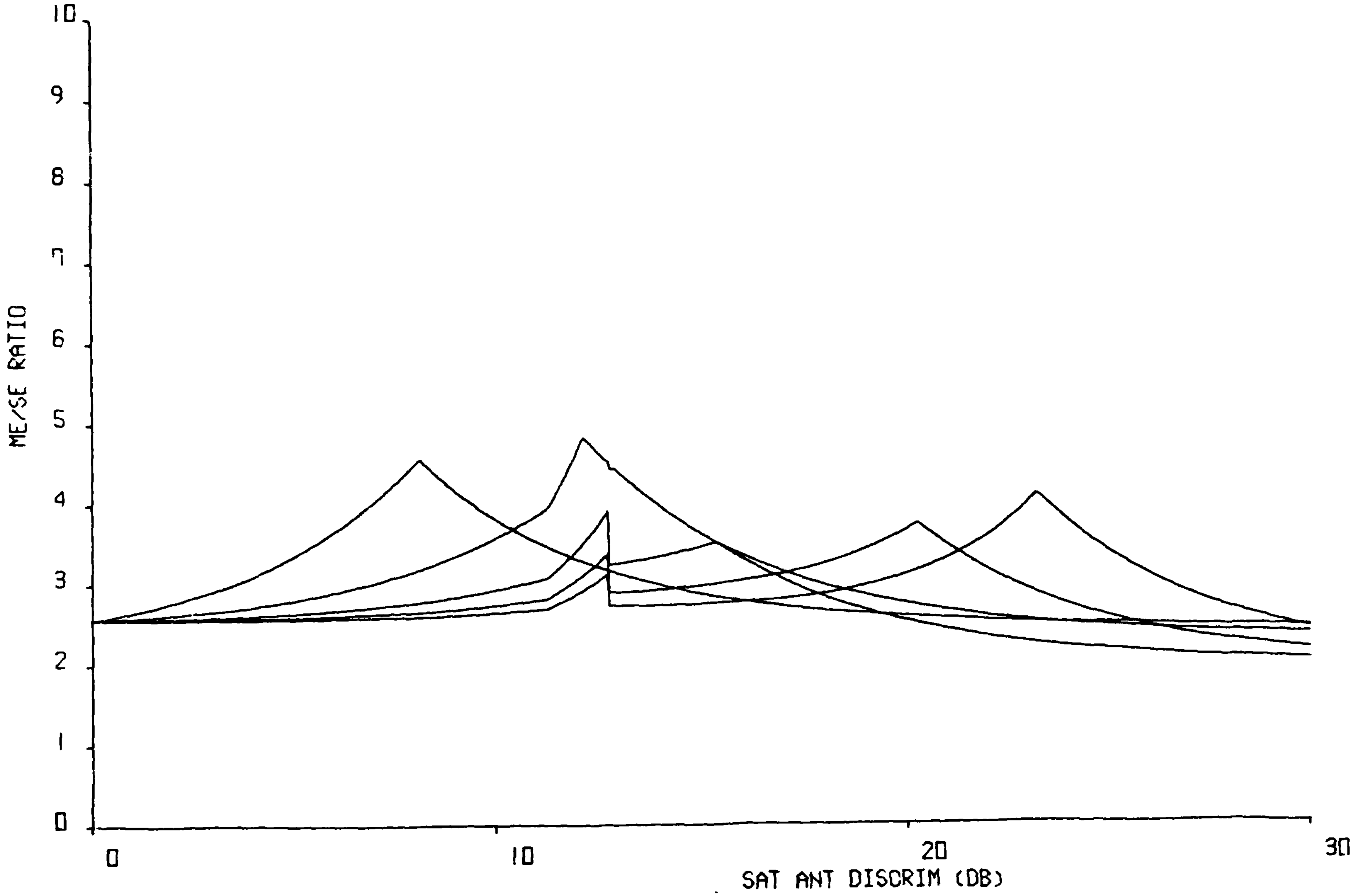
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



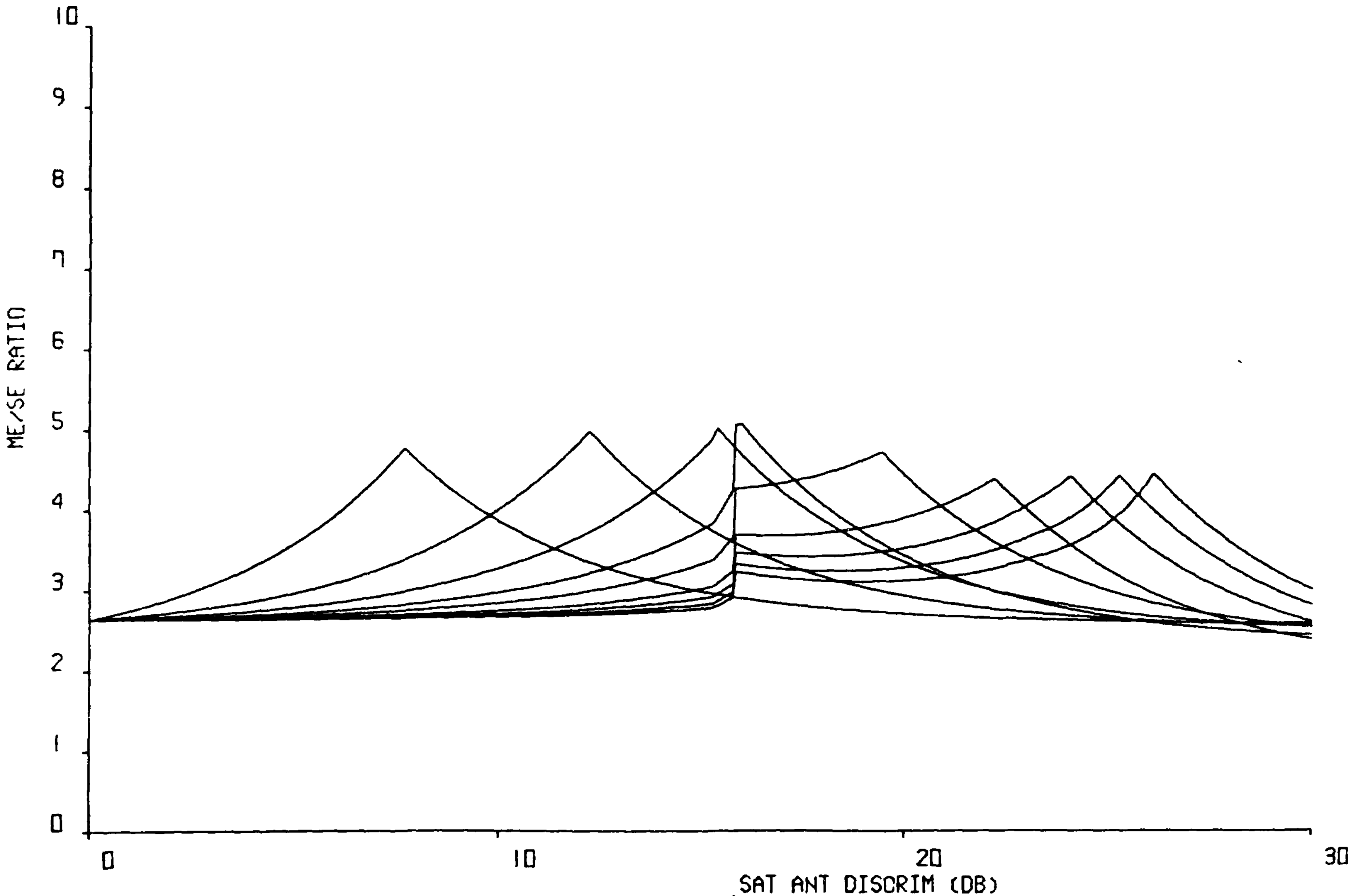
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



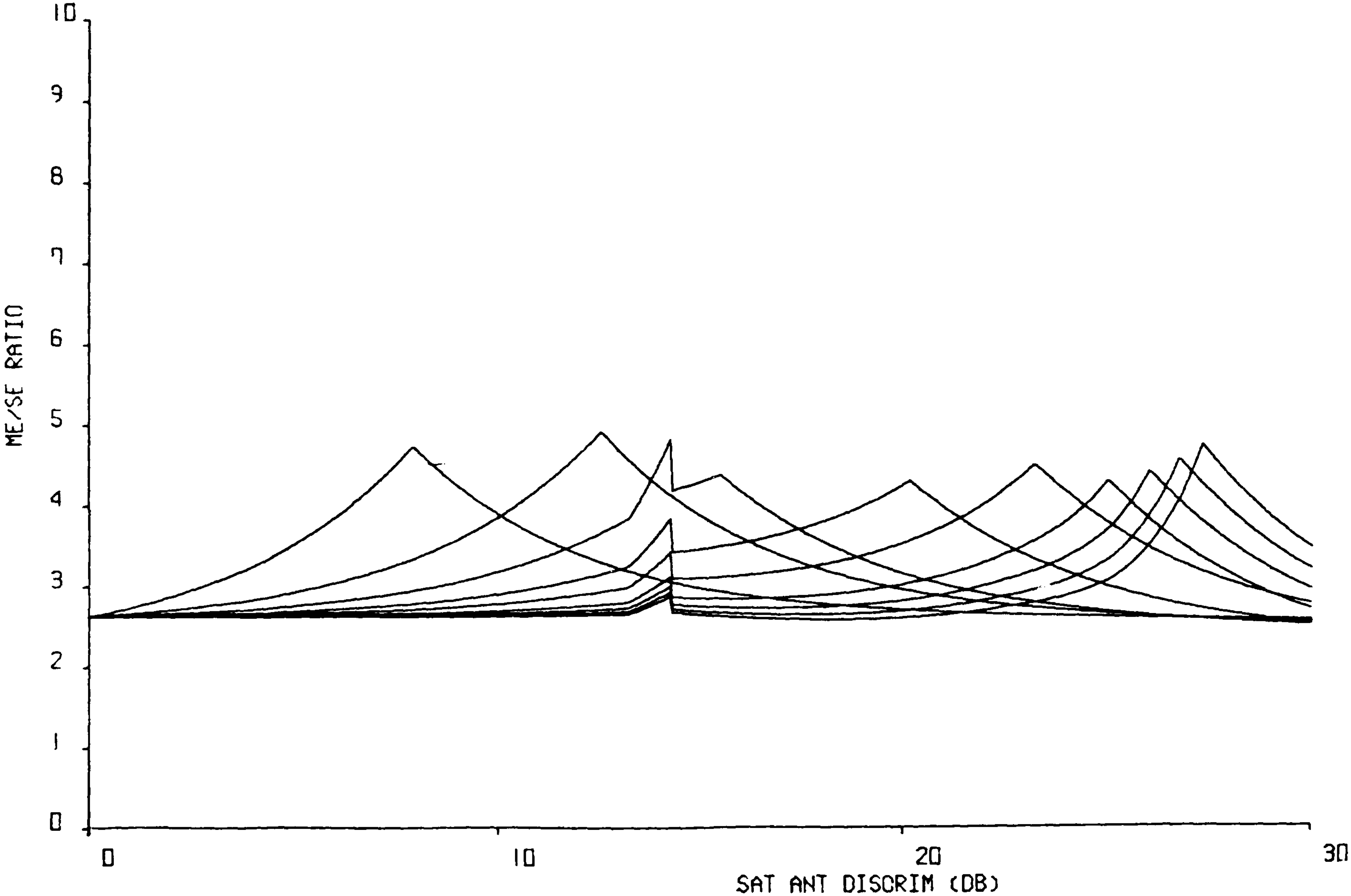
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



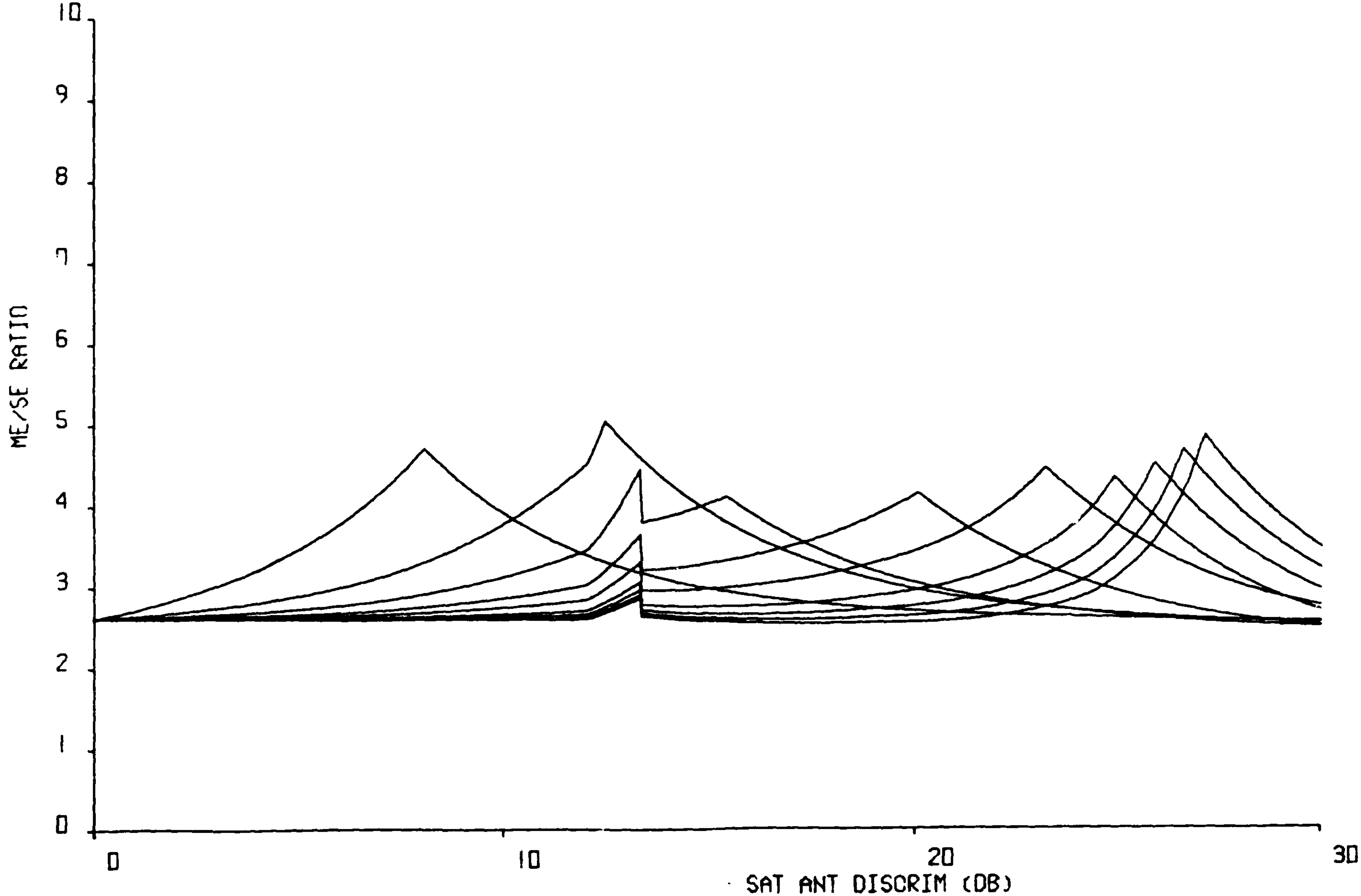
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



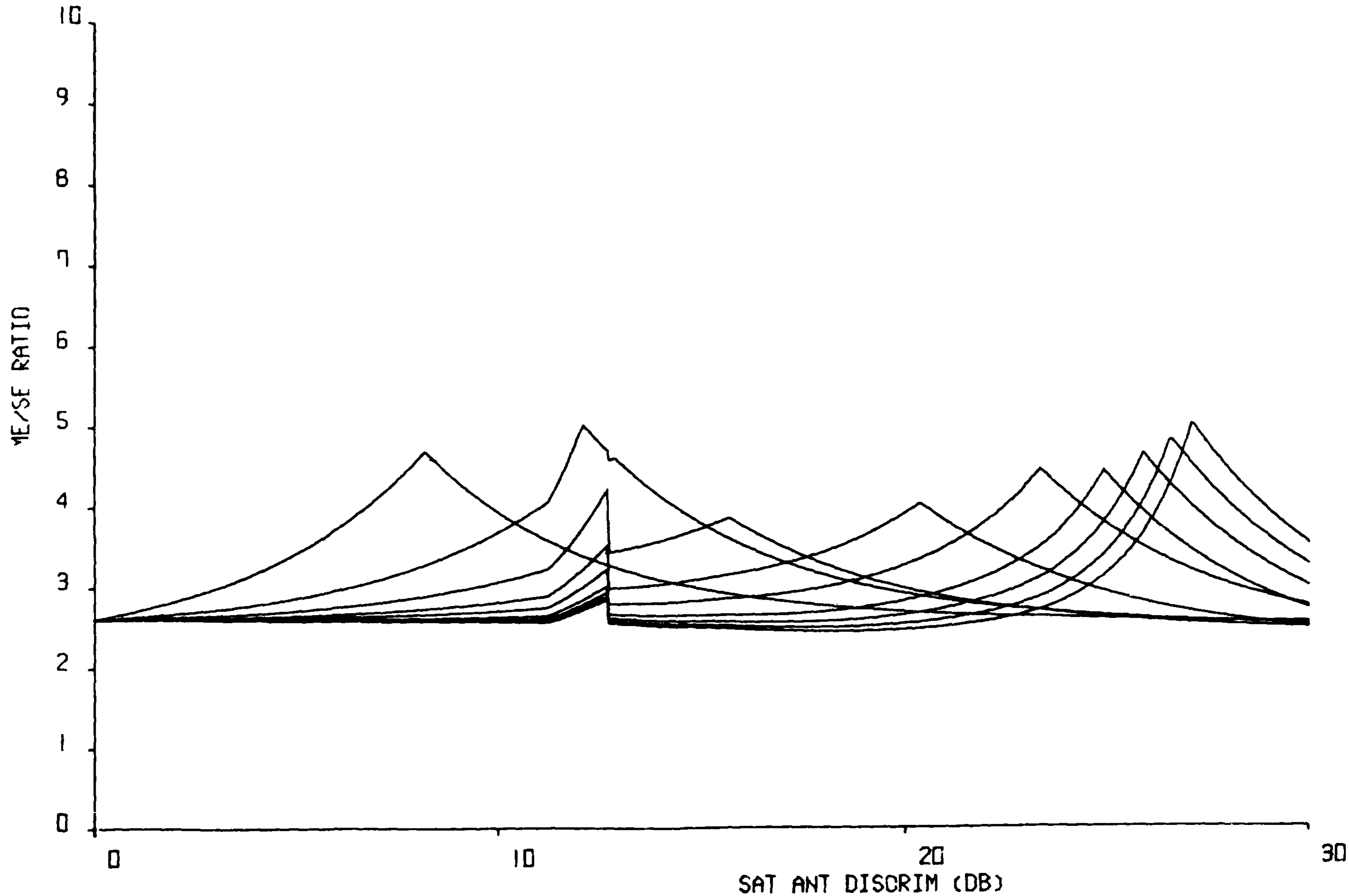
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



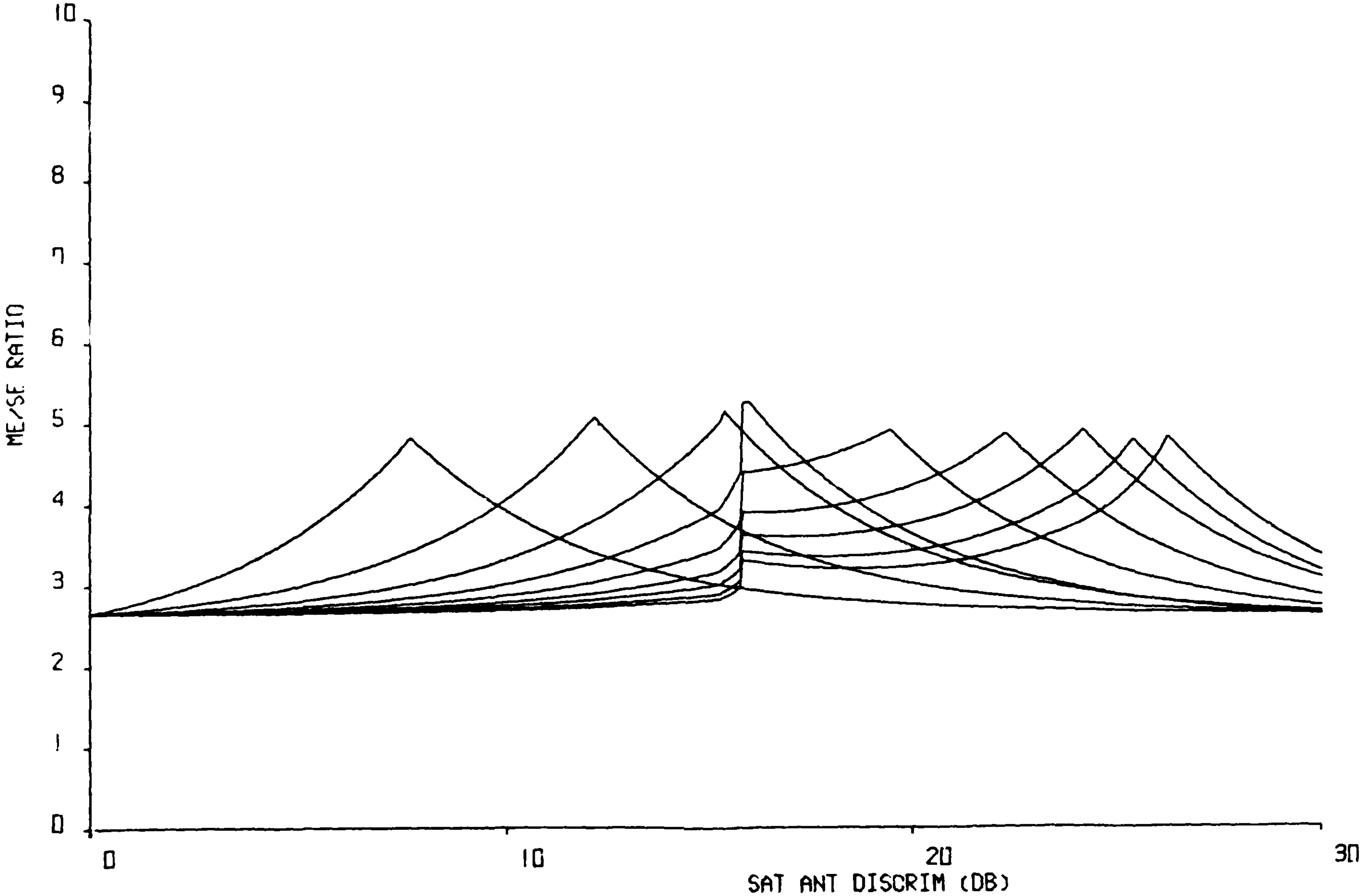
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



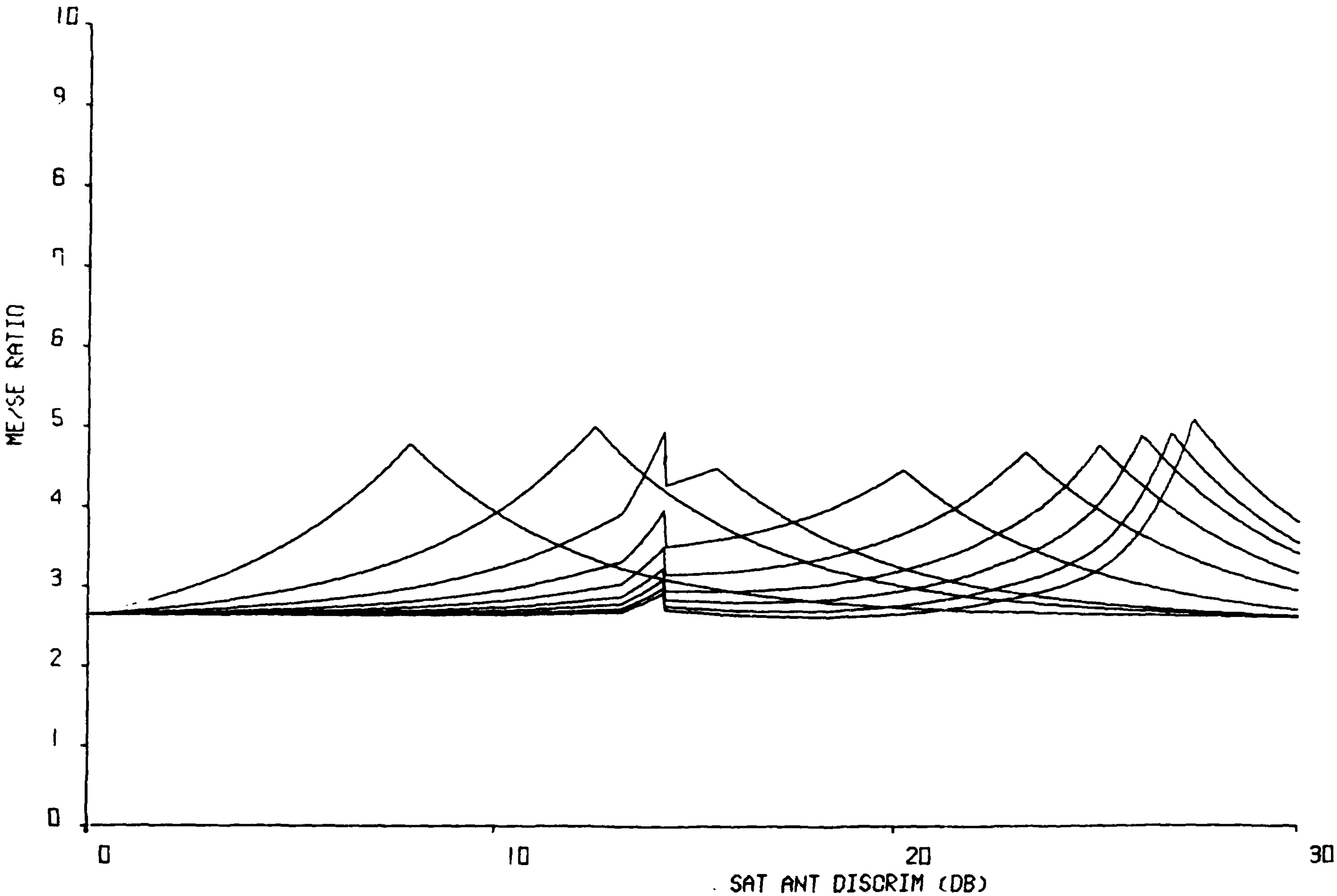
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



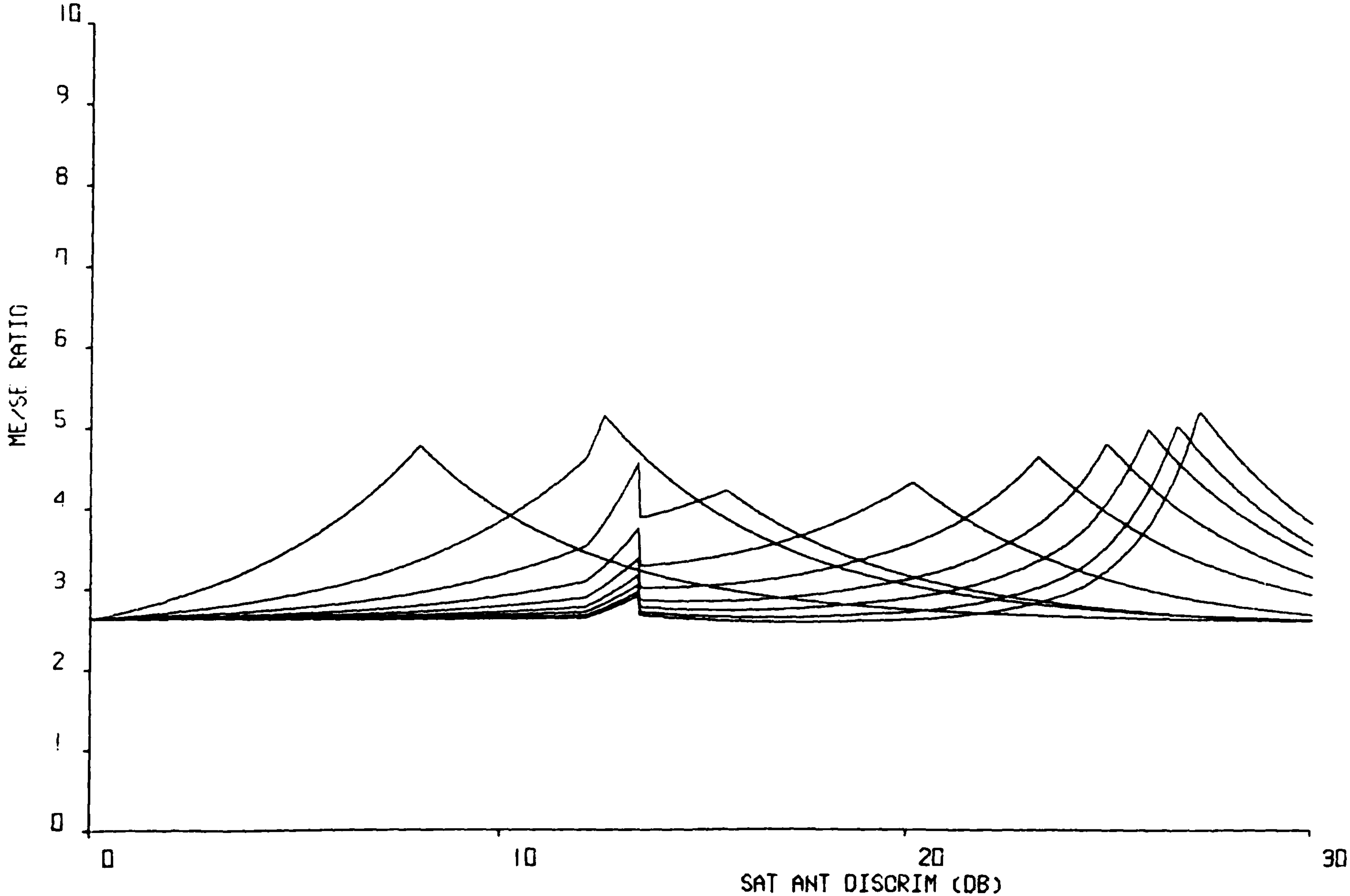
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



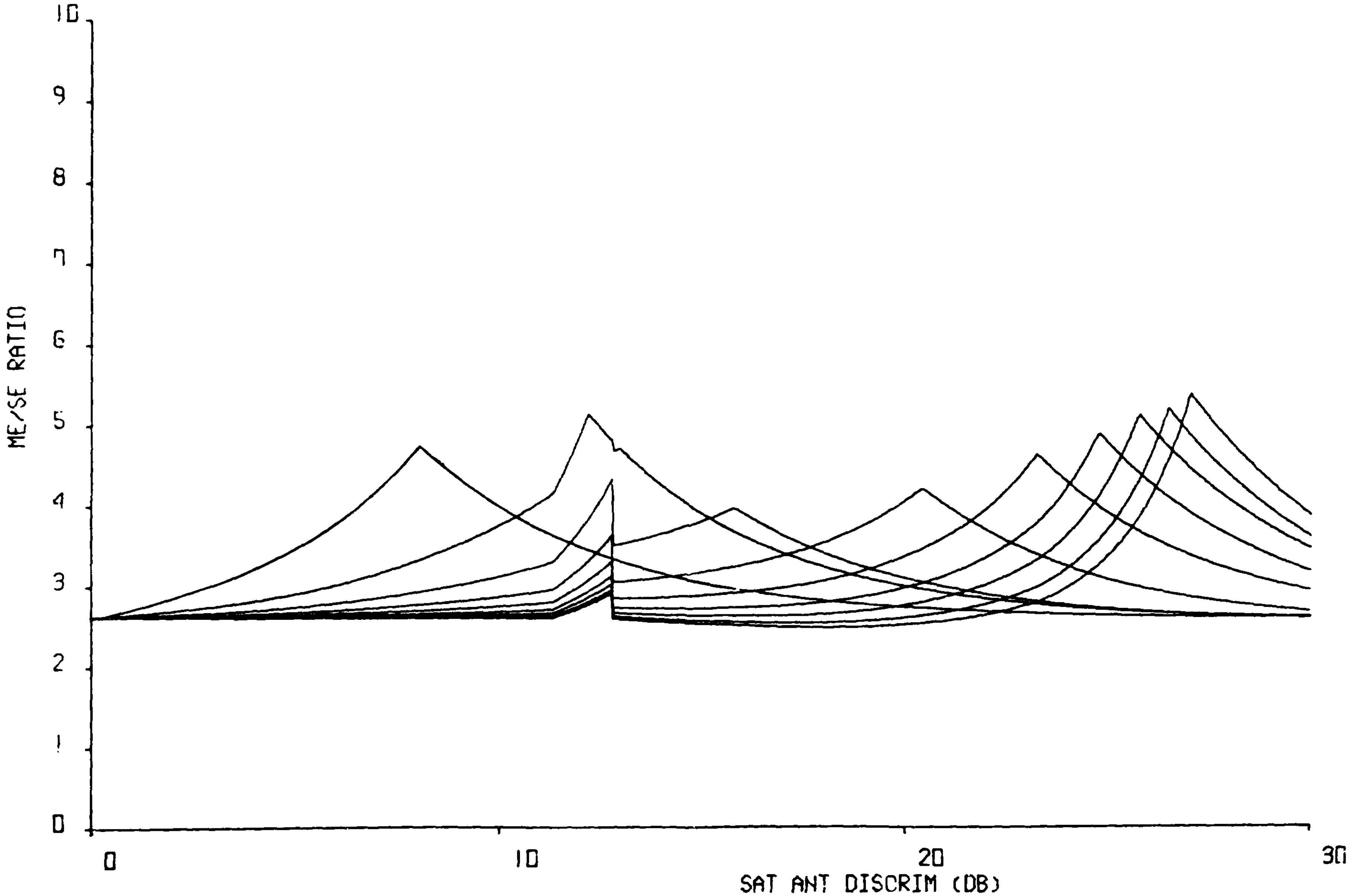
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 24 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 24 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 24 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 24 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.

EAST-WEST STATIONKEEPING ERROR WORST CASE ME/SE RATIO

- Assumptions
  - Victim satellite is moved toward the worst case interferring satellite to maximize SE
  - All other satellites are moved 0.1 degrees toward the victim satellite
  - Appendix 29 antenna patterns are used

WORST CASE ME/SE RATIO—APPENDIX 29 PATTERNS

- Worst case single entry interference is given by

$$SE_{wc} = \alpha KG(\theta_R' - 0.2)$$

- Worst case multiple entry interference is given by

$$ME = K\sum_{n=1}^{N} \alpha_n G(n\theta_R - 0.2) + K\sum_{n=-N}^{-1} \alpha_n G(n\theta_R)$$

- The co-coverage satellite is nominally at an angle of θ_R' from the victim satellite where θ_R' is chosen to provide 30 dB of discrimination
- For this case, the adjacent satellite would be located at an angle of θ_R' / n_c giving discrimination of $G(\theta_R' / n_c) + A$ (dB)
- If the adjacent satellite provides less descrimination it is used in the ME/SE ratio and reset at an angle providing 30 dB of discrimination

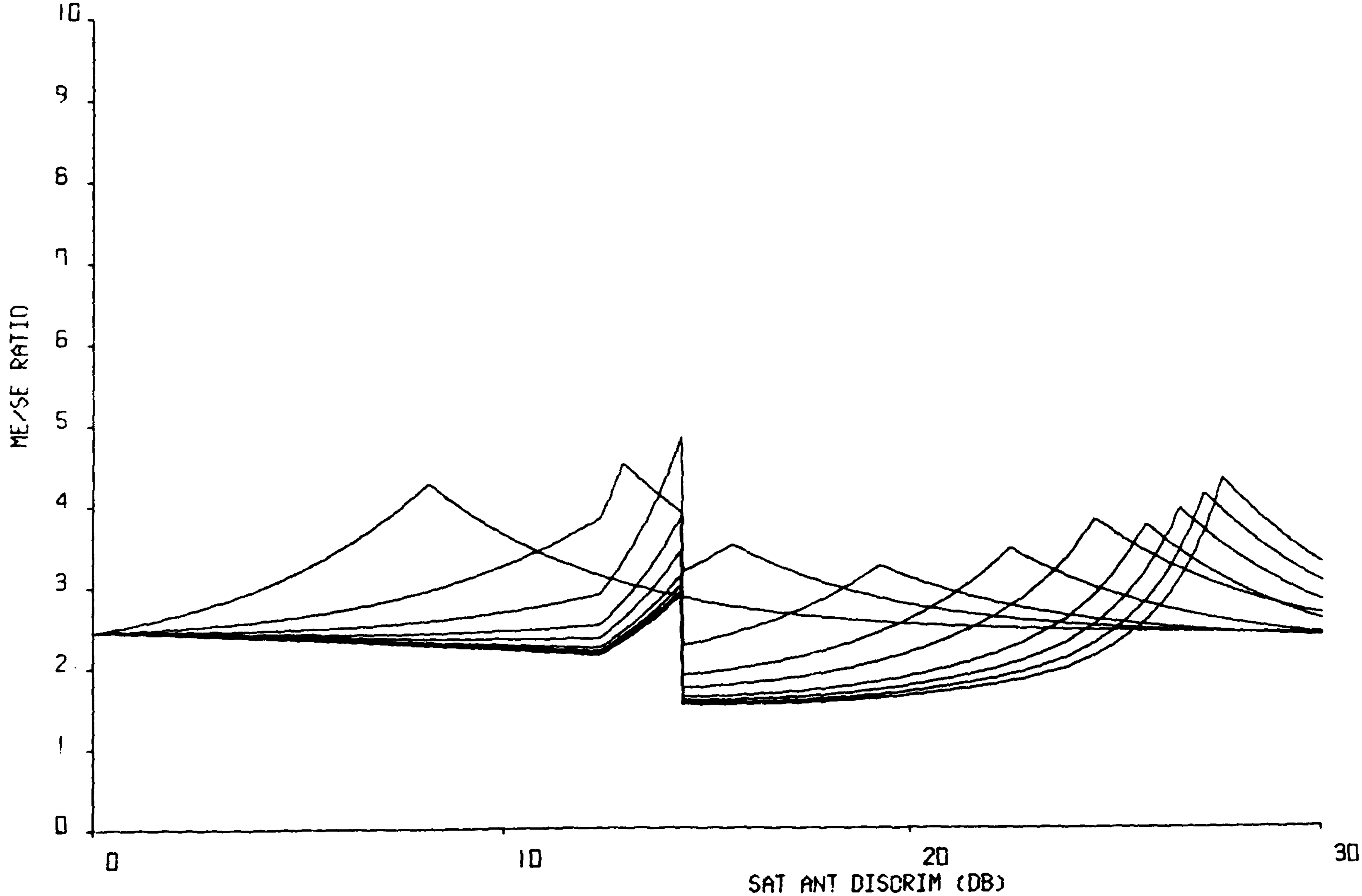
WORST CASE ME/SE RATIO—APPENDIX 29 PATTERNS

- With the worst case Stationkeeping error, the co-coverage satellite has discrimination of $G(\theta_R' \; -0.2)$

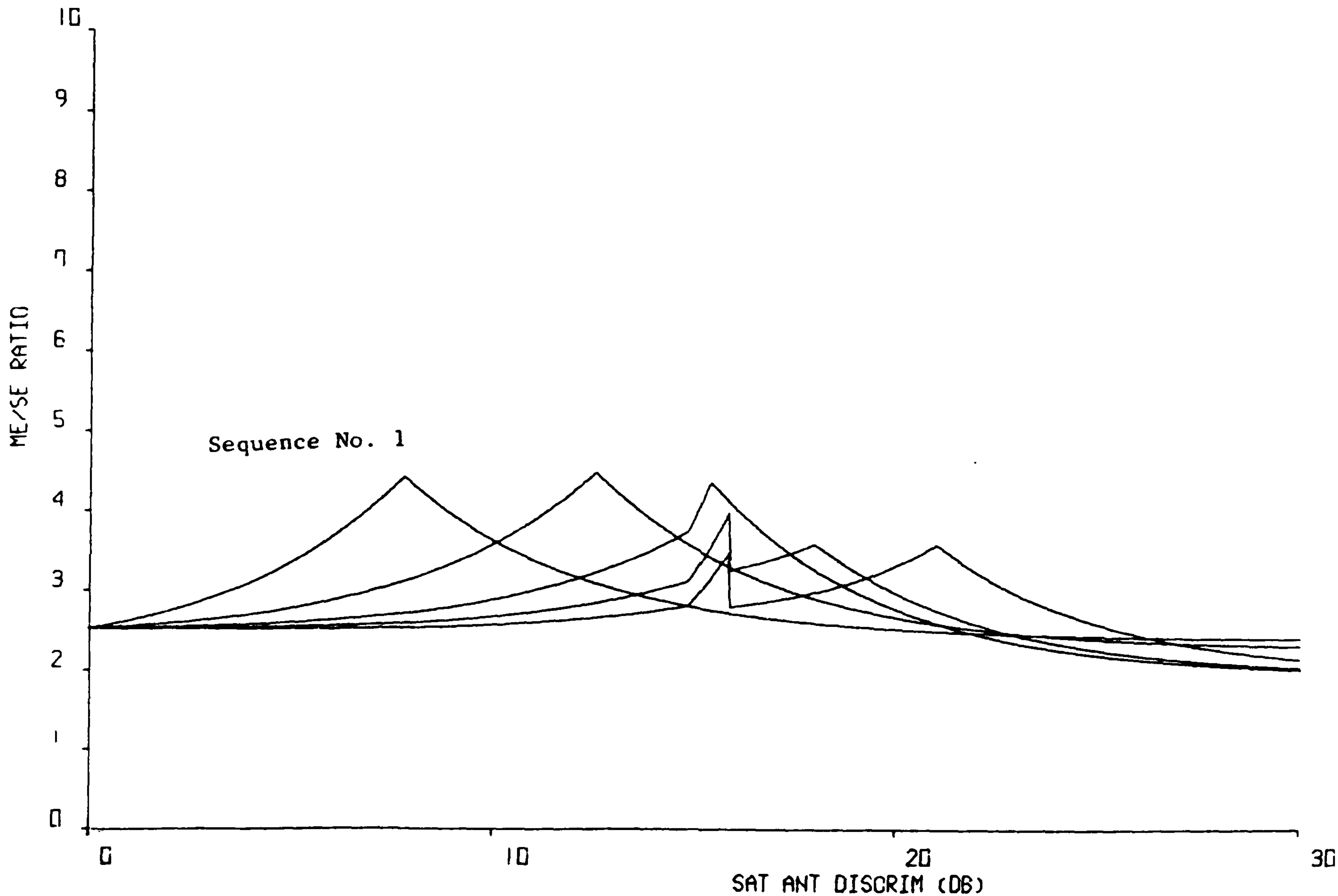
- With the worst case stationkeeping error, the adjacent satellite has discrimination of $G(\theta_R' / n_c \; -0.2) + A$ (dB)

- If the adjacent satellite with the worst case stationkeeping error provides less discrimination than the co-coverage satellite with the worst case stationkeeping error, it is used in the ME/SE ratio and reset to a nominal angle providing 30 dB of discrimination. The SE used would have this nominal angle minus the stationkeeping error.

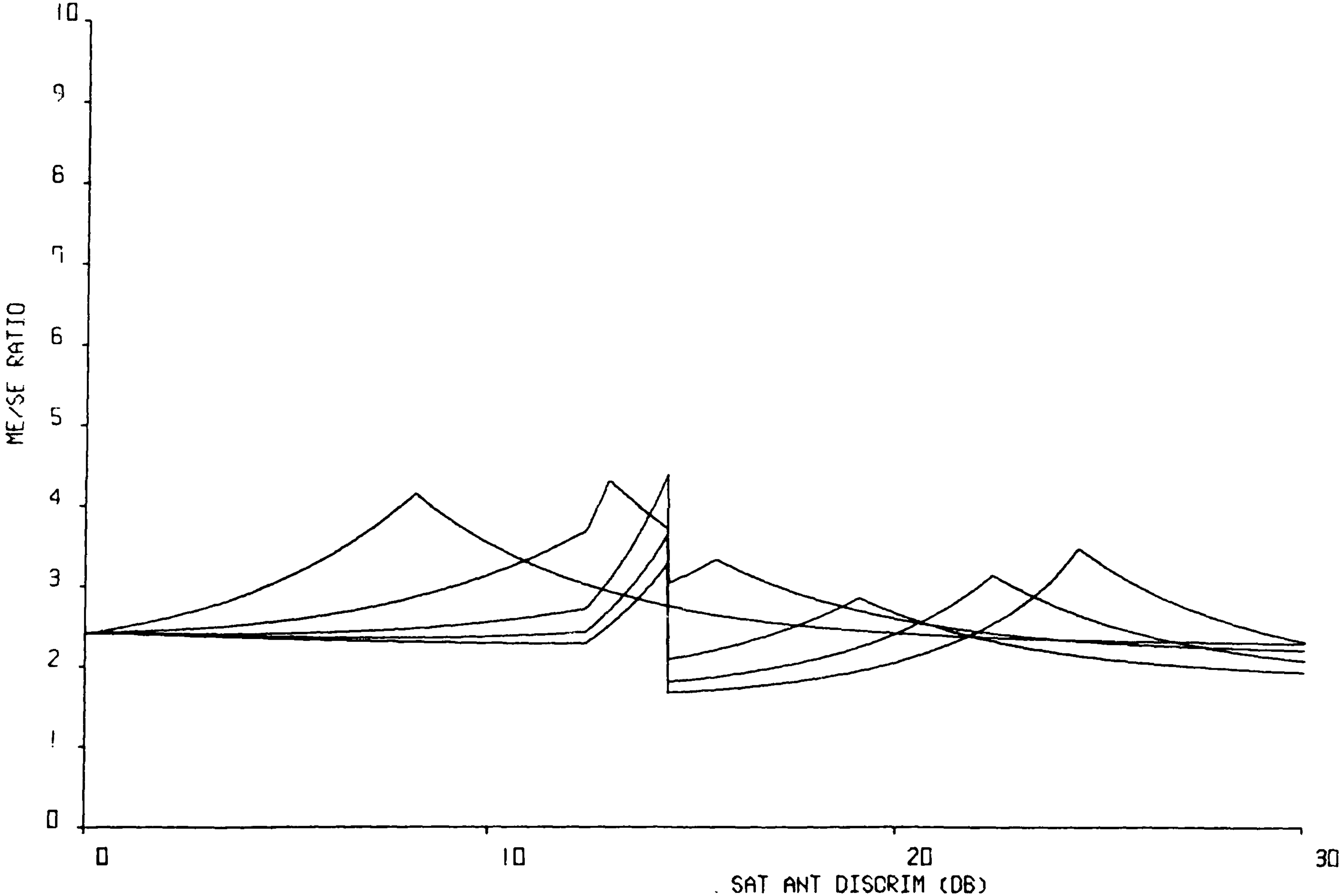
- Plots show the worst case ME/SE ratio for various earth station antenna sizes and for various sequences



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



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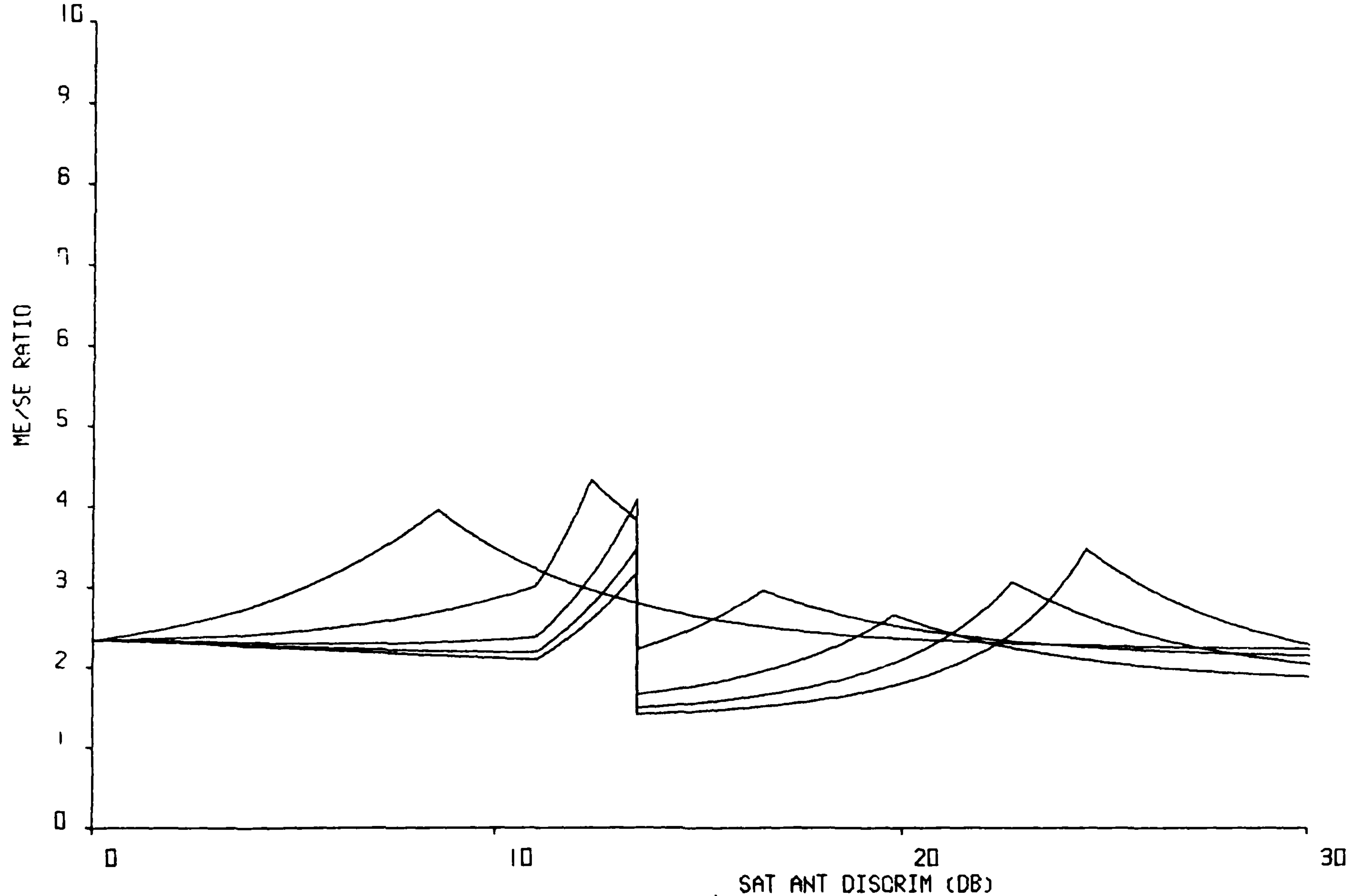
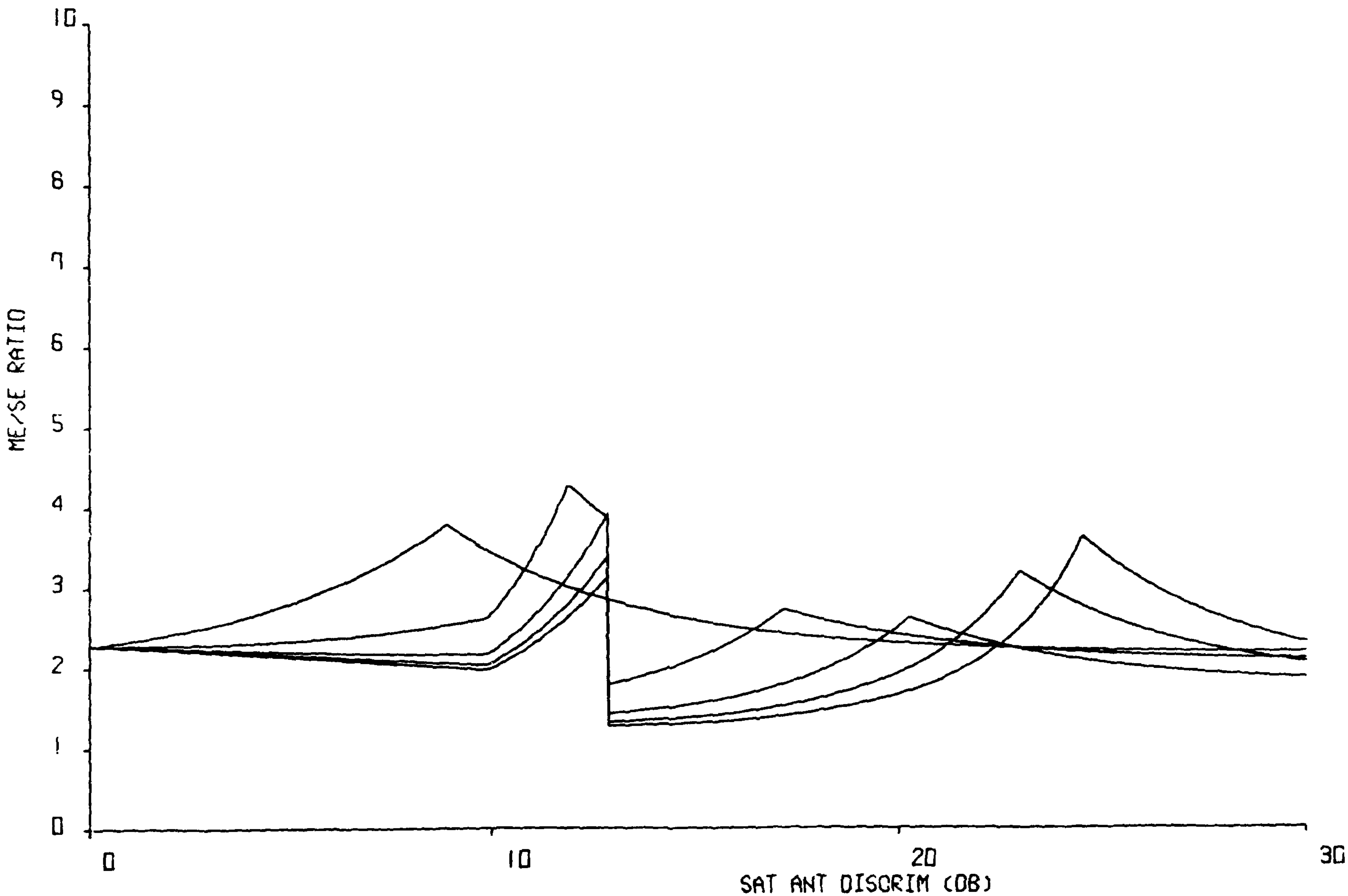
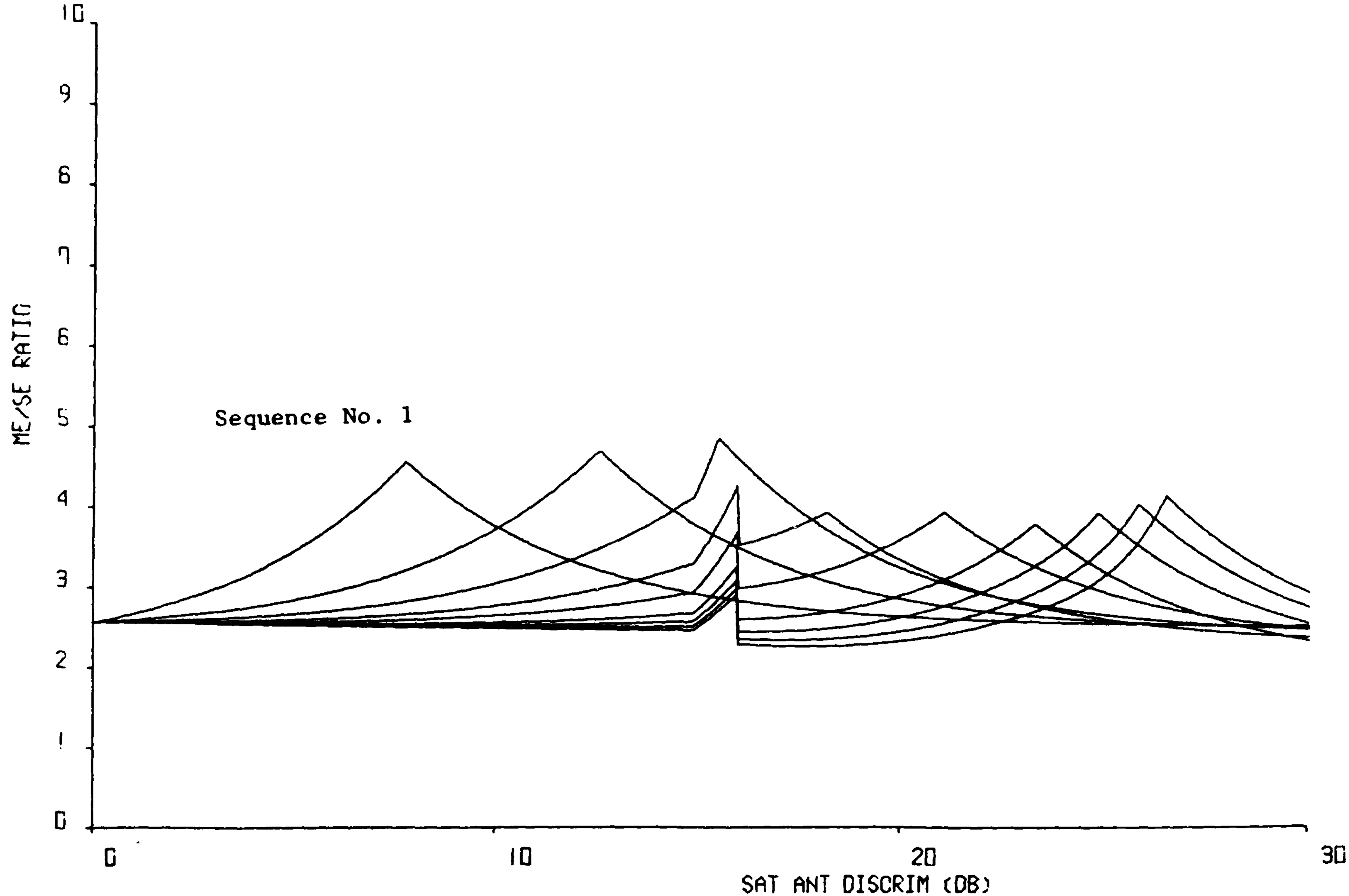


Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



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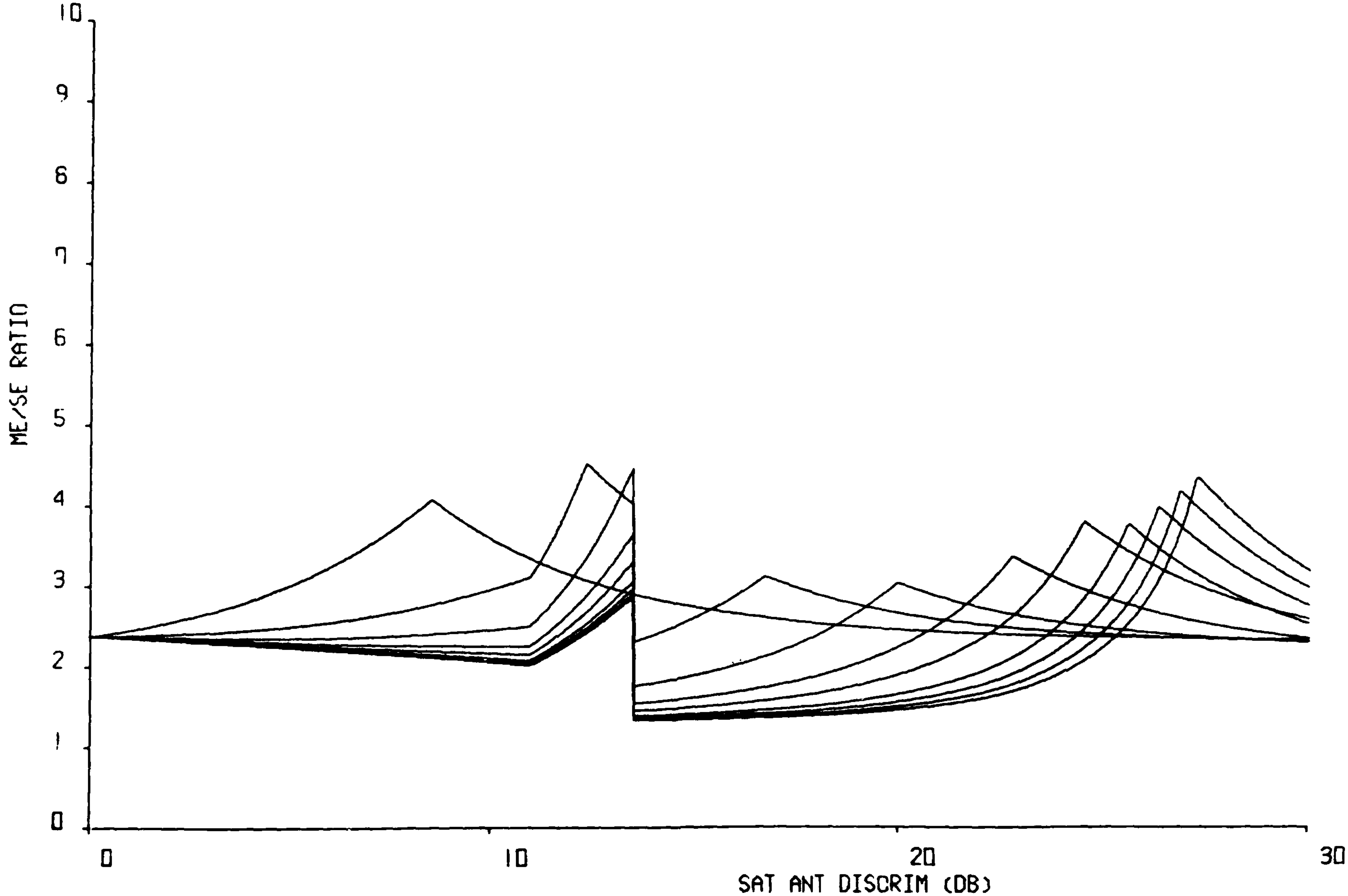

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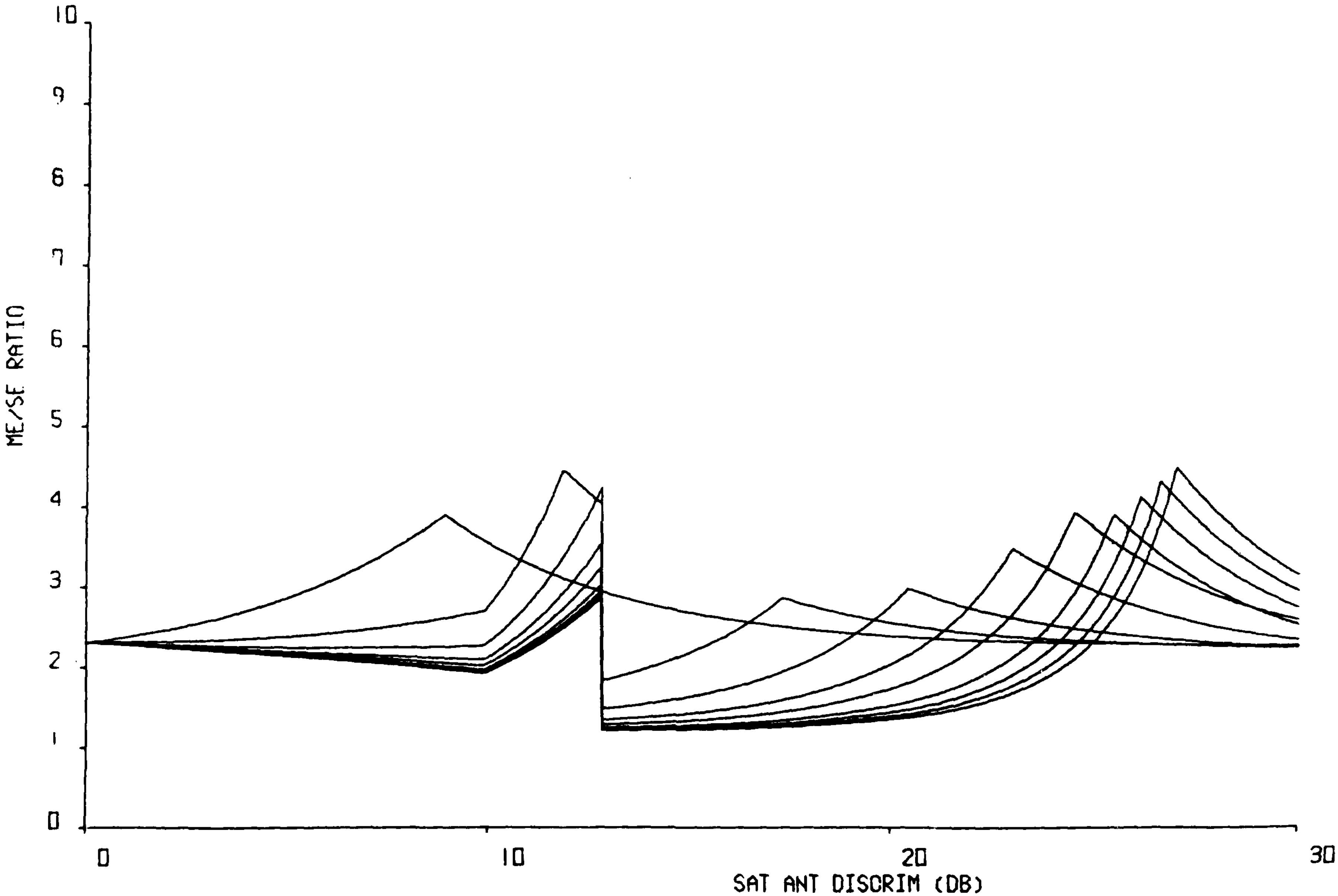
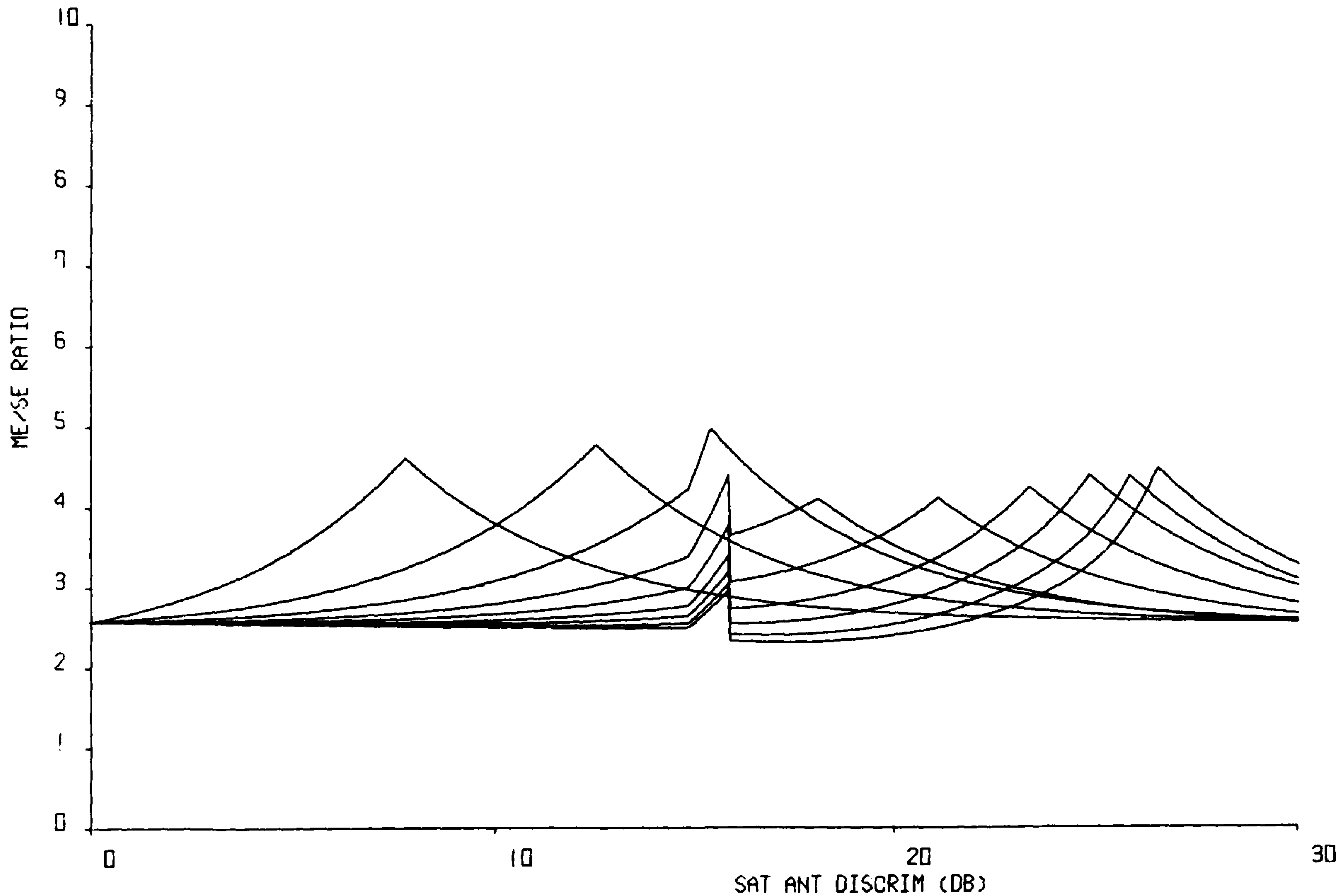
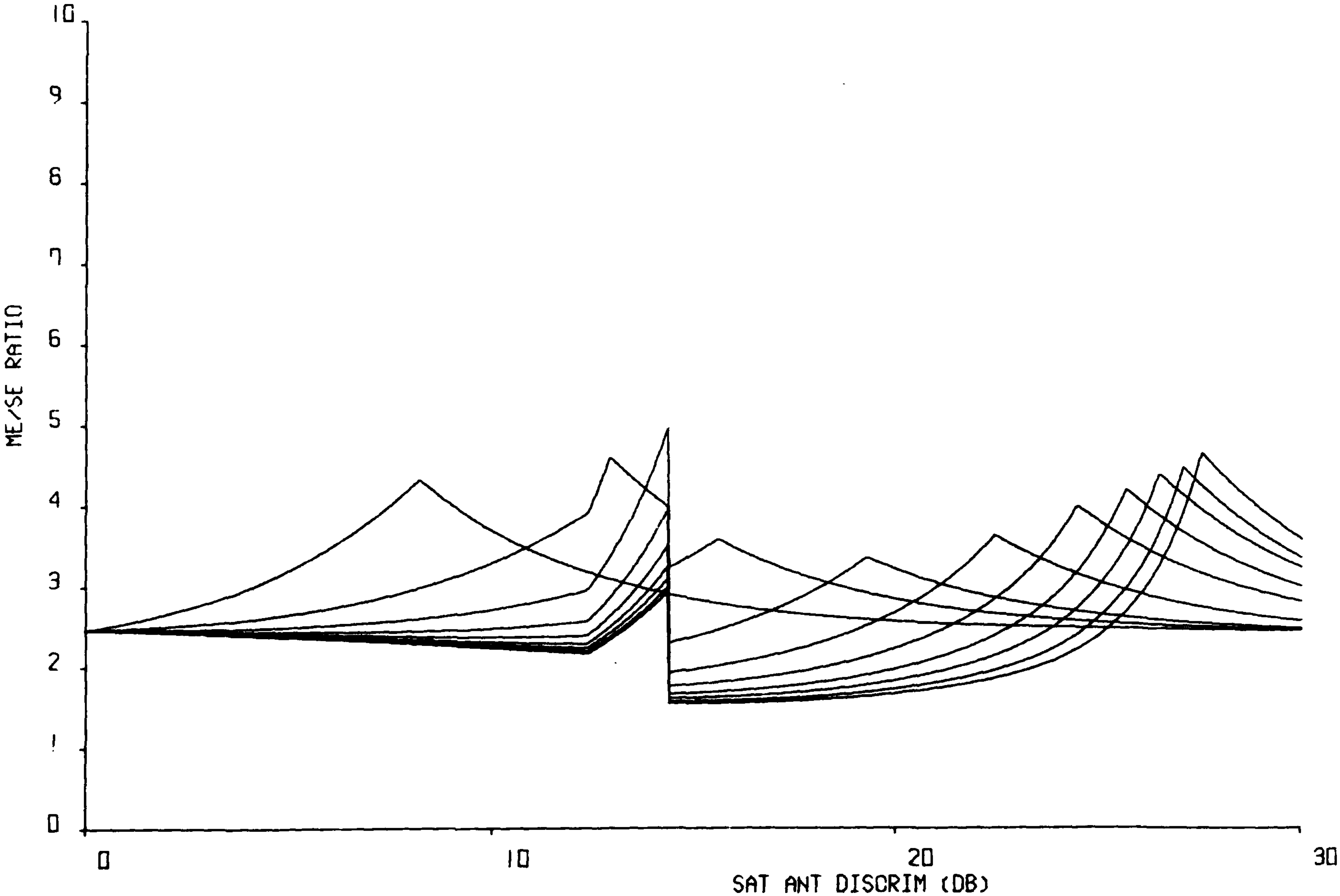


Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth statioh antenna has a diameter to wavelength ratio of 50. There are 24 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.



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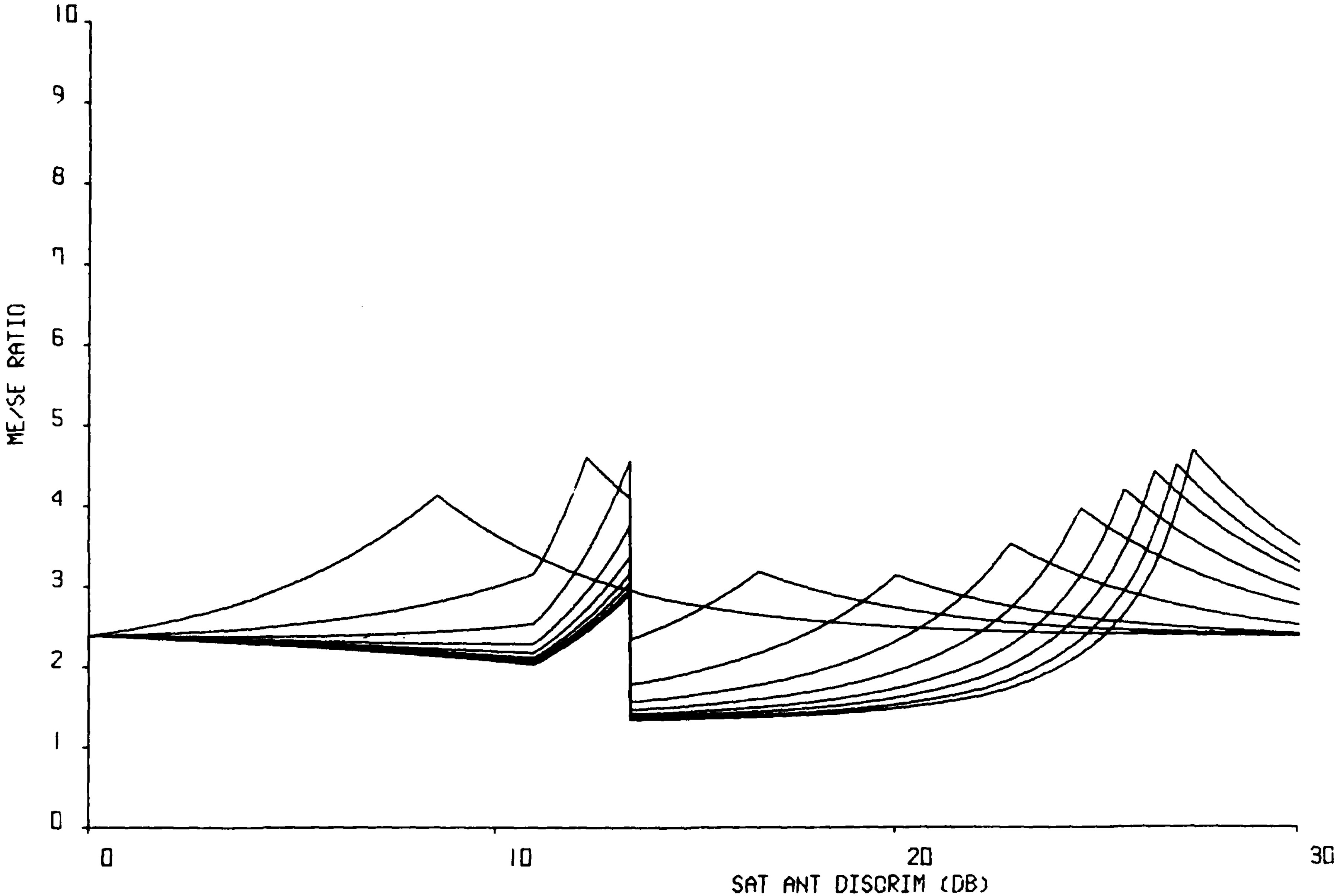

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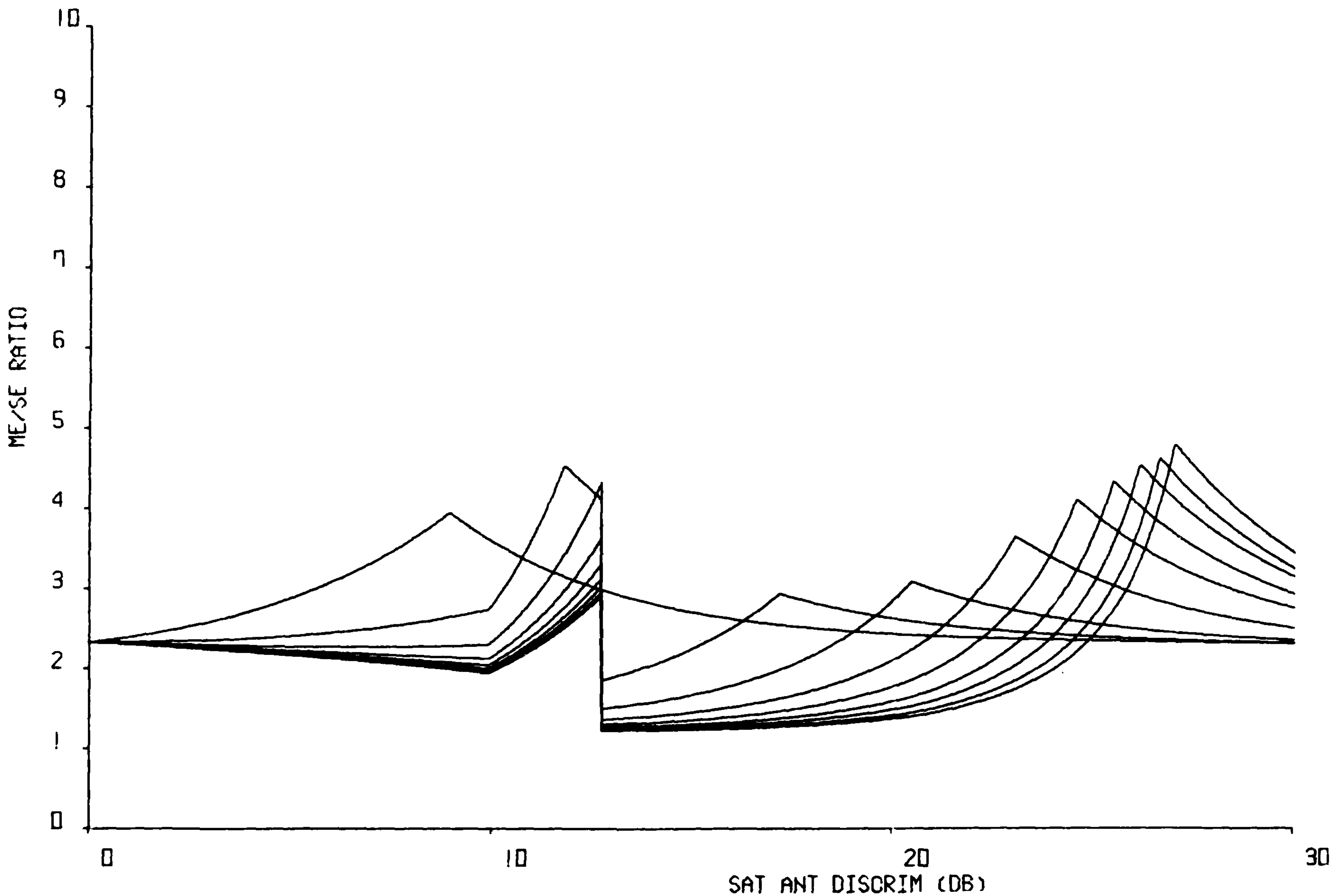


Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 24 satellites on either side of the victim. Required total discrimination is 30 db and the longitudinal stationkeeping error is 0.1 degree.

EAST–WEST STATIONKEEPING ERROR STATISTICAL MODEL

EAST – WEST STATIONKEEPING ERROR

o The East – West station keeping error

- less than $\pm$ 0.1 degrees
- independent among all satellites
- small compared to separation angles
- zero mean

o The angular distance from Satellite 0 to Satellite n is then

$$n\theta + \Delta\theta_n + \Delta\theta_0 \quad ; \quad n > 0$$

$$n\theta + \Delta\theta_n - \Delta\theta_0 \quad ; \quad n < 0$$

where $\Delta\theta_n$ is a random variable representing the East – West stationkeeping error of Satellite n

o The interference from Satellite n is then

$$\alpha_n \left(n\theta + \Delta\theta_n \pm \Delta\theta_0\right)^{-2.5}$$

which for small errors becomes

$$\alpha_n (n\theta)^{-2.5} \left(1 - 2.5\,\Delta\theta_n / n\theta \mp 2.5\,\Delta\theta_0 / n\theta\right)$$

EAST – WEST STATIONKEEPING ERROR (CON

- The expected value of single entry interference

$$E\{SE\} = \alpha_n (n\theta)^{-2.5}$$

based upon the assumption of zero mean stationkeeping errors

- The variance of the single entry interference

$$E\{(SE - E\{SE\})^2\} = \sigma_{SE}^2$$

$$= 12.5\, \sigma_\theta^2 \alpha_n^2 (n\theta)^{-7}$$

where $\sigma_\theta^2 = E\{\Delta\theta_n^2\}$

EAST–WEST STATIONKEEPING ERROR (CONT)

- Based upon the above, the multiple entry interference is for symmetric satellites

$$ME = 2\theta^{-2.5}\sum_{n=1}^{N}\alpha_n n^{-2.5} - 5\theta^{-3.5}\sum_{n=1}^{N}\alpha_n \Delta\theta_n n^{-3.5}$$

- Expected value of multiple entry interference

$$E\{ME\} = 2\theta^{-2.5}\sum_{n=1}^{N}\alpha_n n^{-2.5}$$

based upon the assumption of zero mean stationkeeping errors

- Variance of multiple entry interference

$$E\{(ME - E\{ME\})^2\} = \sigma_{ME}^2 = 25\theta^{-7.0}\sigma_\theta^2\sum_{n=1}^{N}\alpha_n^2 n^{-7.0}$$

EAST-WEST STATIONKEEPING ERROR (CONT)

- The statistics of the ratio, ME/SE, are of interest. The ratio is given by

$$ME/SE = \frac{2\theta^{-2.5}\sum_{n=1}^{N}\alpha_n n^{-2.5} - 5\theta^{-3.5}\sum_{n=1}^{N}\alpha_n \Delta\theta_n n^{-3.5}}{\alpha_m (m\theta)^{-2.5}(1 - 2.5\Delta\theta_m/m\theta - 2.5\Delta\theta_0/m\theta)}$$

$$= \overline{ME}/\overline{SE} - \frac{1}{\overline{SE}}\Big[5\theta^{-3.5}\sum_{n=1}^{N}\alpha_n \Delta\theta_n n^{-3.5}(1 + 2.5\Delta\theta_m/m\theta + 2.5\Delta\theta_0/m\theta)\Big]$$

- Expected value of ME/SE

$$\overline{ME/SE} = \overline{ME}/\overline{SE} - 12.5\,\sigma_\theta^2 (m\theta)^{-2}$$

where bars indicate expected values

EARTH STATION TRACKING ERROR

- o Impact of Earth Station Tracking Error on the ME/SE ratio

- o Performance measures

  - –Worst case ME/SE ratio
    - – –Maximize ME, minimize SE
  - –Worst case SE, worst case ME
    - – –Maximize ME, maximize SE
  - –Expected value of ME/SE ratio
    - – –E{ME/SE}
  - –Expected value of ME/SE ratio given worst case SE

- o Earth Station Tracking Error

  - –less than 1 dB relative gain
  - –independent, zero mean, uniform distribution between 0 and 1 dB

EARTH STATION TRACKING ERROR FORMULATION OF THE PROBLEM

- The angular distance from the victim satellite (number 0) to satellite n is given by

$$n\theta + \delta\theta_n + \delta\theta_o \quad ; n>0$$
$$n\theta + \delta\theta_n - \delta\theta_o \quad ; n<0$$

where $\delta\theta_n$ is a random variable representing the earth station tracking error of satellite n

- The interference from satellite n into the victim satellite 0 is

$$\alpha_n KG(n\theta + \delta\theta_n \pm \delta\theta_o)$$

where the sign depends on n

EARTH STATION TRACKING ERROR

FORMULATION OF THE PROBLEM

- The multiple entry interference is then

$$ME = K \sum_{\substack{n=-N \\ n \neq 0}}^{N} \alpha_n G(n\theta + \delta\theta_n + n/|n|\, \delta\theta_o)$$

- The ME/SE ratio is given by

$$ME/SE = \frac{\sum_{\substack{n=-N \\ n \neq 0}}^{N} \alpha_n G(n\theta + \delta\theta_n + \delta\theta_o\, n/|n|)}{\alpha_m G(m\theta + \delta\theta_m + \delta\theta_o\, m/|m|)}$$

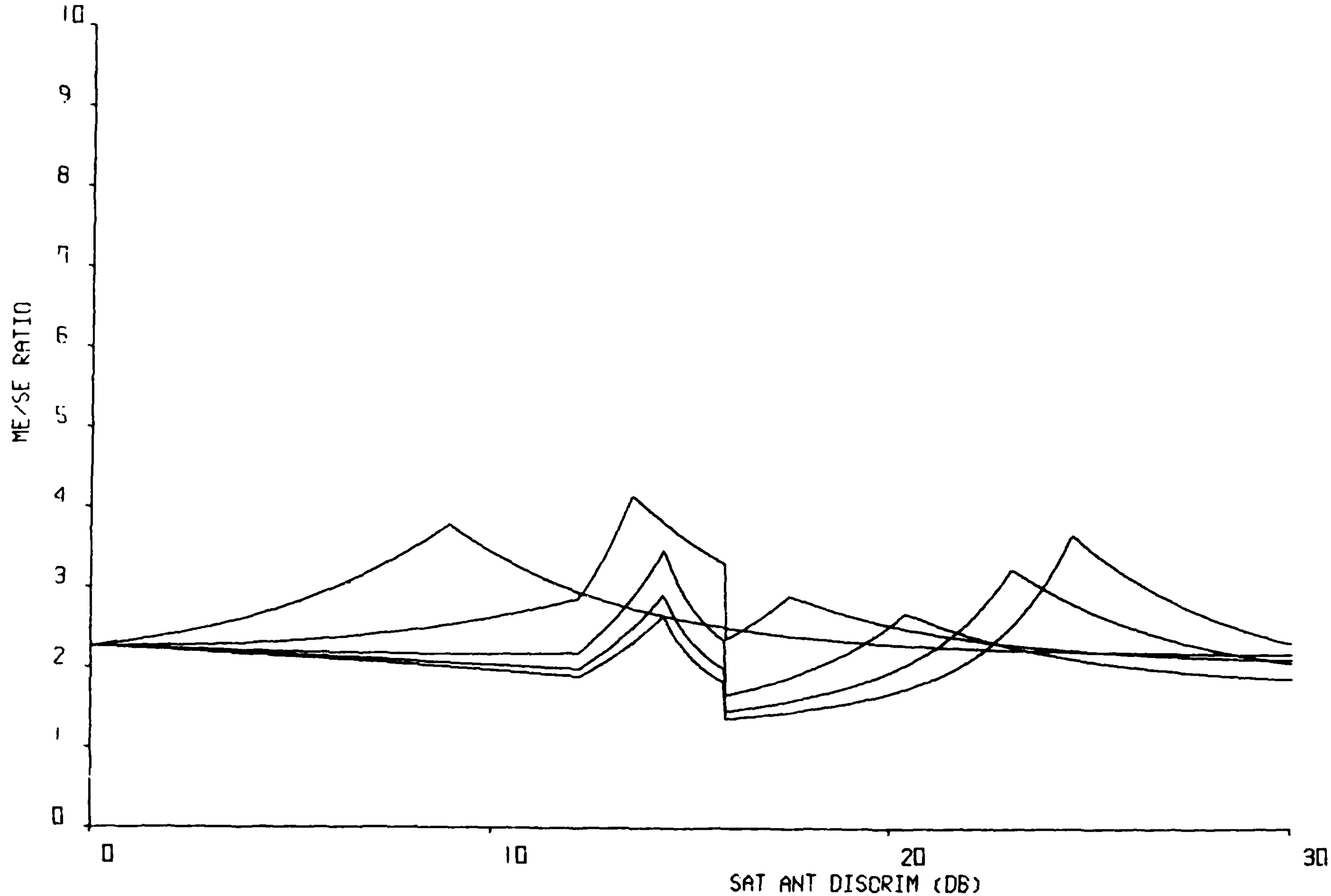
- The angle is found as the −1 dB relative gain point of the earth station antenna

EARTH STATION TRACKING ERROR WORST CASE ME/SE RATIO

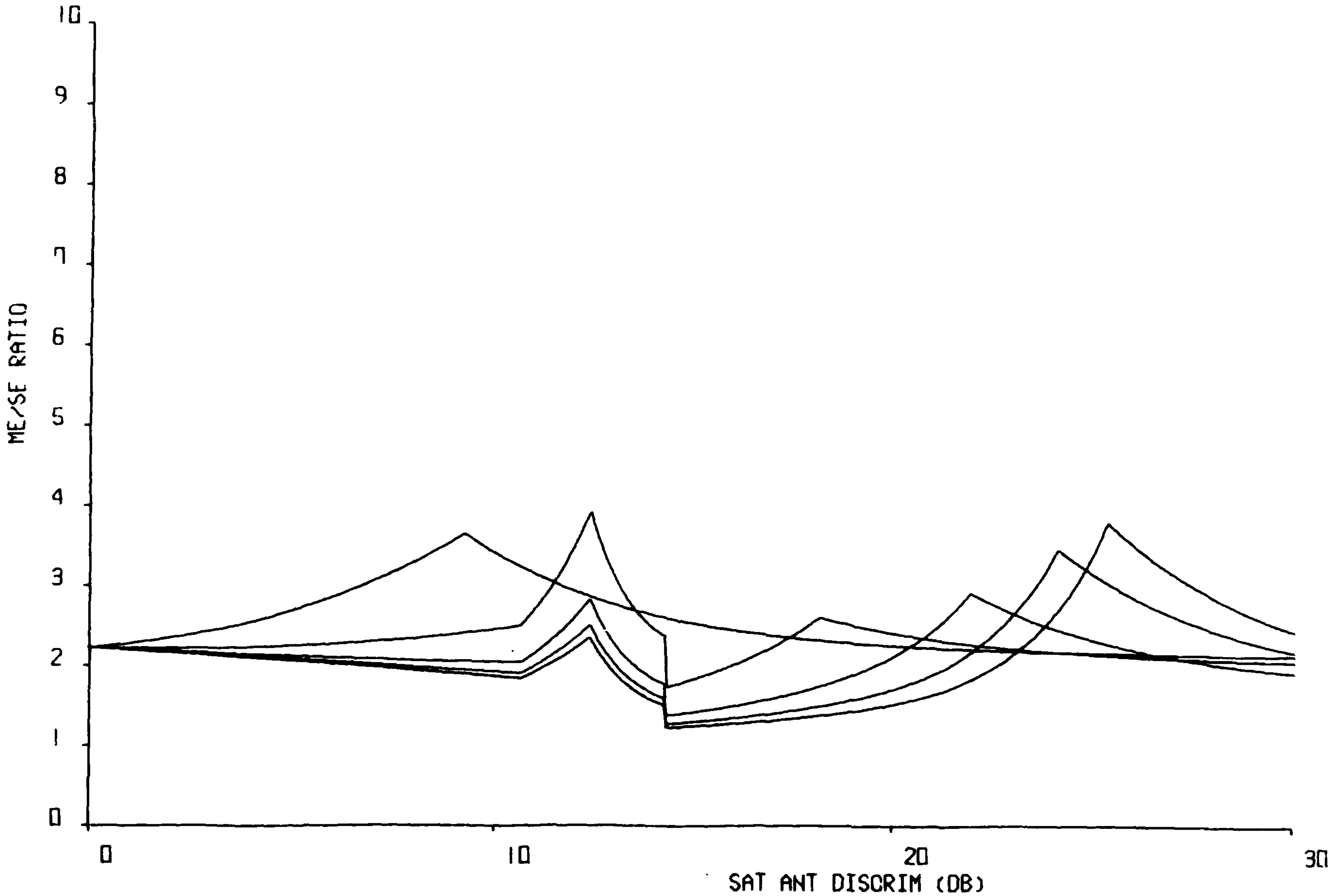
- o Assumptions
  - –Victim satellite's earth station antenna has no tracking error
  - –All other satellite's earth station tracking errors are such as to increase the interference into the victim by moving closer by the angular tracking error
  - –Appendix 29 antenna patterns are used

EARTH STATION TRACKING ERROR WORST CASE ME/SE RATIO

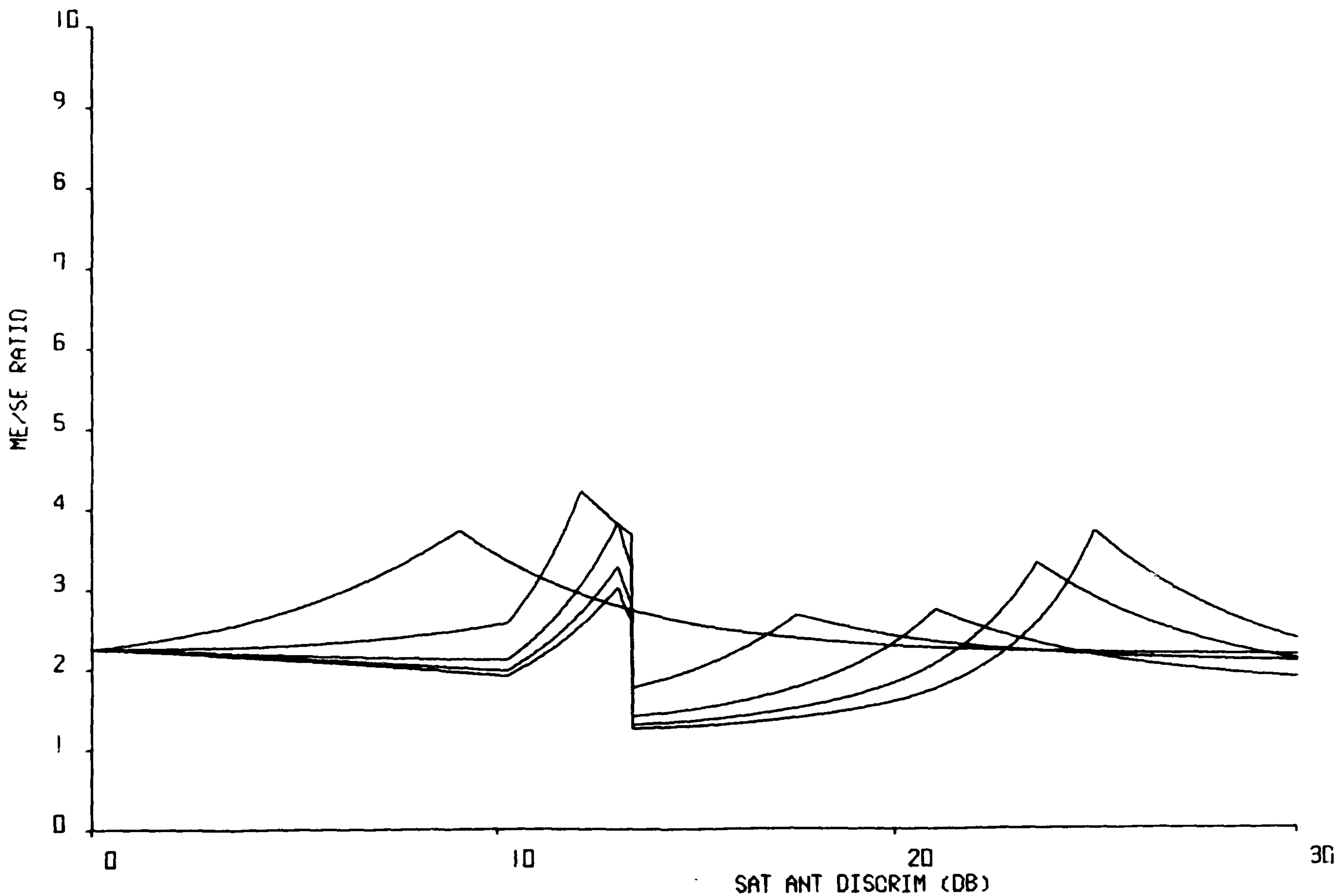
- Assumptions
  - Victim satellite is moved closer to the maximum interfering satellite by the tracking error to maximize SE
  - All other satellites are moved closer to the victim satellite by an amount equal to the earth station tracking error
  - Appendix 29 antenna patterns are used



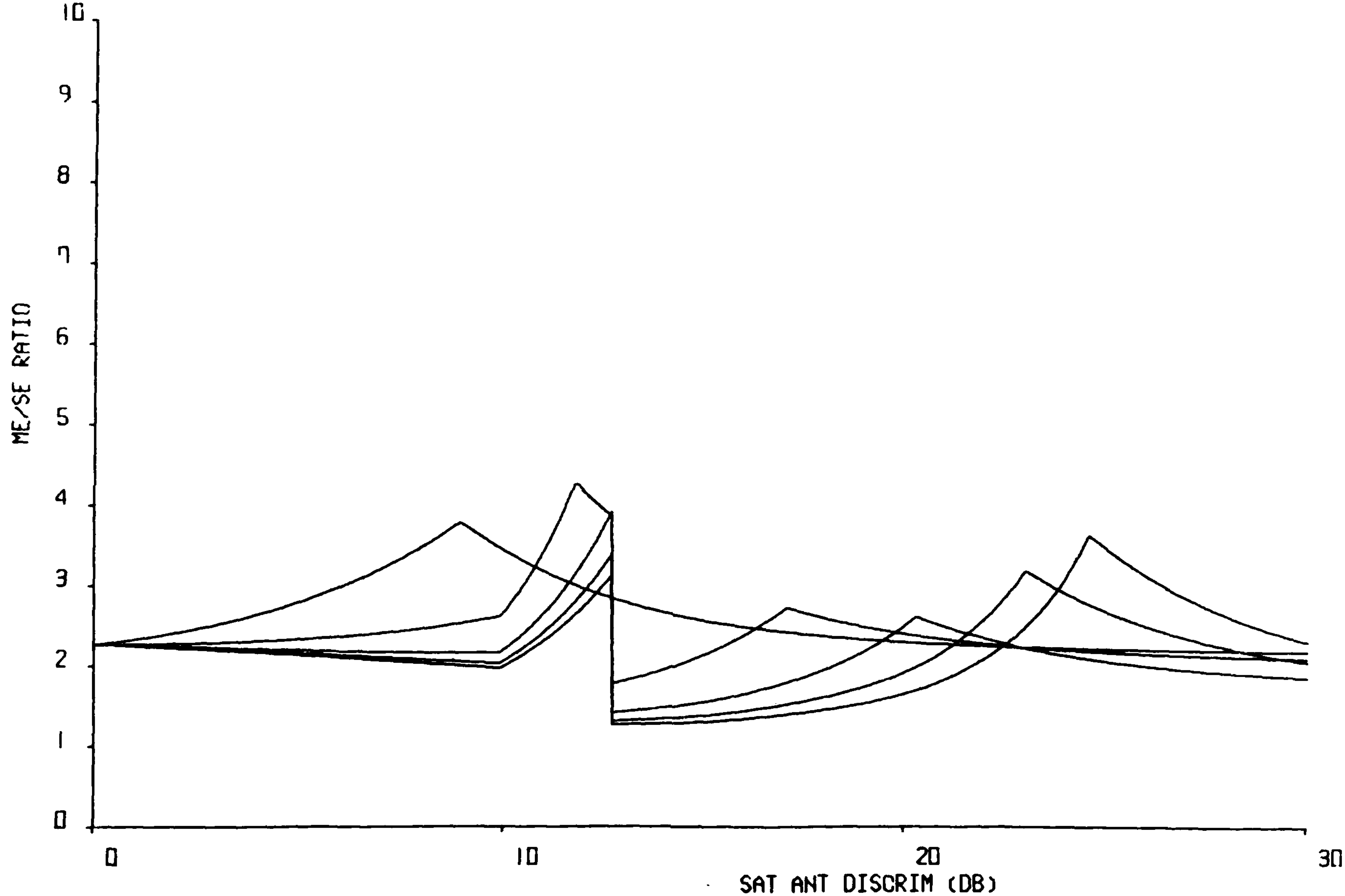
Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1 dB.



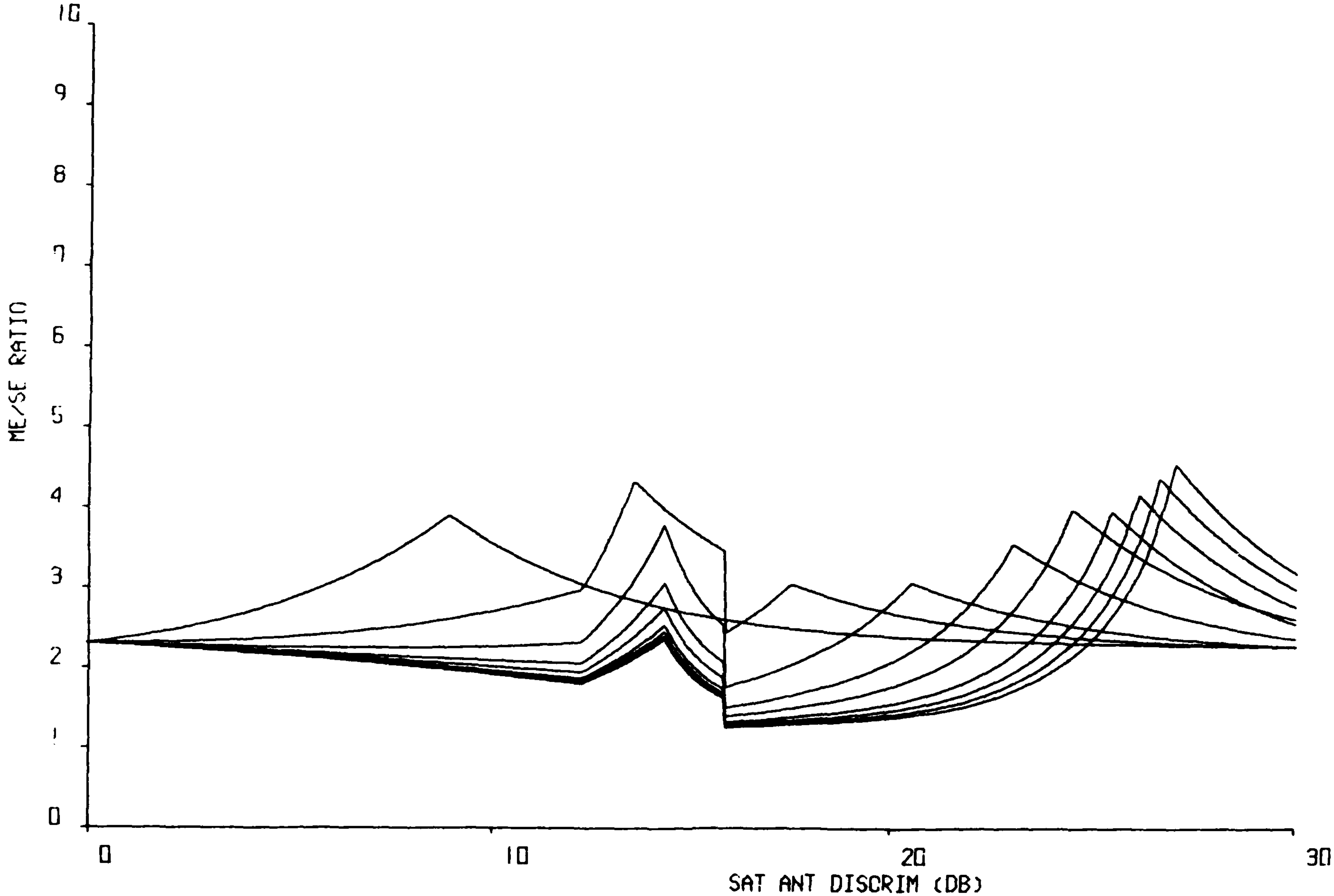
Multiple entry to single entry interference ratio as a function of *satellite antenna* discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1 dB.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1 dB.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1 dB.



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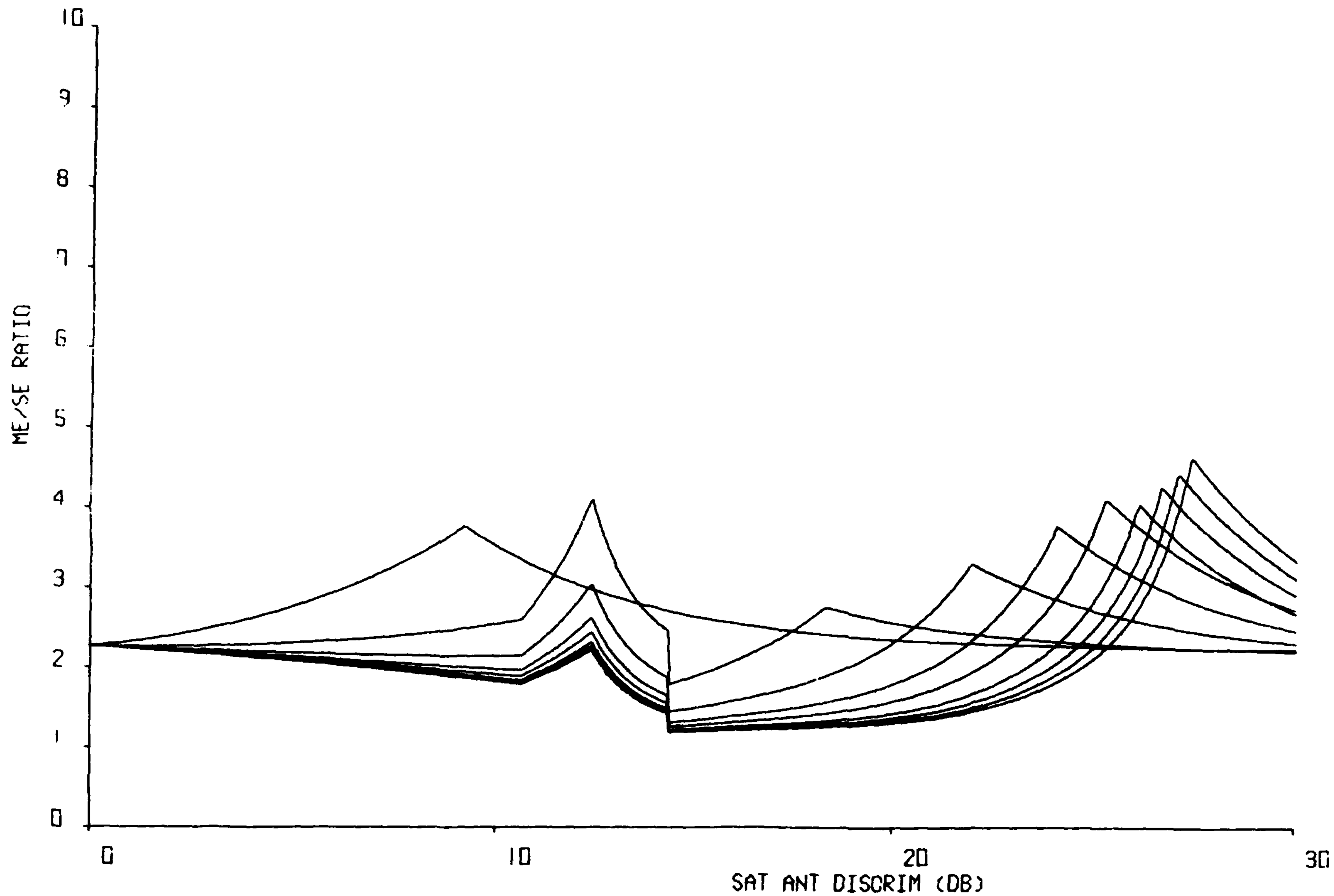
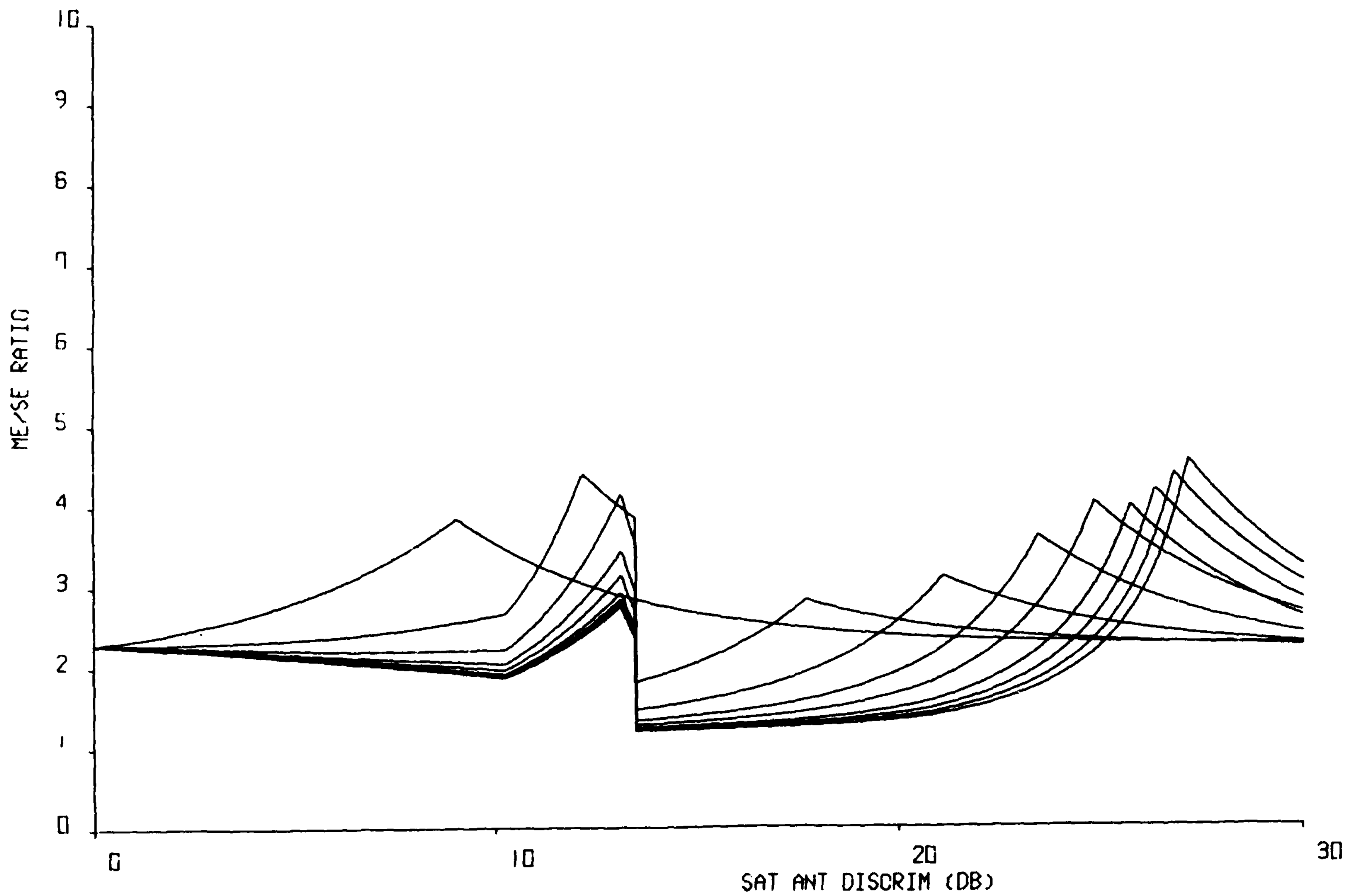
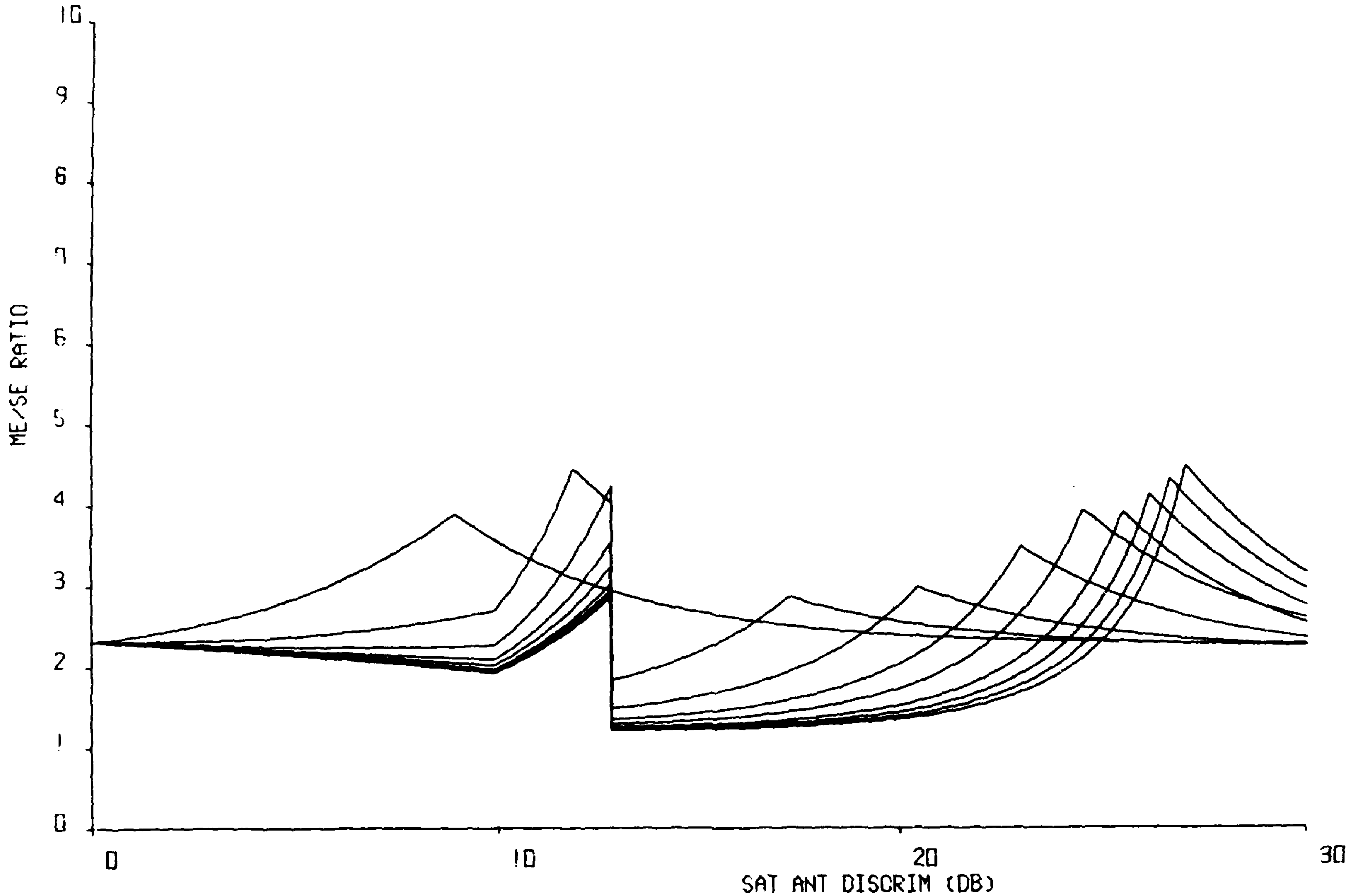


Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 100. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1 dB.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 150. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1 dB.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 200. There are 12 satellites on either side of the victim. Required total discrimination is 30 db and the earth station tracking error is 1 dB.

COMBINED STATIONKEEPING AND TRACKING ERRORS

- The angular distance from the victim satellite (number 0) to satellite n is given by

$$n\theta + \delta\theta_n + \Delta\theta_n + \delta\theta_o + \Delta\theta_o \quad ; n>0$$
$$n\theta + \delta\theta_n + \Delta\theta_n - \delta\theta_o - \Delta\theta_o \quad ; n<0$$

where $\delta\theta_n$ is a random variable representing the earth station tracking error of satellite n and $\Delta\theta_n$ is a random variable representing the satellite stationkeeping error of satellite n

- The interference from satellite n into the victim satellite 0 is

$$\alpha_n KG(n\theta + \delta\theta_n + \Delta\theta_n \pm \delta\theta_o \pm \Delta\theta_o)$$

where the sign depends on n

COMBINED STATIONKEEPING AND TRACKING ERRORS

- The multiple entry interference is then

$$ME = K \sum_{\substack{n=-N \\ n \neq 0}}^{N} \alpha_n G[n\theta + \delta\theta n + \Delta\theta n + n/|n|(\delta\theta o + \Delta\theta o)]$$

- The ME/SE ratio is then given by

$$ME/SE = \frac{\sum_{\substack{n=-N \\ n \neq 0}}^{N} \alpha_n G[n\theta + \delta\theta n + \Delta\theta n + n/|n|(\delta\theta o + \Delta\theta o)]}{\alpha_m G[m\theta + \delta\theta m + \Delta\theta m + m/|m|(\delta\theta o + \Delta\theta o)]}$$

COMBINED STATIONKEEPING AND TRACKING ERRORS

- o Assumptions
  - Victim satellite is moved closer to the maximum interfering satellite by the total of the maximum stationkeeping error and maximum tracking error
  - All other satellites are moved closer to the victim by an amount equal to the total of the maximum stationkeeping error and maximum tracking error
  - Appendix 29 antenna patterns are used
- o Also
  - $\delta\theta n = -\theta t$ = maximum tracking error
    $= -20/\ D/\lambda$ for -1 dB relative gain
  - $\Delta\theta n = -0.1$ = maximum stationkeeping error

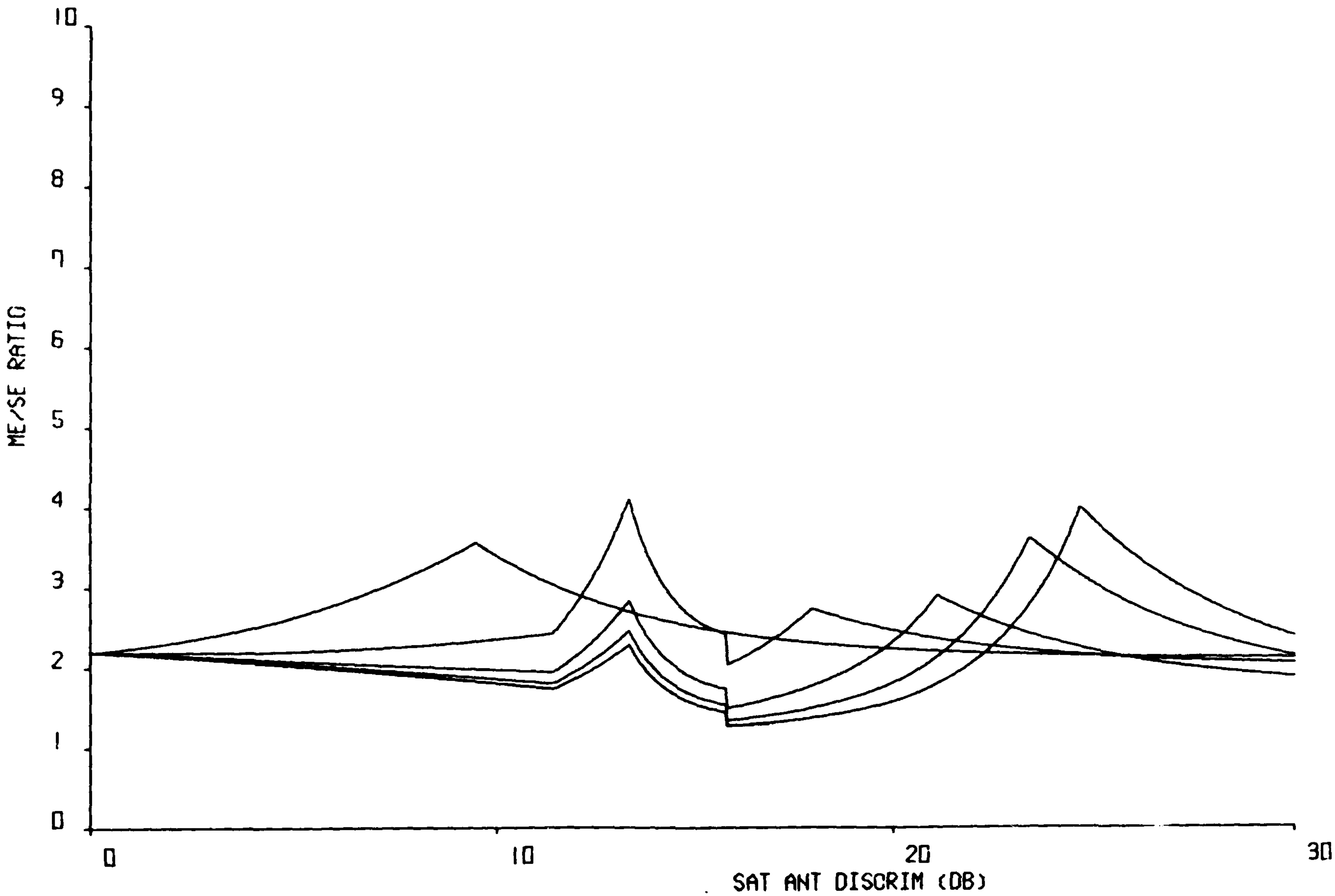
COMBINED STATIONKEEPING AND TRACKING ERRORS

- The ME/SE ratio is then

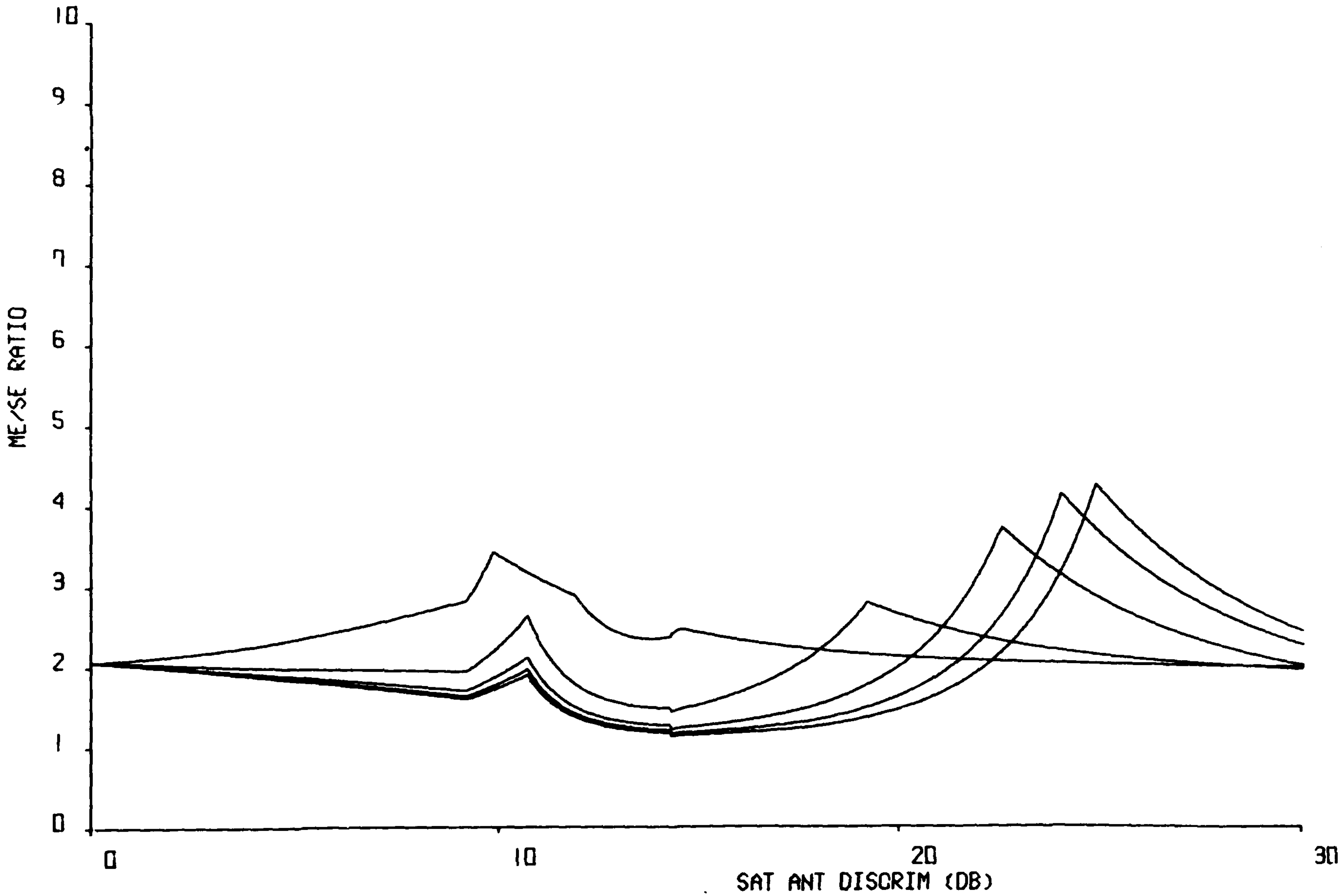
$$ME/SE = \frac{\sum_{\substack{n=-N \\ n\neq 0}}^{N} \alpha_n G[n\theta - (0.1 + 20/D/\lambda)(1 + n/|n|)]}{\alpha_m G[m\theta - 0.2 - 40/D/\lambda]}$$

  where m is selected to be the worst case interfering satellite, i.e., either the adjacent satellite with antenna discrimination or the first satellite without antenna discrimination

- Plots show this worst case ME/SE ratio for various earth station antenna sizes and for various sequences.



Multiple entry to single entry interference ratio as a function of satellite antenna discrimination for various sequences of satellites. The earth station antenna has a diameter to wavelength ratio of 50. There are 6 satellites on either side of the victim. Required total discrimination is 30 db and the satellite stationkeeping error is 0.1 degrees and the earth station tracking error is the -1 dB relative gain points.



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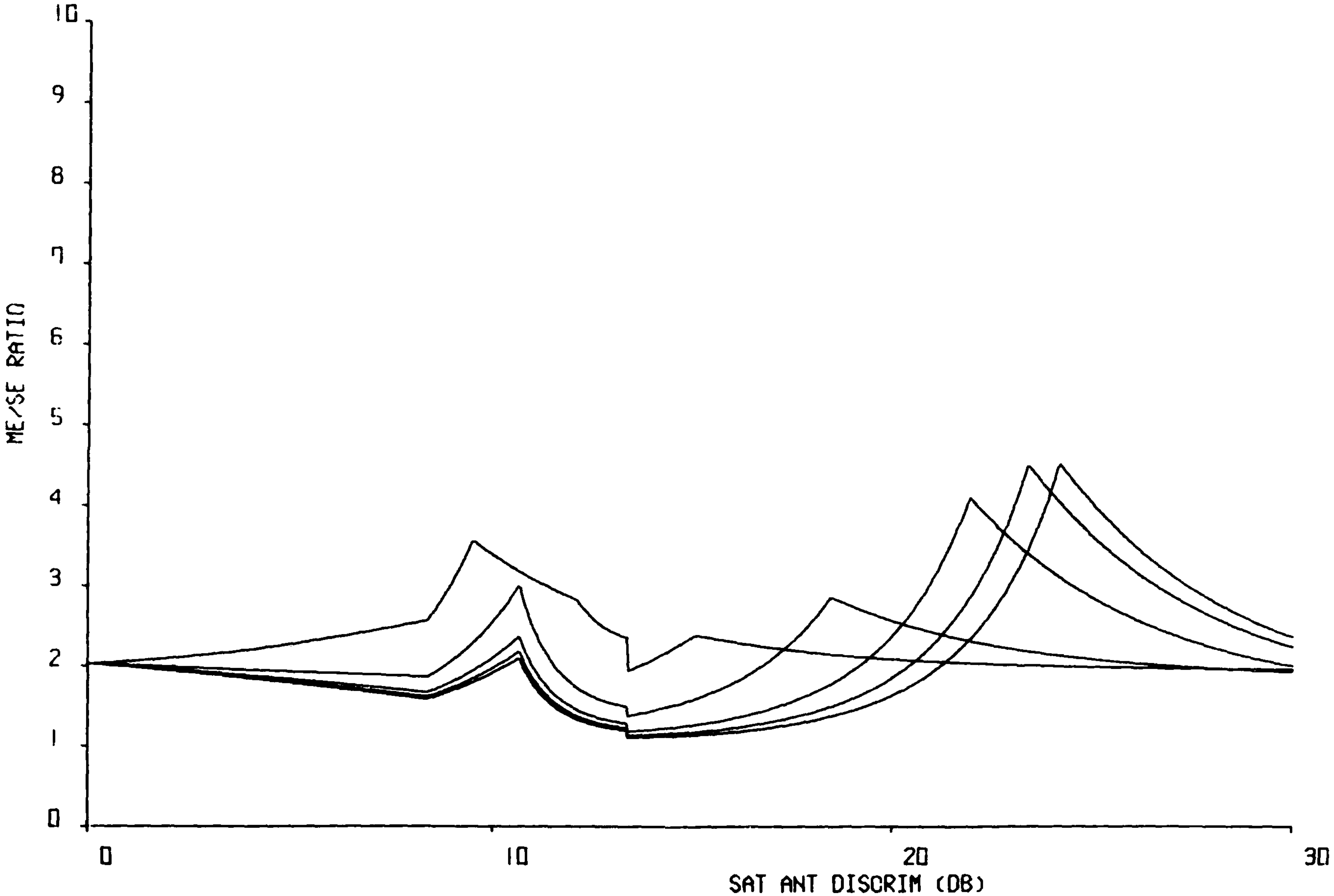

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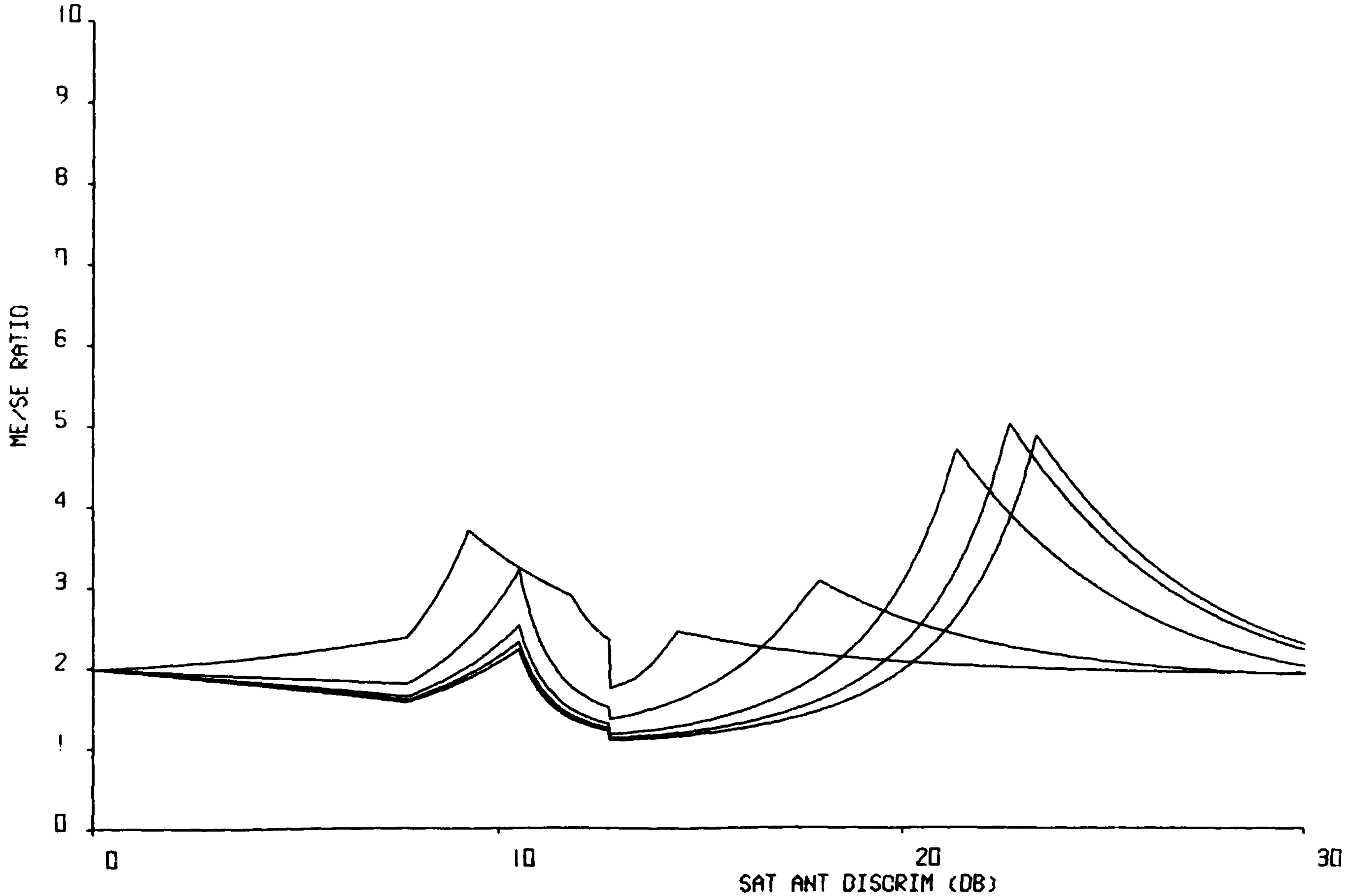

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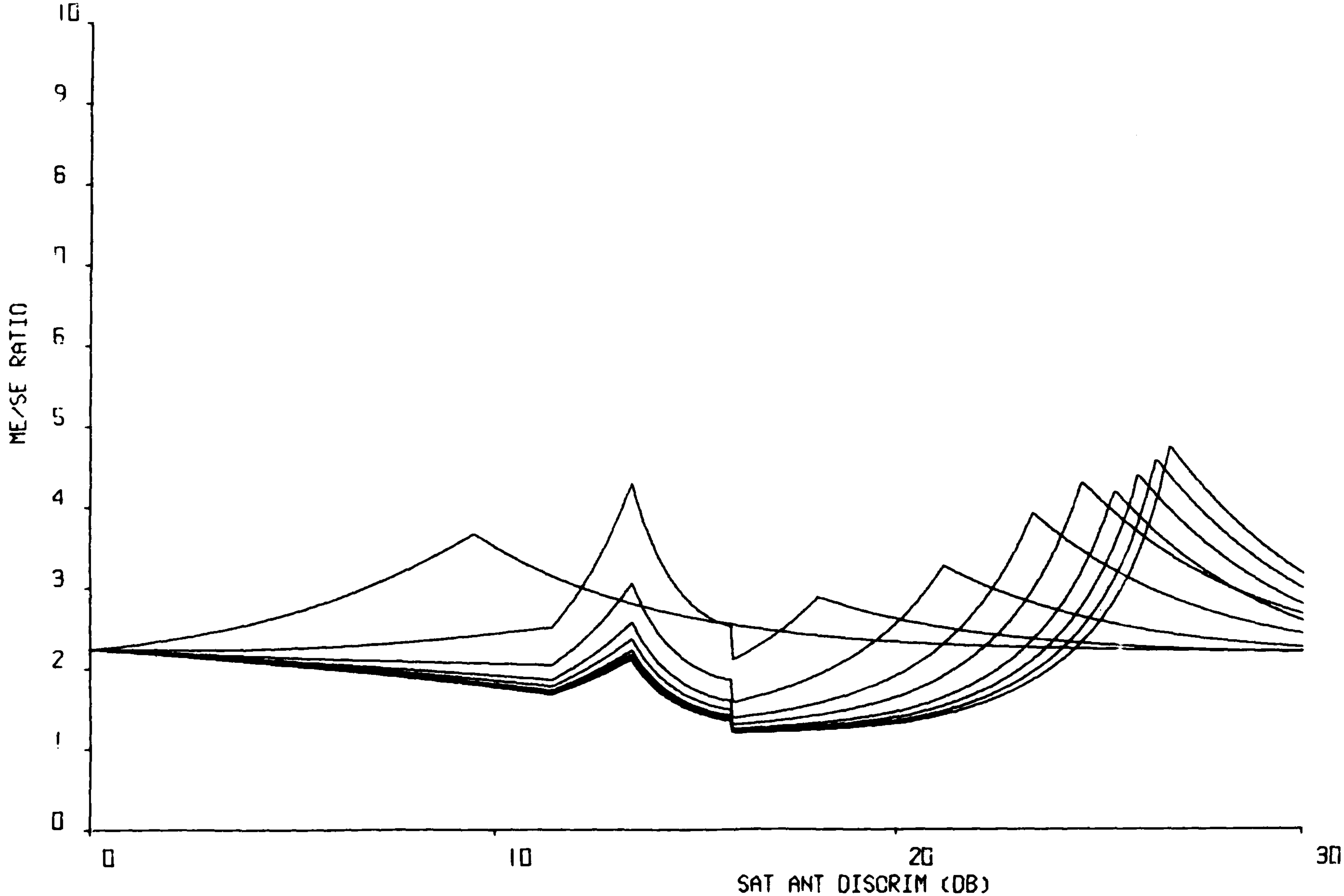

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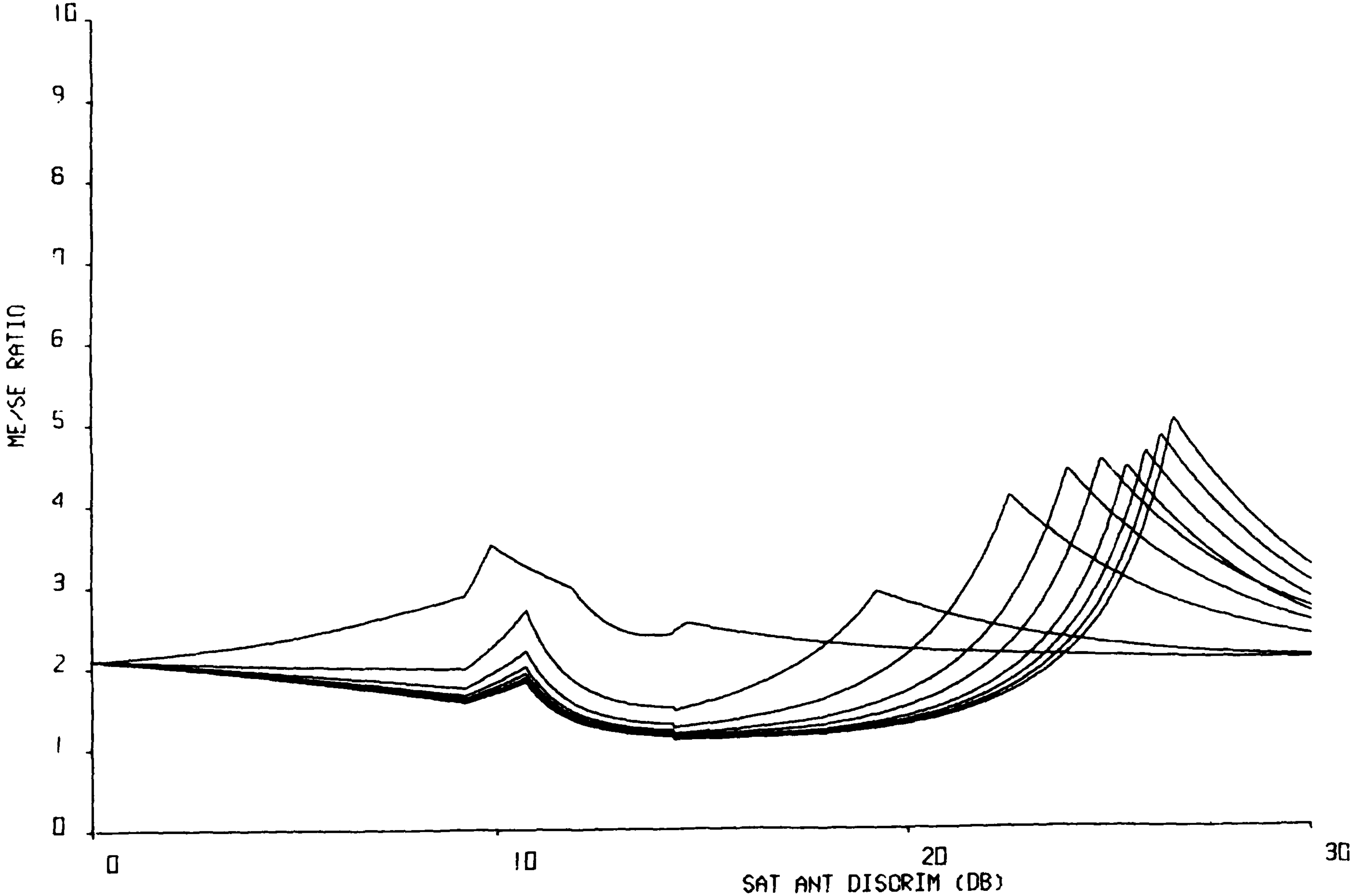

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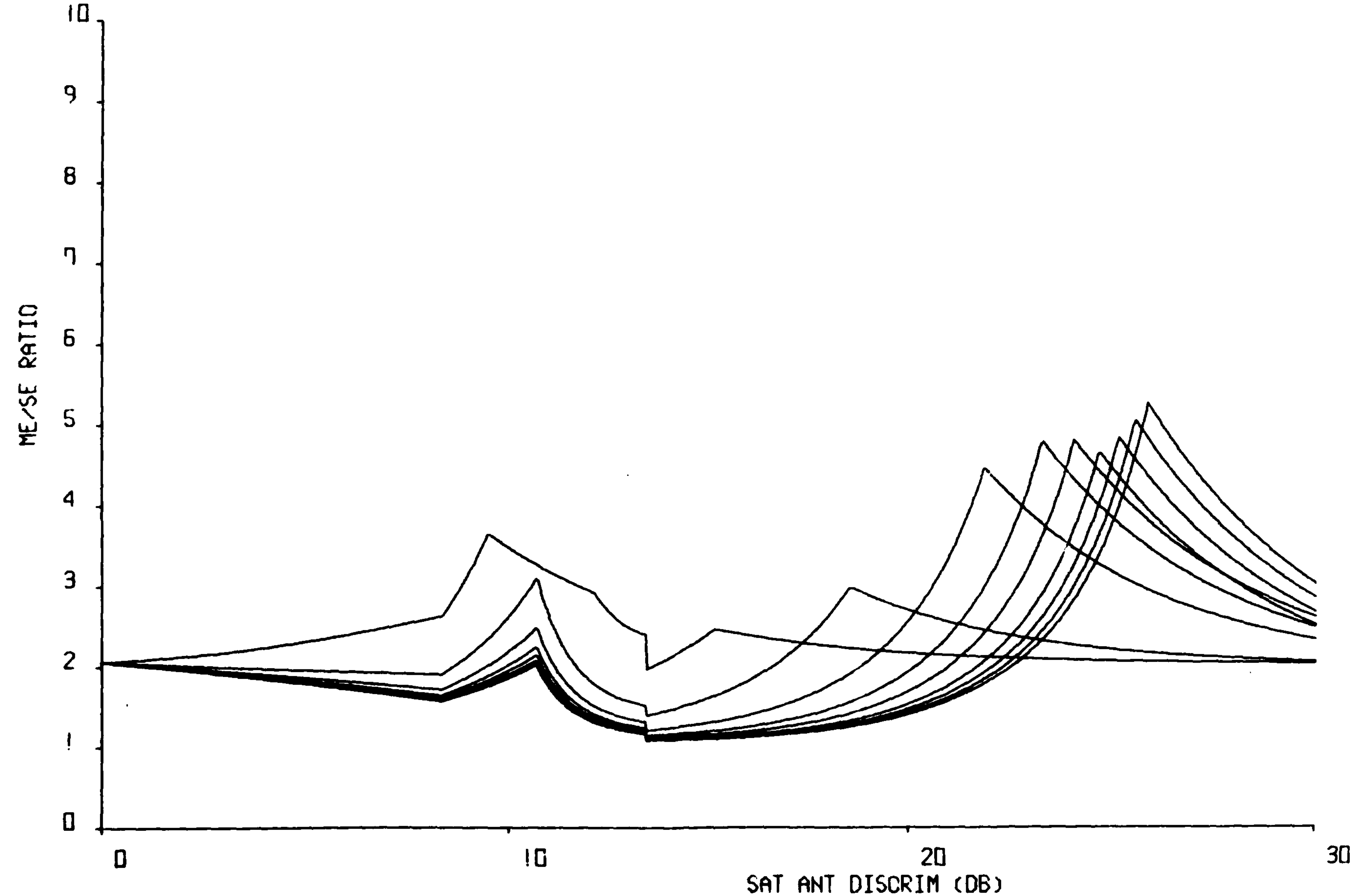

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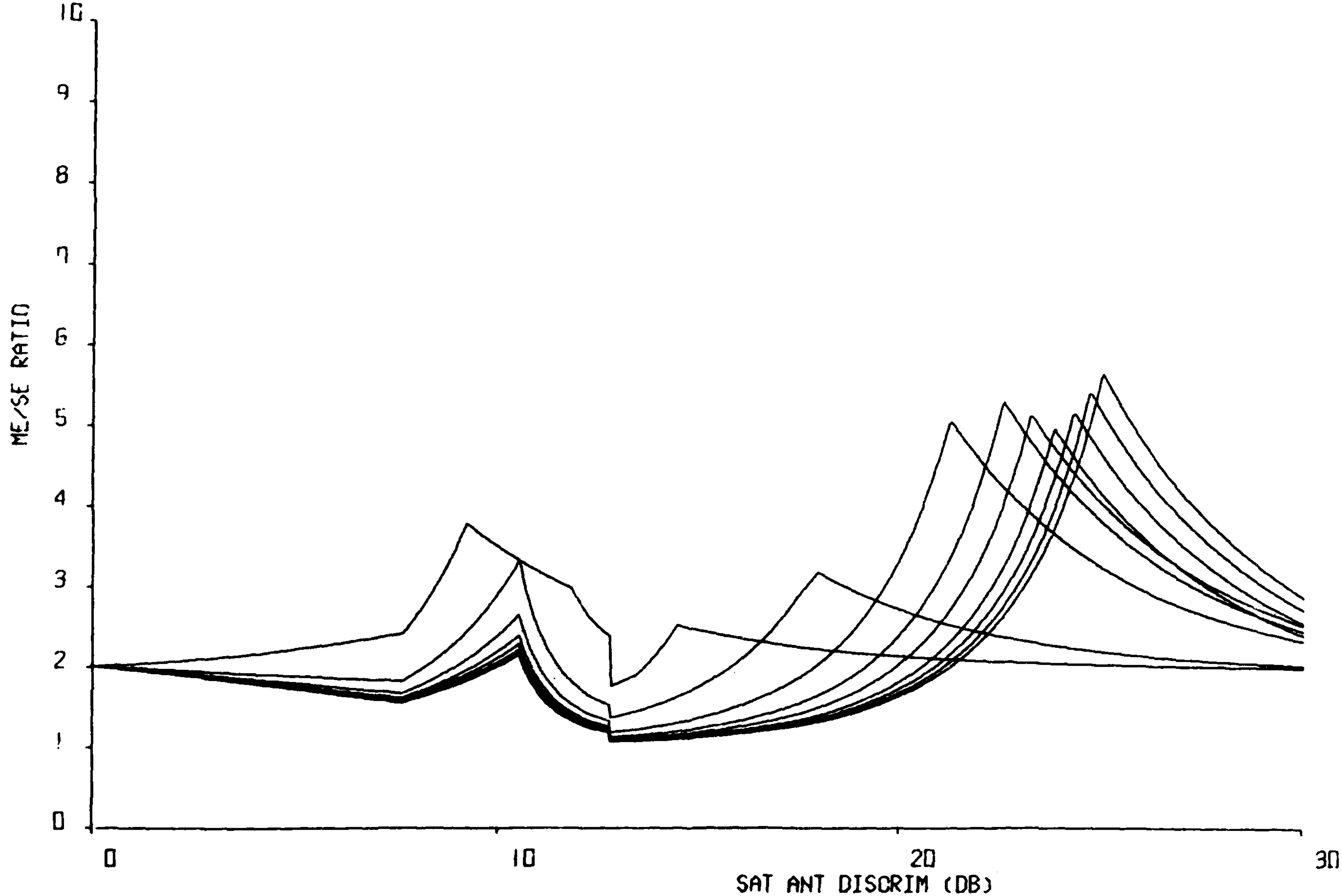

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The objective of this task was to provide support for the development and assessment of DoD proposals, positions and technical analyses related to the WARC-ORB-88 Treaty Conference on Space Telecommunications.

Concepts addressed under this task included:

1. Interference analysis of geostationary satellites in inclined orbits.
2. Bandwidth averaging.
3. Comparison of interference from multiple satellites versus a single satellite.
4. Multiband satellite networks.

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